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Part 1 of 2

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FOREIGN TECHNOLOGY DIVISION

AIR FORCE SYSTEMS COMMAND

EXTERNAL BALLISTICS

By

A. A. Dmitriyevskiy

FOREIGN TECHNOLOGY DIVISION

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By: A. A. Dmitriyevskiy

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U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

Page 2.

External ballistics. Dmitriyevskiy, A. A., M., "Machine-building", 1972, page 584.

In the book are set forth the theoretical bases of external rocket ballistics and artillery shells. Is given the basic information about forces and the moments, which act in flight on rocket and projectile, is given the method of the compilation of the equations of motion of rockets and projectiles, is examined the integration of these equations by analytical and numerical methods with the application/use of electronic computers.

Are proposed the methods of investigation of trajectories and are given the concepts of the optimum solutions of the problems of external ballistics. Is estimated the effect of different perturbation factors on the stability of flight, scattering of trajectories and error for firing.

Are given the bases of the theory of corrections and are set forth the methods of the compilation of the correcting formulas of external ballistics and calculation of ballistic derivatives.

Are given the concepts of the experimental methods of external ballistics.

The book is textbook for the students of schools of higher technical education, and also it can be useful for technical-engineering workers. Table 31, illust. 188, the list of lit. 92 title.

Page 3.

PREFACE.

In the book are presented fundamentals of external rocket ballistics and projectiles, driving/moving in the field of gravity (space flight to other planets here it is not examined. This is independent large and complex theme).

For a period of many years, external ballistics was occupied by the study of motion min and of the projectiles of barrel artillery pieces. With the development of rocket engineering and the perfection/improvement of the theory of rocket flight and projectiles, the series of question of external ballistics considerably was expanded. Appeared works concerning external ballistics of controllable rockets, the including special questions

of optimization and selection of the program of the motion of rocket. Considerable development received the designed ballistic calculations of scattering trajectories with the firing rockets and artillery shells. The application/use of computer technology considerably expanded the possibilities of ballistic investigations.

The material, presented in the book, can be broken into ten thematic sections, which illuminate the bases of external ballistics. In introduction is given definition of the subject and are formulated the basic problems, solved in the course of external ballistics, they are brought information from the history of the development of the theory of flight and external ballistics, are noted the special feature/peculiarities of the flight of different types of rockets and projectiles of artillery pieces.

In Chapters I and II are examined the overall theory of the motion of rockets and projectiles: the forces and the moments, which act on rocket and projectile in flight, trajectories and motion characteristics. The common/general/total theory, presented in these chapters, determines communication/connection of ballistics with aerodynamics and theoretical mechanics. In Chapters III, IV and V is examined solution of one of the basic problems of external ballistics - compilation of the differential equations of motion of rockets and of projectiles. Chapters VI and VII are dedicated to the methods of

integrating the equations of motion. In Chapter VIII is examined stability of motion and the stabilization of rockets and projectiles, in chapter IX - effect of the methods of control on rocket ballistics and projectiles. Chapter X is dedicated to the study of trajectories.

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In Chapter XI is examined the effect of different perturbation factors on the deviations of trajectory elements from their values, calculated for initial data, which correspond to technical specifications for rocket and to the characteristics of standard atmosphere, are given the conclusion/derivations of correcting formulas and the procedures of calculation of corrections into trajectory elements. In Chapters XII and XIII are examined the special feature/peculiarities of the action of the rockets during launching/starting, the errors for firing, the missile dispersion and projectiles. Last XIV Chapter acquaints the reader with the experimental methods of external ballistics.

External ballistics is based on the laws of mechanics, it is closely related with aerodynamics, by gravimetry and the theory of the figure of Earth, by meteorology.

Ballistic calculation gives all the basic data on trajectories

and motion characteristics on the basis of which it is possible to judge the necessary parameters of missile or artillery complex as a whole.

It goes without saying that contents of the book by no means exhausts entire diversity of the problems, confronting external ballistics. The contemporary state of the science of the motion of rockets and artillery shells of different types is such, that many of the examined in the book questions concerning its value can serve as the object/subject of independent theoretical and experimental studies.

The author expresses sincere gratitude to doctors of technical sciences Prof. D. A. Pogorelov and Prof. Yu. V. Chuyev for the valuable councils, expressed by them during preparation of the manuscript for publication, and is expressed gratitude to the doctor of technical sciences N. P. Mazurov and Cand. of tech. sciences docent Sh. Penalty-Kary-Niyazov, that made a series of the useful observations which were taken into account with the modification of the manuscript.

The author thanks the scientific editor engineer S. F. Kol'tsova for work on the editing of the manuscript, and also all comrades, who took part in the discussion of the manuscript.

The designations of physical quantities are given in the book in accordance with the project of the Gost "unity of physical quantities".

All observations and wishes about contents of the book the author requests to guide to: Moscow, E 66, 1st Basmannyy per., 3, publishing house "Machine-building".

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Principal designations.

$\bar{Q}_1$ - momentum vector of the body of variable mass.

$\bar{K}$ - vector of the moment of momentum of the body of variable mass.

T_n - kinetic energy of the body of variable mass.

m - mass of the driving/moving body (rocket, projectile).

v - velocity of the center of mass of body in absolute motion.

a - acceleration of the center of mass of body in absolute motion.

v_r - velocity of the center of mass of missile body in translational motion.

a_r - acceleration of the center of mass of housing in translational motion.

v_r - the velocity of the center of mass of system housing - fuel/propellant - gases relative to missile body.

a_r - the acceleration of the center of mass of system housing - fuel/propellant - gases relative to missile body.

$\Sigma\bar{F}$ - resultant external force.

$\Sigma\bar{F}_p$ - resultant reaction force.

$F_{кор}$ - Coriolis force.

$\bar{\omega}$ - angular velocity vector of the rotation of missile body.

M_r - total moment of external forces relative to center of inertia.

M_p - total moment of reaction forces relative to center of inertia.

$J_{x_1}, J_{y_1}, J_{z_1}$ - the moments of the inertia of rocket relative to the axes coordinate system $Ox_1y_1z_1$.

$J_{x_1y_1}, J_{x_1z_1}, J_{y_1z_1}$ - products of inertia.

Q_i - generalized force.

ϑ - pitch angle.

ψ - yaw angle.

γ - roll attitude.

θ - flight path angle.

Ψ - angle of rotation of trajectory.

γ_c - attitude of roll of the high-speed/velocity coordinate system.

$v_{x_0}=u$ - projection of the velocity of the center of mass on axis Ox_0 of the starting coordinate system.

$\theta_{v_0}-\psi$ - elevation of the velccity cf the center of mass.

α - angle of attack.

β - slip angle.

Π_T - potential of the fcrce cf gravity.

Π_Ω - potential of centrifugal inertial force.

Π - gravitational potential.

g_T - acceleration frcm gravity force.

g - acceleration frcm gravitational force.

Ω - angular rate of rotaticn of the Earth.

φ_r - geographic latitude.

φ_{rn} - geocentric latitude.

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λ - longitude.

λ^* - longitude of nodal line.

t_n - total flying time.

A - azimuth.

ε - angle of elevation.

Φ - geopotential height.

H - altitude.

p - air pressure (kg/cm^2).

ρ - mass air density.

h - air pressure in mm Hg.

τ - virtual temperature.

H(y) - the function of a change in the air density with height/altitude.

q - velocity head.

S - area of maximum cross section.

M - Mach number.

X - drag.

Y - lift.

Z - lateral force.

c_R - aerodynamic coefficient of total aerodynamic force.

c_x, c_y, c_z - aerodynamic coefficients during drag, lift and lateral forces.

m_{z_1} - pitching-moment coefficient.

m_{x_1} and m_{y_1} - rolling-moment coefficients and of yaw.

$M_{\text{ст}}$ - stabilizing moment.

$M_д$ - damping moment.

F(v),G(v),K(v/a) - the force function of air resistance.

c - ballistic coefficient.

a - speed of sound.

π(y) - the function of pressure change with height/altitude.

Γ - torque/moment of surface friction.

P - bench thrust.

X_{p1}, Y_{p1}, Z_{p1} - control forces, which act in the direction of body axes.

$v_ц$ - velocity of the center of mass of target/purpose.

v_p - velocity of the center of mass of rocket.

x_r - horizontal range.

y_s - trajectory height.

L - linear distance over the surface of the Earth.

v_{0I} - orbital velocity.

v_{0II} - escape velocity.

2ψ - range angle.

G - weight of projectile.

$Q_{сек}$ - fuel consumption per second.

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INTRODUCTION.

§ 1. SUBJECT AND PROBLEMS OF EXTERNAL BALLISTICS.

The science of the motion of rockets and projectiles is called ballistics. By the study of the motion of projectile in the bore of artillery instrument is occupied interior ballistics. The sections of interior ballistics, dedicated to the examination of the motion of

projectile immediately after output from bore in the associated jet of that escaping following by the projectile of gas, it is accepted to call intermediate ballistics.

The science of rocket flight and projectiles after the cessation of their power interaction with launching installation is called external ballistics.

Word "ballistics" sympathetic Greek word "throw" ("βάλλω"); from the latter also occurred the name of the first heavy projectile installations which in old times were called ballists. The complex of the questions by which is occupied external ballistics, frequently they began at present to call the theory of flight. For example, name "theory of rocket flight" both is common and name "external rocket ballistics". Subsequently we will utilize both these names. Furthermore, with the presentation of questions of the overall theory of flight under term "projectile" we will understand controlled and unguided rockets, the rotating artillery shell, the spin-stabilized missile, rifle bullet, finned mine and the projectile of smooth-bore system.

The projectile of cannon-type artillery we will call projectile or body of constant mass, and rocket - projectile or body of variable mass.

In spite of the specific difference in the ballistic problems for different classes of rockets and projectiles the setting of these problems and their subsequent solution in many respects remain common/general/total. The motion of rockets and projectiles is subordinated one and the same laws of mechanics and is described by one-type differential equations.

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Most frequently are distinguished the right sides of the equations, which contain the concrete/specific/actual complex of the acting forces and torque/moments. The differential equations, which describe the motion of the guided missiles and projectiles, connect communication/connections, placed by the control system.

The flight of unguided rockets, min and projectiles of barrel artillery pieces represents by itself a special case of the common/general/total task of the mechanics of controlled flight.

External ballistics is occupied by the solution of four basic problems.

/4.

The first task consists of the trajectory calculation of the motion of projectiles along previously known data. For the solution of this problem, it is necessary first of all correctly to determine, which forces act on projectile in flight and know, which will be their value at each moment of time. Further one should comprise the differential equations of action of projectile taking into account all acting forces. As a result of the solution of differential equations, are obtained all the action characteristics: velocity, acceleration, the flight time and coordinate of the center of mass on which can be constructed the trajectory. The first task occasionally referred to as basic or direct problem of external ballistics. The number of forces, which act on projectile during motion, the character of their change in the process of motion, and also the number of equations, which describe motion, and their form they depend on the designation/purpose of projectile, its construction, method of stabilization in flight and the predicted trajectory of action.

The second, or the so-called reverse/inverse, task consists of determining of the designed ballistic data of action along the assigned tactical and technical data of rocket or artillery piece. The second task is direct-connected with the ballistic design of system, important stage of which is finding the optimum states of action and flight trajectories.

The calculation of the attitude control of different designation/purpose and the determination of the conditions of their controllability is the third task of external ballistics. If rocket or projectile are unstable in flight, then, obviously, it cannot be it will be expected that they correctly will fly in the assigned direction.

Usually ballistic calculations are conducted in several approach/approximations. First during ballistic design are determined the characteristics of the ideal trajectory of the center of mass of projectile taking into account the weight of warhead and predicted firing distances (maximum and minimum). As a result of these calculations, is establish/installed the advisability of the selected method of control, the form of trajectory, its curvature, the values of tangential and normal accelerations. Are determined the motion characteristics of projectile upon its rendezvous for target/purpose and the characteristics of scattering.

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In the process of design and manufacture of missile (artillery) complex, ballistic calculations are repeated with introduction in

them the new specified data on projectile, the system of control and stabilization. The application of the selected methods of control and stabilization of rockets and projectiles, which ensure the high accuracy of firing, is the part of common/general/total stability problem of motion, in which external ballistics most closely comes into contact with questions of control of flight. Furthermore, external ballistics gives basic information for developing of rules and reception/procedures of aiming and firing.

Designed trajectory calculations are conducted, as a rule, for the ideally carried out projectile under the average/mean meteorological conditions, accepted as nominal. However, appears in actuality a number of factors, calling the deviation of projectile in flight from calculated trajectory. Scattering the trajectories of separate shots can depend both on structural/design and technological reasons (for example, from caused by them in the rockets eccentricity of thrust) and on the deviations of flight conditions on the calculated, for example, from a change in the weather factors, the nonuniform erosion/climax of nozzle throat and of the jet vanes, inadequacy of the system of control, etc. The study of the factors, which affect scattering of the trajectories of projectiles, and the examination of the methods of decreasing of scattering and increase of the accuracy of firing they are the fourth task of external ballistics.

The complex questions, which require separate examination, they are the theory of rocket flight as elastic body and the theory of rocket flight taking into account the motion of liquid filler (fuel/propellant). In these sections the theory of flight closely comes into contact with vibration theory [1, 32].

During the solution of the problems of the theory of rocket flight and projectiles, the larger number of acting factors with smaller number of assumptions can be taken into account when conducting of calculations in the electronic calculating and analog computers. Is sufficiently widely utilized also the universal, but very laborious method of numerical integration with the application/use of manual calculating automatic machines and only small class of comparatively simple tasks it can be solved by tabular and analytical methods. Purely kinematic methods are applied in essence for the qualitative studies of guided-missile control, intended for dealing with moving targets or which start from mobile units.

§ 2. From the history of the development of the theory of flight.

By its contemporary highly developed state jet-propulsion and

artillery technology is due to a considerable extent to Russian science.

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The first generalized information about rockets and projectiles of cannon-type artillery it is possible to find in the book the "regulations of the military, cannon and other bodies, which concern with military science", written to Onisim Mikhaylov and final in 1620.

At the time of the appearance of the first scientific works in the region of rocket engineering external ballistics of the projectiles of barrel artillery pieces was located on the sufficiently high level of development. The first theoretically substantiated work according to the trajectory calculation the freely projectile is written by G. Galileo (1564-1642) and it is printed in Bologna in 1638. From this work it became it is known that if we do not consider the air resistance, but the acceleration of gravity to take constant on value and direction, then the trajectory, described by projectile, will be parabola.

It is obvious that to use parabolic theory is possible only when the adopted during her conclusion/derivation assumptions

significantly do not affect the results of calculations. First of all this is related to the possibility of neglect of the air resistance.

End of the XVII century and beginning XVIII signify themselves by the increased interest in the study of the effect of the air resistance on the flight of the rapidly flying bodies. The first works on the study of the effect of medium on those moving in it it is thawed they belong to English scholarly I. Newton (1643-1727). Newton's works were related to the low speeds of the motion of bodies and were partially confirmed by later investigations.

The experiments, connected with the measurement of the initial velocity of projectile, were conducted for the first time in Russia in 1727, and the first description of experiments regarding air resistance to motion of spherical gun bullets for considerable for those times velocities (520 m/s) can be found in the book of the Englishman of B. Robinsa, who left in 1742.

The first solution of the problem of the motion of projectile taking into account the air resistance was made in 1753 by a member of the russian Academy of Sciences by L. Euler and it is published in 1755 [63]. Later, in 1775, to them in famous work the "common/general/total principles of the motion of liquids" was placed the beginning to aerohydrodynamics.

The first scientific investigations in the field of rocketry belong to Russian artilleryman to General K. I. Konstantinov (1818-1871), who headed since 1849. Petersburg rocket institution and much made for an improvement in the organization of production and technology of the manufacture of rockets. Constantine is described the physical essence of the motion of rocket and is noted equality increases in the momentum of rocket and momentum of the escape/ensuing from it gas.

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By him is also made important the conclusion that eccentricity of reaction force is the basic reason, which deflects rocket from the initial direction of motion, and is shown the advisability of the cranking of fin-stabilized rockets for purpose of an improvement in the accuracy of fire.

The greatest development rocket artillery of that time achieved in the first half the XIX century. Subsequently the poor closely grouped fire of rockets and the successes in this respect of barrel firearms led to the fact that for a prolonged time the combat missiles were completely removed from the armament of the armies of

all countries. The theory of flight is developed in essence in application to the projectiles of cannon-type artillery, but it is later, from end/lead the XIX century, and in application to the demands of the begun violently to be developed aviation.

In 1820 in Russia, was instituted the artillery school, converted in 1855 into artillery academy. The development of external ballistics in many respects is connected with these educational institutions. In particular, by the professor of the artillery school V. A. Ankudovichem was written the first textbook on external ballistics, published in 1836. In artillery academy from 1855 through 1858 of lecture on external ballistics, read noted Russian mathematician M. V. Ostrogradskiy, for the first time in general form who solved the complex problem of the motion of the spherical rotating projectile in air.

Since 1858 the school of Russian ballistics headed by talented scientist designer N. V. Mayevskiy, who much made for developing the Russian artillery and, in particular, in the field of study of the air resistance at high rates of the motion of projectiles, and also the study of the rotary motion of oblong projectiles. In his work "about the effect of rotary motion on the flight of oblong projectiles in air", published in 1865, N. V. Mayevskiy demonstrated for the first time the existence of the oscillatory motion of the

longitudinal axis of projectile in flight and he investigated the properties of this motion. Under his management were created many Soviet artillery pieces, very modern for those times. N. V. Mayevskiy's works continued his student, well-known scholar N. A. Zabudskiy (1853-1917).

In 1881 revolutionary and member of the "Narodnaya Volya", N. I. Kibal'chich, being located in the prison before execution, created the first in the world project of rocket craft for a manned flight.

Historical interest represent the works of the Petersburg inventor of A. P. Fedorov and the published by it article the "new method of flight, which eliminates the atmosphere as supporting/reference medium". Article appeared in the eightieth years of past century and drew researchers's attention to questions of the theory of rocket flight.

The bases of contemporary rocket dynamics were laid in the works of Russian scholar I. V. Meshcherskiy and K. E. Tsiolkovskiy.

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The outstanding teacher-scientist, professor Ivan Vsevolodovich Meshcherskiy (1859-1935) in his works on theoretical mechanics formed

an equation of the motion of the bodies of the variable mass to which one should relate rocket. He compiled the equation of the vertical motion of rocket.

The bases of the theory of cosmonautics and rocket engineering placed our great compatriot Constantine Eduardovich Tsiolkovskiy. In his early works K. E. Tsiolkovskiy figuratively explained the essence of reactive motion based on the example of the displacement/movement of ship under the action of recoil force that stand on besides the rapid-firing, continuously shooting gun. After the science fiction narratives "on moon", "dreams about the earth/ground and the sky and the effects of gravitational attraction" K. E. Tsiolkovskiy published in 1903 work the "investigation of outer space by reactive instruments". In this work is given known formula for determining the maximum velocity of the motion of rocket on the assumption that air resistance and gravitational force are absent.

After foreseeing development of jet/reactive technology and relying on his theoretical studies, K. E. Tsiolkovskiy introduced a series of the valuable propositions, realized considerably later on the higher level of development of world science and technology. This is related to his ideas of the use of a liquid fuel for the jet engines of the rockets during flights up to large distances, applying the jet vanes for rocket control, which act, also, in the vacuum

where usual aircraft air vanes do not give necessary effect. Tsiolkovskiy also proposed for obtaining high velocities to utilize compound/composite multi stage rockets. Without the realization of this idea, would be at present unthinkable the flight of nose cone up to large distances. Is widely utilized during our days the idea of the automation of control of motion of highspeed aircraft and rockets, proposed to Tsiolkovskiy.

At present the introduction of automatic flight control of rockets made it possible to attain the high accuracy of firing. Beginning to extreme solutions in the theory of flight, in particular, to setting the optimum states of action of rockets, was placed by the solution of the so-called second task of Tsiolkovskiy, consisting of the determination of such laws of a change in the mass of rocket and its velocity in time, at which it is possible to expect greatest climbing range of rocket.

During the study of the effect of air resistance on the driving/moving body K. E. Tsiolkovskiy focused attention on the problem of heating the bodies, flying in air with high rates, known now by the name problems of aerodynamic heating and having enormous value.

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Among Soviet researchers, who worked in the field of rocket engineering in the period of its origin/conception/initiation, one should call/name one of the first students K. E. Tsiolkovskiy gifted engineer F. A. Tsander (1887-1933) and inventor of Yu. V. Kondratyuk. For the popularization of the jet/reactive principle of motion, much made N. A. Rynin, who published during the years 1928-1932 works "interplanetary flights", Ya. I. Perel'man, F. A. Tsander.

It is logical that under conditions of tsarist Russia, and what is more during the weak development of technology of idea of K. E. Tsiolkovskiy and his students did not obtain the development, but his transactions - proper acknowledgement. Only with the Soviet regime, which pays enormous attention to new technology and to the guide the work of many scientists, design and plant collectives to the solution of foremost scientific-technical problems, it was possible to attain enormous and known to entire peace/world results.

In 1918 on the initiative of Vladimir Ilyich Lenin, was initiated the organization of central aerohydrodynamic institute (TsAGI [Central Institute of Aerohydrodynamics im. N. Ye Zhukovskiy]). The founder of TsAGI was the largest Russian scientist-aerodynamicist Nicholas Yegorovich Zhukovskiy, who

developed the bases of the aerodynamic designs cf flight vehicles and flight dynamics.

The first experimental works on aerodynamicist by N. Ye. Zhukovskiy conducted in Moscow University and Moscow highestest technical school (MVTU [Moscow Higher Technical School]). Known to entire world scholar academicians A. N. Tupolev, B. N. Yur'yev, A. A. Arkhangel'skiy, B. S. Stechkin, V. V. Golubyov and a whole series of the designers and scientists were the students of N. E. Joukowski, but many of them - by members of the scientific student small circle, organized by N. Ye. Zhukovskiy in MVTU. On the initiative of N. Ye. Zhukovskiy in 1919, was organized the Moscow institute of the engineers of the air fleet, converted in 1922 into the air force academy, now bearing his name.

The contemporary science of flight dynamics in many respects is due to scientists of N. Ye. Zhukovskiy professors V. P. Vetchinkin, I. V. Ostoslavskiy, V. S. Vedrov, V. S. Pyshnov, the professors of the air force engineering academy D. A. Venttsel', G. F. Burago, academician V. S. Pugachev and many others.

To the pen of N. Ye. Zhukovskiy belongs the work "about the strength of motion", written by him in 1882. This investigation together with noted Russian mathematician A. M. Lyapunov's work

"common/general/total task of stability of motion", written in 1892, they initiated to the development of the Soviet science of stability of motion and stabilization of flight vehicles.

In 1918 by the decision of Soviet government, was also created the constantly acting board of special artillery experiments (KOSARTOP), which charged the solution of the problems, connected with the creation of artillery pieces.

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In the commission fruitfully worked the greatest scholars-artillerymen V. M. Trofimov, N. F. Drozdov, G. P. Kisnemskiy, academicians N. Ye. Zhukovskiy, A. N. Krylov, S. A. Chaplygin. So, by academician A. N. Krylov during the years 1917-1918 was developed the method of the numerical integration of the differential equations of motion of projectiles, utilized at present, also, for the case of determining the motion characteristics of the rockets of different designation/purpose.

By the founder of the new branch of aerodynamics - gas dynamics rightfully is counted academician Sergey Alekseyevich Chaplygin (1869-1942) one of the most talented students of N. Ye. Zhukovskiy. Headed by S. A. Chaplygin more than 10 years central aerohydrodynamic

institute rapidly exceeded the one-type scientific research institutes of Europe and America on the spread/scope of works, to equipment and their scientific significance. To the end/lead of his life, S. A. Chaplygin remained the scientific leader of Soviet gas aerodynamics. Its many problems, connected with the theory of flight, are solved by the scientists of S. A. Chaplygin's school, by academicians M. V. Keldysh, M. A. Lavrentyev, L. I. Sedov, S. A. Khristianovich and by many others.

The development of external ballistics is inseparably connected with the solution of practical questions of the creation of rocket and artillery pieces. The beginning of planned theoretical and experimental investigations in the USSR in rocket engineering were works gas-dynamic of the laboratory, organized in 1921 in Moscow. In 1927 the laboratory was relocated into Leningrad and is was been called name GDL. During the years 1931-1932 are created the groups of the study of the reactive action (GIRD), in which worked the enthusiasts of rocketry F. A. Tsander, S. P. Korolev, M. K. Tikhonravov, Yu. A. Pobedonostsev et al. In GIRD were designed, constructed, made and tested in flight the first Soviet rockets. In 1933 GDL and GIRD were united into the first in the world state jet/reactive scientific research institute (RNII). In the 30's engineer-scholar N. I. Tikhomirov, V. A. Artem'yev, B. S. Petropavlovsiy, G. E. Langemak, etc. designed the first Soviet combat

rocket projectiles on solid fuel. These projectiles with certain modification were used in the Great Patriotic War 1941-1945. The achievements of contemporary external ballistics were utilized also during the creation of artillery pieces. Large successes attained design collectives, led by scholars-artillerymen V. G. Grabin, B. I. Shavyrin, I. I. Ivanov, F. F. Petrov and other Soviet designers.

In the beginning of the Second World War when firing distances the small unguided rockets (rocket projectiles) were small and on the structural/design form of rocket they reminded projectile or mine, system of equations, describing the motion of their center of mass, it differed little from system of equations, describing the motion of the projectile of cannon-type artillery. For determining the air resistance, it was of sufficient use standard functions of air resistance or the so-called laws of air resistance.

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With an increase in the flying ranges, and respectively also the rates of motion, with the introduction of control and the complication of constructions and forms of rockets, equation, which describe their motion, considerably they were complicated. Was required use in the wide scales of contemporary aero- and gas dynamics, highly developed in application to the demands of aircraft

construction. To the solution of the tasks, connected with the construction of rockets, they began to be drawn scholars-specialists in the region of dynamic stability, automatic regulation, pilotless control and induction to target/purpose.

The theory of the flight of the guided ballistic missiles, intended for a firing to very long range, the Earth satellites and interplanetary vehicles requires the widest participation also the astronomers, specialists in the field of celestial mechanics. The elliptical theory of the motion of planets, which began to be developed from the times of I. Kepler (1571-1630) and of I. Newton, to our time is considerably improved and successfully is applied for the calculation of the motion of satellites and space routes. Considerable role in the creation of the overall theory of the motion of rockets belongs to Soviet scientists, the professors of the Moscow state university A. A. Kosmodemiansky and D. E. Okhotsimskiy.

Is interesting the history of the development of the theory of the flight of the guided aimed missiles, intended for dealing for the fast moving target/purposes or started from the fast-moving carriers (ships, aircraft, tanks). For the first time the task of the motion of point along the curve of "pursuit" was placed even in the sixteenth century Leonardo De Vinchi; however, its this work remained unknown to the end/lead of the nineteenth century. Beginning from the

first decades of the eighteenth century a series of the scientists in many countries was examined the kinematics of the motion of two points during their approach along the curve of "pursuit" and three-point curved.

The idea of control of moving objects at a distance arose together with the beginning of development electric coupling in the beginning of the nineteenth century. At the end of the nineteenth century, was proposed a series of the systems, radio-controlled. It is later, beginning with the Second World War, for the trajectory calculation of the rocket projectiles with remote control, which are aimed for the mobile objectives, it was required the creation of special theory. Here as the basis of initial investigations, lay the known method of the firing of cannon-type artillery into set forward point and the theory of dogfight, detailed in connection with the actions of fighter aviation. It is logical that the first works in this direction bore also purely kinematic character. The complex dynamic problems of the theory of the flight of controllable rockets, intended for dealing for the fast moving target/purposes, successfully are solved at present by Soviet scientists.

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Great difficulties are encountered during the solution of the

reverse problem when appear themselves the most advantageous optimum states of motion of rockets. Basic works in this direction are published by Soviet scientists by A. A. Kosmodemiansky, D. Ye. Okhotsimskiy, T. M. Eneyev, D. A. Pogorelov, R. F. Appazov, I. N. Sadovskiy et al.

The creation of the contemporary multistage rockets, which are technical the base of space flights, required the enormous creative effort/forces of the large collectives of the scientists, designers, engineers and workers. Leader of one of the collectives, who prepared the assault of space, was academician Sergey Pavlovich Korolev (1906-1966) - the creator of many Soviet rockets.

carrier rocket with space ship "Soyuz" in flight is represented in Fig. 0.1. Figures 0.2 shows a version of surface-to-air missile on troops's parade army's Soviet. Artillery weapons are shown on Fig. 0.3.

§3. SPECIAL FEATURES OF FLIGHT OF DIFFERENT TYPES OF ROCKETS.

Firing distance, the trajectory of motion, the method of stabilization and control and other ballistic and structural/design characteristics of rockets are direct-connected with their designation/purpose. Depending on the location of starting device and

target/purpose, it is accepted to divide rockets into four class: "surface-to-surface" (" surface-to-surface", "ship-to-ship", "ship-to-shore" and "shore-to-ship"); "surface-to-air" ("surface-to-air", "ship-to-air"); "air-to-air" ("aircraft-to-aircraft"; "aircraft-to-rocket") and "air-to-surface".

Furthermore, rockets divide into controlled and unguided. Controlled rocket is supplied with the control system and possesses property to forcedly change motion characteristics in the process of flight. When for some reason control is switched off, rocket flight becomes unguided. But if rocket does not have a control system, then it is called unguided. unguided are the rocket projectiles, the tactical missiles, anti-tank, zenith, aviation and others. Figures C.4 depicts the design concept of aviation unguided rocket with the opened tail assembly. The unguided flight can be realized with operating and working engine. Unguided is the flight of the spin-stabilized missiles and projectiles of barrel artillery pieces and mortars.

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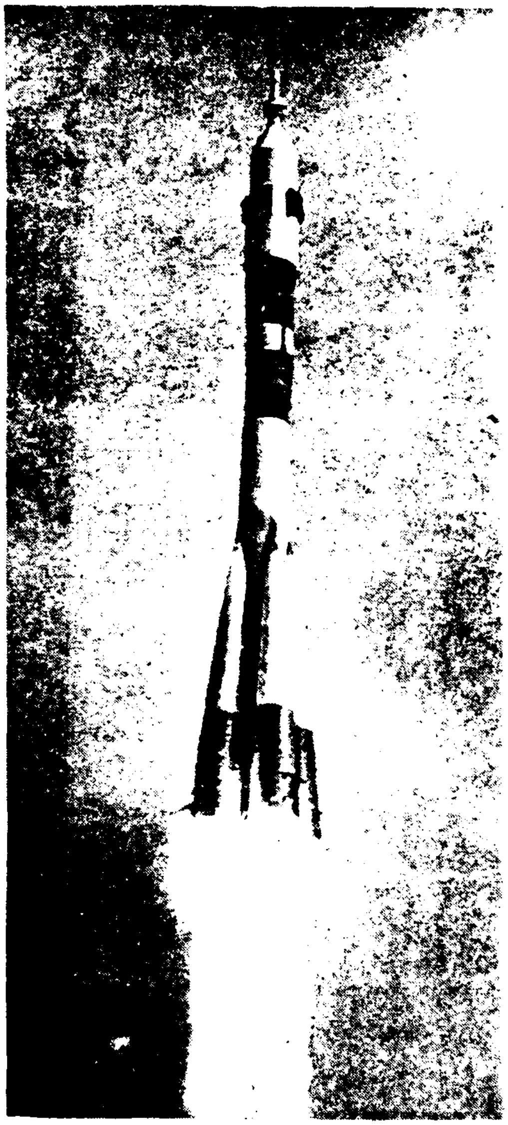

Fig. 0.1. Carrier rocket with space ship "Soyuz" in flight.

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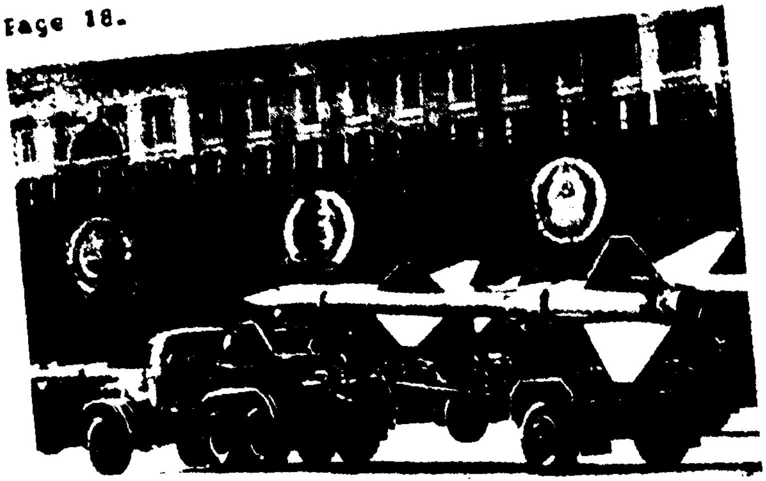

Fig. 8.2. Surface-to-air missile on parade.

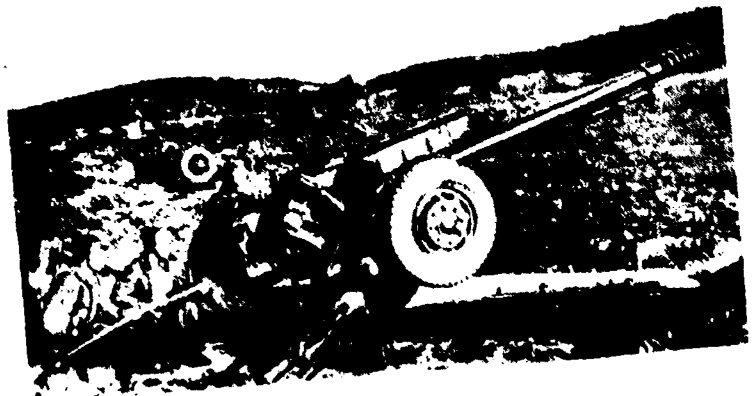

Fig. 8.3. Artillery instruments on firing position.

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In the theory of flight, it is accepted to consider the unguided flight as special case of controlled flight, when control forces and torque/moments are equal to zero.

If we do not examine the possible patterns of the technical realization of control, it is possible to count that the controlled flight is of two forms. These are the flight according to program, which occurs with the observance of the predetermined law of change of some one or several motion characteristics, and flight without the predetermined laws of characteristic change, determining the trajectory of motion.

Most frequently flight according to program is realized if necessary to strike from fixed starting device a fixed target, to derive rocket to the given point of space or to have in the specific time the assigned motion characteristics. In accordance with the basic designation/purpose of rocket, previously can be programmed: the change in the time of the angles of the slope of axis of rocket to coordinate axes in vertical and horizontal planes, the change in the coordinates, which ensures obtaining the specific parameters of trajectory, change in the amount of the thrust and rate of motion, a change in the forces and torque/moments, accelerations and g-forces.

As an example of rockets with programmed control, it is possible to give the rockets of class "surface-to-surface", majority of which is control/guided by a change in the angle of the slope of longitudinal axis to the horizon, which lays out of the trajectory of motion in vertical plane. Figures 0.5 gives pattern of such ballistic missile with engine on solid fuel, intended for flight to relatively long range. As controls serve air and jet vanes.

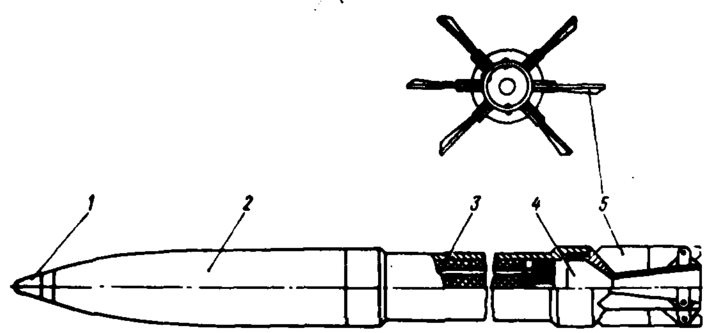

Fig. 0.4. Aviation unguided rocket with exposed tail assembly: 1 - fuse; 2 - warhead; 3 - rocket camera/chamber with grains; 4 - nozzle exit; 5 - opened tail assembly.

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The typical trajectory of the motion of the ballistic missile during starting/launching from the surface of the Earth is shown on Fig. 0.6a. The schematic of the trajectory of airborne ballistic rocket, controlled according to program, is given to Fig. 0.6b. Figures 0.7 shows the trajectories of the motion of winged missile and rocket glider, that have programmed control. Winged missile (Fig. 0.8), as a rule, starts from inclined guides, then changes into horizontal constant-level flight to target area where it transfer/converts into dive to target/purpose. Previously by program is assigned flight altitude. Of rocket glider the program must ensure takeoff along the

predetermined trajectory and, upon reaching of optimum height, gradual transition to the conditions/mode of the gliding descent.

Flight according to program does not eliminate the possibility of the correction of motion characteristics in the process of the countering of the disturbance/perturbations, which attempt to deflect rocket from the assigned programmed trajectory. For example, the starting/launching of the artificial Earth satellites and the spacecraft is conducted according to the previously designed program, carried into on-board program unit. All the deviations from design characteristics of motion are detected with means ground-based control/check, and command radio transmitter transmits correction for the elimination of these deviations through the on-board receiver to the system of the flight control. A similar correction of the programmed controlled flight provides the high accuracy of the conclusion/derivation of flight vehicle into the fixed point of space and obtaining the assigned motion characteristics.

During controlled flight without program, almost always is assumed either the induction of rocket to the moved target/purpose, or firing from the driving/moving starting device. In some cases can be moved the starting device and target/purpose as, for example, with the firing the air-to-air missiles; in other cases target/purpose or starting device are fixed. Figures 0.9 shows the schematic of the controlled aircraft rocket, intended for a dogfight.

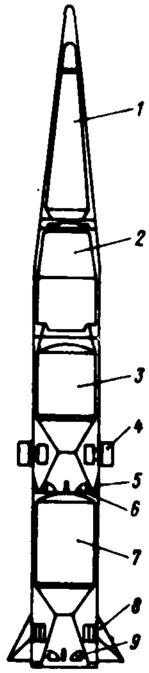

Fig. 0.5. Ballistic solid fuel rocket: "Pershing": 1 - nose cone with warhead; 2 - section with the equipment of guidance and control system; 3 - second-stage engine; 4 - aerodynamic controls of the second step/stage; 5 - system of stage separation; 6 - gas governing controls of the second step/stage; 7 - first-stage engine; 8 - aerodynamic governing controls of first stage; 9 - gas governing controls of first stage.

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The action characteristics of such guided missiles depend on the action characteristics of target/purpose and carrier of starting device. Communication/connection between the action characteristics of rocket and target is accomplished by the control system which must ensure the conclusion/derivation of rocket up to the distance with which the work of its warhead will ensure kill. The trajectories of such guided missiles were called the trajectories of induction. Trajectories of induction - complex space curves.

Is known the combination of both principles of rocket control.

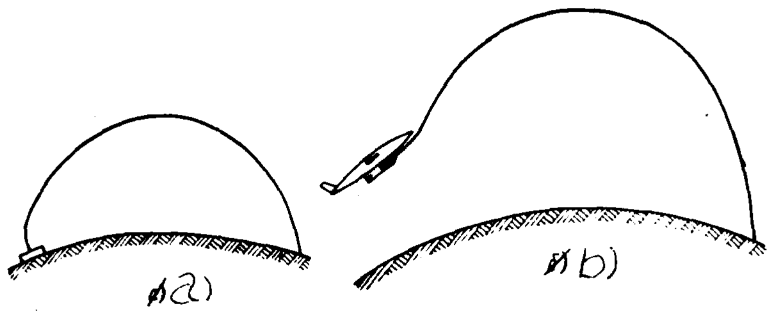

Fig. 0.6. the forms of the trajectories of different ballistic missiles: a) the trajectory of the ballistic missile, which starts from the surface of the Earth; b) the trajectory of airborne ballistic rocket.

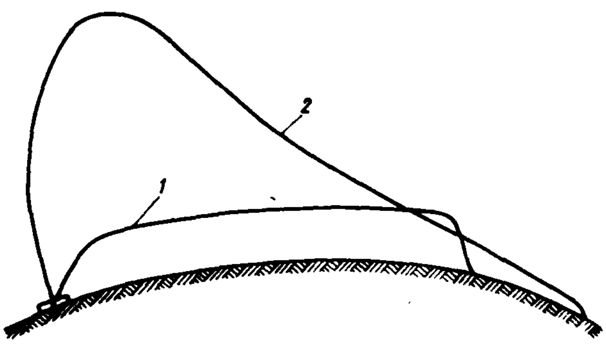

Fig. 0.7. Standard trajectories: 1 - winged missile; 2 - planning/gliding ballistic cruise missile.

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For example, it is possible to itself to present AA guided missile (antiaircraft guided missile), in the work of first stage of which the motion is realized according to the program, which assigns the specific form of trajectory; in the work of the subsequent step/stages, the motion is realized along the trajectory of induction, which depends on the action characteristics of target/purpose. Schematic of antiaircraft guided missile is shown on Fig. 0.10.

The systems of the flight control of rockets work in the majority of the cases according to the principle of servo systems. This principle is instituted on the comparison of the assigned and actual values of the governing parameter. The difference between them serves as the source of the error signal which after conversion acts on controls of flight prior to the torque/moment when the error signal becomes equal to zero.

Considering rocket solid body, in the most general case it is possible in a specific manner to change in the time of the characteristic of its motion along six degrees of freedom. In actuality for the solution of the practical problems to control/guide motion characteristics along all six degrees of freedom it is not required, since many parameters of motion are interconnected between themselves. For example, changing the angle between direction of

velocity vector and by the axis of rocket, it is possible to change in time of the coordinate of the center of mass of rocket.

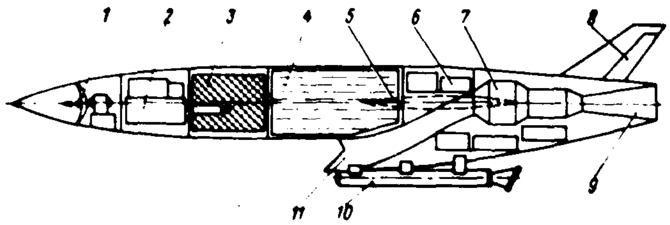

Fig. 0.8. The winged missile: 1 - locator of homing system; 2 and 6 - equipment for control; 3 - warhead; 4 - fuel tank; 5 - wing; 7 - sustainer engine; 8 - rudder; 9 - engine nozzle; 10 - booster engine; 11 - air intake of engine.

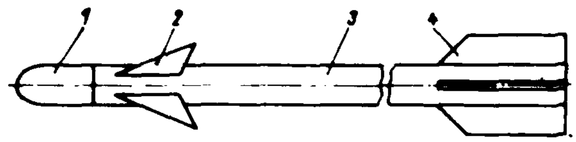

Fig. 0.9. Guided aircraft rocket "Sidewinder": 1 - self-homing head; 2 - governing controls; 3 - missile body with engine; 4 - stabilizer.

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In real constructions for the governing parameter, is accepted or the angle between the axis of rocket and any known direction (or plane), either the position of the center of mass of rocket of the relatively predetermined direction of motion, or any other characteristic.

Each servo system of IMET [Institute of Metallurgy im. A. A. Baykov]: the measuring device, which determines eigenvalue of the governing parameter; the device (unit) of comparison of the assigned and actual values of the governing parameters, consumption/production/generation of the error signal and control commands; actuating elements in the form of steering organ/controls and drives for them (control actuators). Besides these basic parts, control system, have various kinds the amplifiers, which integrate, converting and compensators. The basic correction of servo systems is directed toward a reduction/descent in the ill effect of the inertia properties of system. For this, with the formation of control command, are introduced the supplementary signals, proportional to velocity and the acceleration of a change in the governing parameter. Furthermore, into the control system are introduced the devices, which lower the effect of the various kinds of interference/jammings.

In accordance with two principles of controlled flight presented subsequently, let us divide rockets into controlled according to program and controlled according to guidance method into target/purpose.

The control systems can be also divided into two large groups - into autonomous and command systems. Under autonomous let us understand such systems, whose steering is concentrated on board

rocket. Track-command guidance assumes the presence of the separate point/item, which develops commands and which transmits by their one or the other method onboard of rocket. In principle autonomous and command systems can realize programmed flight and flight along the trajectory of induction.

Autonomous systems for programmed flight must have the onboard equipment, which determines the position of rocket of the relatively previously selected coordinate system. At present are well known autonomous systems with gyroscopic orientation.

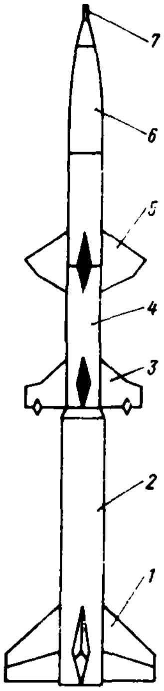

Fig. 8.10. Antiaircraft guided missile: 1 - stabilizer of first stage; 2 - first stage of rocket; 3 - stabilizer of the second step/stage; 4 -second stage of rocket; 5 - rotary governing wing; 6 - warhead; 7 - fuse.

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The gyroscope or the gyroscopes, establish/installed on rocket, realize fixed coordinate system, relative to which is determined the

position of axis of rocket, and is establish/installed its deviation from direction, by predetermined program; gyroscopes develop the error signal which after conversion enters the steering organ/controls, which change the missile heading until the axis of rocket engages the position, establish/installed by program.

Autonomous inertial system is called the system, which has on board acceleration pickups (accelerometers). The integration of the continuously measured accelerations in the onboard computing equipment makes it possible to determine the velocity of motion and displacement/movement of rocket. During the comparison of these values with programmed, are developed the control signals. are known the mixed systems, which have the gyroscopic and inertial sensors of the disagreement/mismatch of the governing parameters.

The autonomous control systems, which realize target homing without the predetermined program of action, must have devices, which determine automatically the position of rocket relative to target/purpose and correspondingly changing heading. Such devices are called homing heads. The source of the error signal can be, for example, angle ε between the axis of rocket and the direction in target/purpose (Fig. 0.11). In the heads of active homing, is utilized some form of radiant energy whose source is establish/installed on rocket. The reflected from target/purpose

beams recover by self-homing head and are converted into steering impulses.

With the command control of rocket to target/purpose from the separate point/item, arrange/located out of rocket, is determined the relative position of rocket and target/purpose. Control commands in the form of the signals are accepted on rocket, they are converted and are transferred to actuating elements. Since the target/purpose, as a rule, does not fly evenly and rectilinearly, but it maneuvers, the process of induction continues always to damage to target.

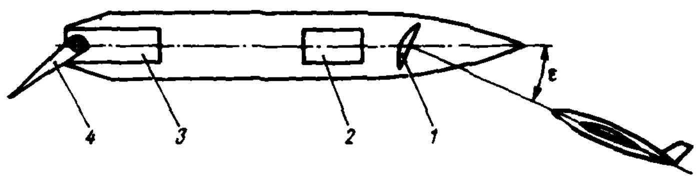

Fig. 8.11. The schematic of the sighting of target/purpose by self-homing head: 1 - head-locator, that determines the value of angle ε and developing the error signal; 2 - unit of generation of commands; 3 - steering drive; 4 - control.

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§4. Special feature/peculiarities of the flight of projectiles and min of barrel artillery pieces.

Depending on method of stabilization in trajectory, the ammunition of barrel artillery pieces are divided into two large groups. The first group - is the projectiles of threaded/cut artillery instruments, which are stabilized in flight because of the high-spin motion of relatively longitudinal axis (Fig. 0.12); second group comprise the unrotated fin-stabilized projectiles (mines) (Fig. 0.13. The initial velocity, necessary for obtaining of the assigned distance of firing, is reached during the action of projectile (mine) along the bore of artillery piece under the action of force of pressure solid-reactant gases. Mortars and some threaded/cut artillery instruments (for example, howitzer) have the alternating/variable charge because of charge in which is changed the initial velocity of missile and, consequently, also the firing distance.

The projectiles of artillery instruments are supplied with the driving band (bands), which, cutting into into the screw threads of bore, communicates to the driving/moving projectile the necessary for

stabilization angular rate of rotation. The mines of smooth-bore mortars have a tail assembly - stabilizer, which ensures stable flight.

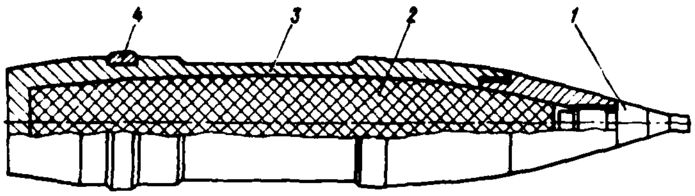

Fig. 0.12. The projectile of the artillery instrument: 1 - fuse; 2 - warhead; 3 - housing of missile; 4 - the driving band.

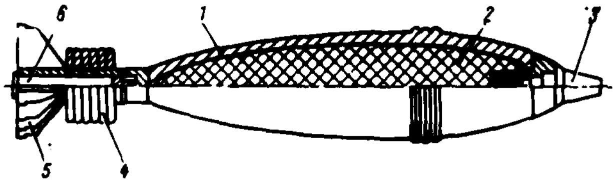

Fig 0.13. Mine of smooth-bore mortar: 1 - housing; 2 - warhead; 3 - fuse; 4 - booster charges; 5 - stabilizer; 6 - basic ammunition/cartridge of propellant charge.

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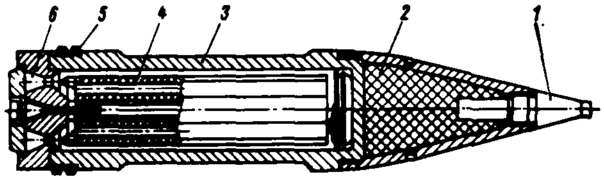

Fig. 8.14. Rocket projectile (ARS): 1 - fuse; 2 - warhead; 3 - rocket camera/chamber; 4 - charge of rocket propellant; 5 - driving bands; 6 - nozzle unit.

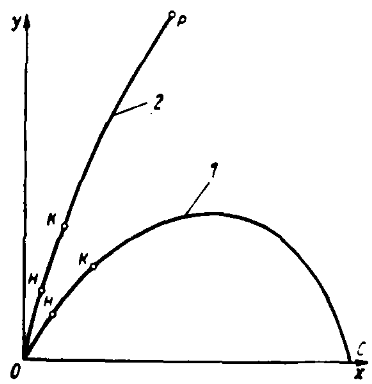

Fig. 8.15. Schematic of trajectory of active-reaction projectile: 1 - ground-based system; 2 - antiaircraft ARS.

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Is known the method of the combined report/communication to velocity to artillery shell (or mine). This the so-called

active-reactive principle of motion. Rocket projectile (ARS), for example, American 127-mm rocket projectile, or active-reactive mine (ARM) have the rocket camera/chamber, which contains the charge of rocket propellant (Fig. 0.15). During charge gun burning in the bore of artillery piece ARS (or ARM) is communicated initial (muzzle) velocity. During the burning of the charge of rocket propellant of ARS, is obtained an additional increase in the velocity. Depending on specific conditions, jet engine of rocket projectile (mine) can be included in work either in the bore of artillery piece, or at previously outlined point in the trajectory.

The trajectory of motion ARM (or ARS) is represented in Fig. 0.16. Complete trajectory can be broken into three section: ON - the free-flight phase from the muzzle end face of shaft to the connection/inclusion of jet engine, NK - a section of motion while the motor is running (active section), KS - free flight from the place of the end/lead of the engine operation to impact point (or, with antiaircraft firing, to the point of rupture R). During firing of engine in the bore of artillery piece complete trajectory one should divide/mark off into two section.

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Chapter I.

OVERALL THEORY OF THE MOTION OF ROCKETS AND PROJECTILES.

The overall theory of the motion of rockets and projectiles is based on the known theorems of the classical mechanics: about the motion of the center of mass of body; about the motion of the arbitrary point, which belongs to body; about changes in the momentum, of moment of momentum and kinetic energy of body. Motion characteristics are determined taking into account the forward motion of the center of mass of body and rotary motion relative to the center of mass. The theory of the action of rockets utilizes concepts and equations of the dynamics of the body of variable mass. The theory of the motion of artillery shells is based on simpler positions of the dynamics of a rigid body of constant mass.

During ballistic calculations the rocket and the projectile of artillery piece usually are accepted as solid undeformable body.

§ 1. Fundamental equations of the dynamics of the body of variable mass.

The body of variable mass is considered as sum of the points of variable mass. For the body of variable mass, momentum vectors and moment of momentum of the relatively fixed axes of coordinates are determined that on the formulas:

$$\bar{Q}_1=\sum_{\nu=1}^{n} m_\nu \bar{v}_\nu; \qquad (1.1)$$

$$\bar{K}=\sum_{\nu=1}^{n} (\bar{\varrho}_\nu \times m_\nu \bar{v}_\nu). \qquad (1.2)$$

Kinetic energy of the body of variable mass is equal to

$$T_n=\sum_{\nu=1}^{n} \frac{m_\nu \bar{v}_\nu^2}{2}. \qquad (1.3)$$

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In the given formulas: m_ν - variable mass of point ν at the moment of time t; $\bar{v}_\nu$ - velocity of point ν of relatively fixed coordinate system; $\bar{\varrho}_\nu$ - radius-vector of particle, carried out from the beginning of fixed coordinate system.

Utilizing a concept of a change in the momentum of the body of variable mass, are obtained the equations of the forward motion of the center of mass of rocket. Utilizing a concept of a change in the

moment of momentum, find the equations of the rotary motion of the body of variable mass of the relatively instantaneous position of the center of mass.

Let us form an equation of the forward motion of the center of mass of rocket.

In the general case let us examine simultaneous separation and the connection of particles to the basic variable mass of body. (As characteristic example of the model in question can serve the motion of flight vehicle with jet engine, through partition diffuser of which enters relative wind, necessary for the engine operation. Simultaneously with air intake the fuel combustion products escape/ensue with large velocity of engine nozzle back/ago, creating thrust). In the process of connection and separation of particles, the mass of body changes continuously. Let us assume that the velocities of connection and separation of particles do not depend on the rate of the motion of body.

Let at the moment of time t in question the body have mass $m+dm_2$ and it moves at a rate of $\bar{V}$. For time interval dt, the mass of body will change because of the connection of the elementary mass dm_1 and of the separation of mass dm_2 (Fig. 1.1). According to the hypothesis, accepted by I. V. Meshcherskiy, the connection and the

separation of particles occurs for infinitesimal time interval, similar to impact/shock. After connection the particle moves with the speed of bulk of body, and the separate/liberated particle, after obtaining velocity, immediately loses interaction with bulk of body. This the so-called hypothesis of contact interaction.

On the system of three masses in question act the external forces whose resultant ΣF. As a result of cooperating between themselves the masses m, dm_1 and dm_2 and under the action of force ΣF the velocity of the connected mass $m+dm_1$ will be equal to $\bar{v}+d\bar{v}$. The absolute velocity of the motion of mass dm_1 before the connection let us designate $\bar{u}_1$, while the absolute velocity of mass dm_2 after separation - $\bar{w}_2$.

Let us find a change in the momentum of the system of masses m_1, dm_1 and dm_2 in time interval dt and will equate to its momentum/impulse/pulse of the external forces:

$$m(\bar{v}+d\bar{v})-m\bar{v}+dm_1[(\bar{v}+d\bar{v})-\bar{u}_1]+dm_2(\bar{w}_2-\bar{v})=\sum\bar{F}dt.$$

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Disregarding the term of the second order of smallness $dm_1d\bar{v}$, after dividing both parts of the equation to dt and after conducting conversions, we will obtain the equation of motion of the body of variable mass in the form

$$m\frac{d\bar{v}}{dt}+\frac{dm_1}{dt}(\bar{v}-\bar{u}_a)+\frac{dm_2}{dt}(\bar{w}_a-\bar{v})-\sum\bar{F}=0, \quad (1.4)$$

or

$$m\frac{d\bar{v}}{dt}=\sum\bar{F}+\frac{dm_1}{dt}(\bar{u}_a-\bar{v})-\frac{dm_2}{dt}(\bar{w}_a-\bar{v}). \quad (1.5)$$

Let us note that during the separation of particles from mass m derivative itself dm_2/dt has minus sign.

If we designate how this it was conducted by I. V. Meshcherskiy, through $\dot{x}, \dot{y}, \dot{z}$ the projections of the velocity of bulk m on the axis of rectangular coordinate system, through X, Y, Z - projection of the resultant of all forces on the same coordinate axis, through α_1, β_1, γ_1 - projection of the speed of the connected particle, and through α_2, β_2, γ_2 - the projection of the speed of the separate/liberated particle, then, by projecting equation (1.4) on the coordinate axis, we will obtain:

$$\left.\begin{aligned} m\ddot{x}+\frac{dm_1}{dt}(\dot{x}-\alpha_1)-\frac{dm_2}{dt}(\dot{x}-\alpha_2)-X=0;\\ m\ddot{y}+\frac{dm_1}{dt}(\dot{y}-\beta_1)-\frac{dm_2}{dt}(\dot{y}-\beta_2)-Y=0;\\ m\ddot{z}+\frac{dm_1}{dt}(\dot{z}-\gamma_1)-\frac{dm_2}{dt}(\dot{z}-\gamma_2)-Z=0.\end{aligned}\right\} \quad (1.6)$$

These equations are published by I. V. Meshcherskiy in 1904 and are named its name on professor A. A. Kosmodemianskiy's proposition [42]. Entering equations (1.6) terms $\frac{dm_1}{dt}(\dot{x}-\alpha_1)$, $\frac{dm_2}{dt}(\dot{x}-\alpha_2)$ and so forth, I. V. Meshcherskiy call/named projections on the coordinate axes of "additional/surplus force".

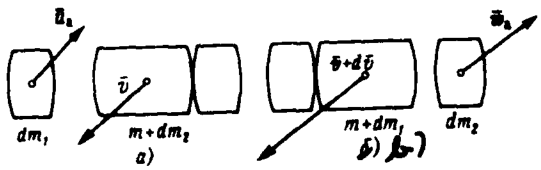

Fig. 1.1. The schematic of a change in the mass of the system: a - composition of system before connection and separation of particles; b - composition of system after connection and separation of particles.

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Thus, I. V. Meshcherskiy showed, that the equation of motion of the body of variable mass it is possible to write just as the equation of motion of the body of constant mass, after include/connecting in the number of acting forces "additional force".

If we in (1.5) designate the relative velocities of the connection of particles $\bar{u}_{отн}=\bar{u}_a-\bar{v}$ and the separations of particles $\bar{w}_{отн}=\bar{w}_a-v$, then this equation will take the form

$$m\frac{d\bar{v}}{dt}=\sum\bar{F}+\frac{dm_1}{dt}\bar{u}_{отн}-\frac{dm_2}{dt}\bar{w}_{отн}. \quad (1.7)$$

The variable mass of the driving/moving body is equal to

$$m=m_0+\int_0^t\frac{dm_1}{dt}dt-\int_0^t\left|\frac{dm_2}{dt}\right|dt.$$

If there is no connection of particles, i.e., $\int_0^t\frac{dm_1}{dt}dt=0$, then of (1.7) we will obtain the equation of motion of flight vehicle with usual type jet engine

$$m\frac{d\bar{v}}{dt}=\sum\bar{F}+\left|\frac{dm}{dt}\right|\bar{w}_{отн}. \quad (1.8)$$

where $\left|\frac{dm}{dt}\right|=|\dot{m}|=-\frac{dm_2}{dt}$ -- mass flow rate of work substance per second.

Second term of right side is accepted to call reaction force. Subsequently let us designate it ΣF_p.

For rectilinear motion of rocket vertically upward I. V. Meshcherskiy gave equation [1].

$$m\ddot{x}=-mg+p_r-\frac{dm}{dt}w_{отн}-R(\dot{x}), \qquad (1.9)$$

where g - acceleration of gravity; $R(\dot{x})$ - air resistance.

FOOTNOTE [1]. In this case vertical axis I. V. Meshcherskiy designated Ox. ENDFOOTNOTE.

After obtaining equation (1.9), I. V. Meshcherskiy did not open the content of term p_r, after call/naming his "pressure gases" [see further obtaining (2.115)].

In the more detailed examination of the motion of the center of mass of rocket, it is necessary to keep in mind following: the forces, which act on rocket, are applied to missile body, at the same time the center of mass of an entire system (housing -

fuel/propellant - gases) it is moved relative to housing because of the flow rate working medium/propellant (burnout and the outflow of gases). Let us designate velocity and the acceleration of the center of mass of an entire system in absolute motion through $\bar{v}$ and $\bar{a}$.

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The motion of housing and rigidly connected with it parts (i.e. and of that point of body, from which at the given instant it coincides the center of mass) relative to fixed coordinate system it will be movable. Velocity and the acceleration of the center of mass of housing in translational motion let us designate through $\bar{v}_e$ and $\bar{a}_e = \frac{d\bar{v}_e}{dt}$.

Velocity and the acceleration of the center of mass of system (housing - fuel/propellant - gases) relative to missile body let us designate through $\bar{v}_r$ and $\bar{a}_r$. From the mechanics of the body of variable mass, it is known that the product of the mass of body for the translational acceleration of its center of mass equal to the resultant all external and reaction forces, which act on the body, i.e.

$$m\bar{a}_e = \sum \bar{F} + \sum \bar{F}_p. \qquad (1.10)$$

Velocity and the acceleration of the center of mass of rocket in absolute motion are respectively equal to

$$\bar{v} = \bar{v}_e + \bar{v}_r; \quad \bar{a} = \bar{a}_e + \bar{a}_r + 2(\bar{\omega} \times \bar{v}_r),$$

where $\bar{\omega}$ - an angular velocity vector of the rotation of missile body

of relatively fixed coordinate system (with the beginning, which coincides with the center of mass).

From last/latter equality let us determine $\bar{a}_e$ and let us substitute in (1.10). Then is the equation of motion of the center of mass of system housing - fuel/propellant - gases, written in vector form, we will obtain in this form

$$m\bar{a}=\sum\bar{F}+\sum\bar{F}_p+m\bar{a}_r+2m(\bar{\omega}\times\bar{v}_r). \qquad (1.11)$$

When deriving the equations (1.5) and (1.11) it was assumed that interaction of basic body with the connected or separate/liberated particles occurs instantly, similarly to impact/shock. In actuality the process of cooperating the flight vehicle with the mobile connected or separate/liberated gas flows is more complex. Of rockets with engines on liquid and solid fuel, the separate/liberated particles obtain the relative velocity even in the combustion chamber of engine to the torque/moment of the output of particle for the plane of the external cross section of nozzle, i.e., to the loss of communication/connection with bulk of rocket. Furthermore, of liquid fuel rockets combustible and oxidizer they are moved in the process of the engine operation within missile body. During interaction of the driving/moving flows with the housing, which varies in transverse direction, appears the Coriolis force. Let us designate it in general form through $F_{кор}$ and will write the equation of motion of the center of mass of rocket taking into account this force

$$m\bar{a}=\sum\bar{F}+\sum F_p+\bar{F}_{кор}+m\bar{a}_r+2m(\bar{\omega}\times\bar{v}_r).$$

Let us add into last/latter equation term, that considers the transiency of the motion of masses within rocket.

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Let the momentum of fuel/propellant and gases, which are moved within housing, at the moment of time t be equal to $\bar{q}_{вар}$, but at torque/moment t+dt, is equal to $\bar{q}_{вар}+\delta\bar{q}_{вар}$. It is obvious, for time interval dt, a change in the momentum of moving masses is equal to $\delta\bar{q}_{вар}$ and the equation of motion of rocket will be written in the more complete form:

$$m\bar{a}=\sum\bar{F}+\sum\bar{F}_{р}+\bar{F}_{кор}+m\bar{a}_{r}+2m(\bar{\omega}\times\bar{v}_{r})+\frac{\delta\bar{q}_{вар}}{dt}. \quad (1.12)$$

That comprise $\frac{\delta\bar{q}_{вар}}{dt}$ is conventionally designated as variation force.

During the compilation of the equations of the rotary motion of rocket, is utilized the theorem about a change in the moment of momentum of the body of variable mass. In accordance with the conclusions of the works [31] time derivative of the moment of momentum, calculated relative to the center a of the masses of body, during the acceptance of hypothesis about contact interaction of body with the rejected particles, is determined by the formula

$$\frac{d\bar{K}}{dt}=\bar{M}_F+\bar{M}_p+\sum_{\nu=1}^{n}\left[\bar{\varrho}_\nu\times\frac{dm_\nu}{dt}(\bar{\omega}\times\bar{\varrho}_\nu)\right], \qquad (1.13)$$

where $\bar{M}_F$ and $\bar{M}_p$ - the moments of all external and reaction forces relative to the center of mass; $\sum_{\nu=1}^{n}\left[\bar{\varrho}_\nu\times\frac{dm_\nu}{dt}(\bar{\omega}\times\bar{\varrho}_\nu)\right]$ - moment of momentum of the particles, reject/thrown by body per unit time, in their motion of the relatively forward/progressively moved unrotated axes whose beginning coincides with the center of mass of rocket.

The given formula does not consider the action of fuel/propellant and gases within rocket. For the account of this action, one should use the formula

$$\frac{d\bar{K}}{dt}=\bar{M}_F+\bar{M}_p+\bar{M}_{кор}+\sum_{\nu=1}^{n}\left[\bar{\varrho}_\nu\times\frac{dm_\nu}{dt}(\bar{\omega}\times\bar{\varrho}_\nu)\right]+\frac{\delta\bar{K}_{вар}}{dt}. \qquad (1.14)$$

Here $\bar{M}_{кор}$ - moment of Coriolis forces, determined by the motion of fuel/propellant and gases within the oscillating rocket; $\frac{\delta\bar{K}_{вар}}{dt}$ - moment of variation forces.

Equations (1.12) and (1.13) are explained well by the principle of solidification [13].

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Content of missile body power-on is changed in time and rocket will be the system of variable composition. For the principle of solidification the equation of motion of the body of variable

composition can be written in the form of the equation of motion of the body of the constant composition, which has the instantly fixed (hardened) mass. In the number of forces, which act on body in the torque/moment in question, are included: external forces, reaction forces, forces of Coriolis and variation forces.

It was above noted that the variation forces and torque/moments reflect the transiency of the motion of masses within missile body. However, in the majority of the cases the process of the displacement of working medium/propellant within rocket can be accepted as quasi-stationary and variation forces, in view of smallness, not to consider.

Coriolis's forces, caused by the motion of masses within missile body and by its oscillation/vibrations, for the motion of the center of mass barely have effect. Coriolis's forces, which appear in the examination of relative motion of the rocket in the connected with the Earth coordinate system, have noticeable effect on its flight only with firing at velocities, which exceed 2000-3000 m/s and will take into account we below.

If we do not consider the named secondary factors, then the motion of the center of mass will be determined by equation (1.8).

They very frequently write the equation of motion of the center of mass in moving coordinate system $Ox_iy_iz_i$, connected with rocket. In this case equation (1.8) is converted according to the rule of passage from fixed coordinate system to mobile and takes the form

$$m\frac{d\bar{v}}{dt}=m\frac{d\bar{v}_i}{dt}+m[\bar{\omega}\times\bar{v}_i]=\sum\bar{F}+\sum\bar{F}_p, \qquad (1.15)$$

where $\frac{d\bar{v}_i}{dt}$ - derivative of a vector of the velocity of the center of mass of rocket in moving coordinate system.

For any rectangular coordinate system $Ox_iy_iz_i$, beginning by which coincides with the center of mass of flight vehicle, on the basis of (1.15) it is possible to write three scalar equations of motion of the center of mass of the rocket

$$\left.\begin{aligned}\dot{v}_{x_i}+\omega_{y_i}v_{z_i}-\omega_{z_i}v_{y_i}&=\frac{\sum F_{x_i}}{m}+\frac{\sum F_{px_i}}{m};\\ \dot{v}_{y_i}+\omega_{z_i}v_{x_i}-\omega_{x_i}v_{z_i}&=\frac{\sum F_{y_i}}{m}+\frac{\sum F_{py_i}}{m};\\ \dot{v}_{z_i}+\omega_{x_i}v_{y_i}-\omega_{y_i}v_{x_i}&=\frac{\sum F_{z_i}}{m}+\frac{\sum F_{pz_i}}{m},\end{aligned}\right\} \qquad (1.16)$$

where $v_{x_i}, v_{y_i}, v_{z_i}$ - projection of the velocity vector of the center of mass of rocket on the axis of the connected with it coordinate system; $\omega_{x_i}, \omega_{y_i}, \omega_{z_i}$ - projection of the angular velocity vector of the rotation of the connected (i-th) coordinate system relative to the coordinate system, also driving/moving with the rocket whose direction of axes constant/invariably in space coincides with the direction of the axes of fixed coordinate system, on the selected i coordinate axis; $\sum F_{x_i}$, $\sum F_{y_i}$, $\sum F_{z_i}$, $\sum F_{px_i}$, $\sum F_{py_i}$, $\sum F_{pz_i}$ - the

projection of the external and reaction forces, which act on flight vehicle, on the axis of the system of coordinates $Ox_iy_iz_i$.

For the compilation of the scalar equations of the rotary motion of rocket relative to the axes, passing through the center of mass and which rotate with respect to the rocket with angular velocity $\bar{\omega}^*$ during the rotation of rocket itself with angular velocity $\bar{\omega}$, it is necessary to use the known equation

$$\frac{d\bar{K}}{dt}=\frac{d^*\bar{K}}{dt}+[(\bar{\omega}+\bar{\omega}^*)\times\bar{K}], \qquad (1.17)$$

where $\frac{d\bar{K}}{dt}$ - derivative of moment of momentum, calculated relative to fixed coordinate system; $\frac{d^*\bar{K}}{dt}$ - derivative of moment of momentum $\bar{K}$ according to time, calculated relative to the i-th system of coordinates $Ox_iy_iz_i$.

If the system of coordinates $Ox_iy_iz_i$ is not moved relative to rocket, then $\bar{\omega}^*=0$ and

$$\frac{d\bar{K}}{dt}=\frac{d^*\bar{K}}{dt}+\bar{\omega}\times\bar{K}. \qquad (1.18)$$

The projections of moment of momentum $\bar{K}$ on the axis of coordinates $x_iy_iz_i$ are equal to

$$\begin{aligned} K_{x_i}&=J_{x_i}\omega_{x_i}-J_{x_iy_i}\omega_{y_i}-J_{x_iz_i}\omega_{z_i};\\ K_{y_i}&=J_{y_i}\omega_{y_i}-J_{y_iz_i}\omega_{z_i}-J_{y_ix_i}\omega_{x_i};\\ K_{z_i}&=J_{z_i}\omega_{z_i}-J_{z_ix_i}\omega_{x_i}-J_{z_iy_i}\omega_{y_i}, \end{aligned} \qquad (1.19)$$

where $J_{x_i}, J_{y_i}, J_{z_i}$ - the moments of the inertia of rocket relative to axes $x_iy_iz_i$; $J_{x_iy_i}; J_{x_iz_i}; J_{y_iz_i}$ — the products of inertia, determined relative to coordinate planes.

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During the determination of axial and products of inertia, can be taken into account the displacement of the center of mass of rocket (origin of coordinates) and the rotation of the coordinate axes relative to body [26]

$$J_{x_i}=\sum_{\nu=1}^{n} m_\nu (y_{i\nu}^2+z_{i\nu}^2);\quad J_{y_i}=\sum_{\nu=1}^{n} m_\nu (z_{i\nu}^2+x_{i\nu}^2);$$

$$J_{z_i}=\sum_{\nu=1}^{n} m_\nu (x_{i\nu}^2+y_{i\nu}^2);\quad J_{x_iy_i}=J_{y_ix_i}=\sum_{\nu=1}^{n} m_\nu x_{i\nu} y_{i\nu};$$

$$J_{x_iz_i}=J_{z_ix_i}=\sum_{\nu=1}^{n} m_\nu x_{i\nu} z_{i\nu};\quad J_{y_iz_i}=J_{z_iy_i}=\sum_{\nu=1}^{n} m_\nu y_{i\nu} z_{i\nu}.$$

Obviously, axial and products of inertia in the process of the operation of engine and motion of rocket will be the variable values, which depend on time.

The equations of rotary motion relative to the center of mass let us write, utilizing formulas (1.18) and (1.19), assuming that the flight vehicle is symmetrical relative to longitudinal axis. For an axially symmetrical body the products of inertia

$$J_{x_iz_i}=J_{y_iz_i}=0$$

and of the equation of rotary motion take the form:

$$\begin{aligned}
&J_{x_i}\dot{\omega}_{x_i}+(J_{z_i}-J_{y_i})\omega_{y_i}\omega_{z_i}-J_{x_iy_i}(\dot{\omega}_{y_i}-\omega_{x_i}\omega_{z_i})=M_{x_i}+M_{p\,x_i};\\
&J_{y_i}\dot{\omega}_{y_i}+(J_{x_i}-J_{z_i})\omega_{x_i}\omega_{z_i}-J_{x_iy_i}(\dot{\omega}_{x_i}+\omega_{y_i}\omega_{z_i})=M_{y_i}+M_{p\,y_i};\quad (1.20)\\
&J_{z_i}\dot{\omega}_{z_i}+(J_{y_i}-J_{x_i})\omega_{x_i}\omega_{y_i}-J_{y_ix_i}(\omega_{x_i}^2-\omega_{y_i}^2)=M_{z_i}+M_{p\,z_i}.
\end{aligned}$$

The right sides of the equations contain the projections of the sum of moments with respect to the center of mass of all forces, which act on rocket, on the appropriate coordinate axes. The moments of Coriolis forces and the supplementary torque/moment, determined by the displacement of the center of mass of the rocket relative to body, during the solution of many ballistic problems in view of smallness are not considered.

If we the moving axes of coordinates combine with principal central inertia axes $Ox_1y_1z_1$, then the equations of rotary motion take the form of the dynamic equations of Euler:

$$\left.\begin{aligned} J_{x_1}\dot{\omega}_{x_1}+(J_{z_1}-J_{y_1})\omega_{y_1}\omega_{z_1}&=M_{x_1}+M_{p\,x_1};\\ J_{y_1}\dot{\omega}_{y_1}+(J_{x_1}-J_{z_1})\omega_{x_1}\omega_{z_1}&=M_{y_1}+M_{p\,y_1};\\ J_{z_1}\dot{\omega}_{z_1}+(J_{y_1}-J_{x_1})\omega_{x_1}\omega_{y_1}&=M_{z_1}+M_{p\,z_1}.\end{aligned}\right\} \quad (1.21)$$

Let us examine the theorem about a change in the kinetic energy.

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Kinetic energy of rocket as bodies of variable composition is determined by the sum of kinetic energies of the points of the variable composition

$$T_n=\sum_{\nu=1}^{n}\frac{m_\nu\bar{v}_\nu^2}{2}.$$

Utilizing equality $\bar{v}_\nu = \bar{v} + \bar{v}_{\nu r}$, after conversions it is possible to obtain [26]

$$T_n = T + \frac{m\bar{v}^2}{2} - m\bar{v}\bar{v}_r, \qquad (1.22)$$

where m - mass of the instantly hardened rocket at the moment of time t, and $T = \sum_{\nu=1}^{n} \frac{m_\nu \bar{v}_{\nu r}^2}{2}$ - kinetic energy of the system of the material points, which belong to rocket, in its motion relative to the center of mass.

The center of mass of rocket is moved relative to body and in many instances proves to be convenient to determine kinetic energy of the rocket through the velocities in movable and relative motions. Since $\bar{v} = \bar{v}_e + \bar{v}_r$, then

$$T_n = T + \frac{m\bar{v}_e^2}{2} - \frac{m\bar{v}_r^2}{2}. \qquad (1.23)$$

Thus, kinetic energy of rocket in its absolute motion, besides T, contains a difference in the kinetic energies of the center of mass, which possesses the mass of the instantly hardened rocket, in its movable and relative motions.

The differential of kinetic energy of the body of variable composition can be obtained by direct differentiation of equality (1.3):

$$dT_n = d\left(\sum_{\nu=1}^{n} \frac{m_\nu \bar{v}_\nu^2}{2}\right) = \sum_{\nu=1}^{n} \frac{dm_\nu \bar{v}_\nu^2}{2} + \sum_{\nu=1}^{n} m_\nu \bar{v}_\nu d\bar{v}_\nu. \qquad (1.24)$$

Bale right and the left of part to dt, we will obtain second

term of the right side of the last/latter equality in this form:

$$\sum_{\nu=1}^{n} m_\nu \bar{v}_\nu \frac{d\bar{v}_\nu}{dt}.$$

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According to the Meshcherskiy equation for the point of variable mass, the equation of motion of particle ν takes the form:

$$m_\nu \frac{d\bar{v}_\nu}{dt} = \bar{F}_\nu + \bar{R}_\nu + \bar{F}_{p\nu},$$

where $\bar{F}_\nu$ - resultant of the external active forces, applied to point ν;

$\bar{R}_\nu$ - resultant of all internal effective forces, applied to point ν; $\bar{F}_{p\nu}$ - the reaction force, applied to point ν.

Disregarding the action of internal forces and carrying out replacement in (1.24), we will obtain

$$\frac{dT_n}{dt} = \sum_{\nu=1}^{n} \bar{F}_\nu \bar{v}_\nu + \sum_{\nu=1}^{n} \bar{F}_{p\nu} \bar{v}_\nu + \sum_{\nu=1}^{n} \frac{dm_\nu}{dt} \frac{\bar{v}_\nu^2}{2}.$$

Representing $\bar{v}_\nu = \frac{d\bar{S}_\nu}{dt}$, where dS_ν - elementary path of particle m_ν, and returning to differentials, let us have

$$dT_n = \delta A_F + \delta A_p + \sum_{\nu=1}^{n} \frac{dm_\nu \bar{v}_\nu^2}{2}, \qquad (1.25)$$

where δA_F and δA_p - elementary works of the external and reaction forces, applied to rocket.

Last/latter term of right side - kinetic energy, determined by the intensity of a change in the mass of the points m_ν, which belong to body, and by their absolute velocities.

Kinetic energy of rocket is examined also when they compile an equation of the motion of rocket in generalized coordinates. If the position of the body of variable mass is determined by the independent generalized coordinates $q_1, q_2, \ldots, q_S$ and

$$\bar{r}_\nu = r_\nu(q_1, q_2, q_3, \ldots, q_S, t),$$

the equation of motion can be written in the form of a Lagrangen equation of the second kind [31]

$$\left.\begin{aligned}
&\frac{d}{dt}\left(\frac{\partial T_{\text{п}}}{\partial \dot{q}_1}\right)-\frac{\partial T_{\text{п}}}{\partial q_1}=Q_1+P_1;\\
&\frac{d}{dt}\left(\frac{\partial T_{\text{п}}}{\partial \dot{q}_2}\right)-\frac{\partial T}{\partial q_2}=Q_2+P_2;\\
&\cdots\cdots\cdots\cdots\cdots\cdots\\
&\frac{d}{dt}\left(\frac{\partial T_{\text{п}}}{\partial \dot{q}_S}\right)-\frac{\partial T}{\partial q_S}=Q_S+P_S,
\end{aligned}\right\} \qquad (1.26)$$

where $Q_\sigma=\sum_{\nu=1}^{n}\bar{F}_\nu\frac{\partial \bar{r}_\nu}{\partial q_\sigma}$ - the generalized force, determined by the external acting factors (σ = 1, 2, ..., S; $P_\sigma=\sum_{\nu=1}^{n}\frac{dm_\nu}{dt}\bar{w}_\nu\frac{\partial \bar{v}_\nu}{\partial \dot{q}_\sigma}$ -- the generalized force, determined by the inflow of mechanical energy to the hardened body of variable mass during the rejection of particles.

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The latter is easy to show, if we replace $\bar{w}_\nu=\lambda(t)\bar{v}_\nu$. Then the function, which characterizes the inflow of mechanical energy, is equal to

$$\Pi=\sum_{\nu=1}^{n}\lambda(t)\frac{dm_\nu}{dt}\frac{\bar{v}_\nu^2}{2}$$

and

$$P_\sigma=\frac{\partial \Pi}{\partial \dot{q}_\sigma}.$$

§2. Fundamental equations of the dynamics of the body of constant mass.

The motion characteristics of rockets (on inactive legs), of min and projectiles of barrel artillery pieces are determined on the basis of the overall dependences of the dynamics of the solid undeformable body of the constant mass, which are special cases of the given above dependences.

The equation of motion of the center of mass of the projectile of constant mass, written in vector form, takes the form

$$m\frac{d\bar{v}}{dt}=\sum\bar{F}.$$

The equation, which determines the derivative of moment of momentum, let us write on the basis (1.13), after drop/omitting in right side all torque/moments, except the sum of the moments of external forces relative to the center of mass

$$\frac{d\bar{K}}{dt}=\sum\bar{M}. \qquad (1.27)$$

For the system of coordinates $Ox_1y_1z_1$, of the combined with principal central inertia axes, equation of the rotary motion of the projectile of constant mass, they will be written in the form of the usual equations of Euler

$$\left.\begin{aligned}J_{x_1}\dot{\omega}_{x_1}+(J_{z_1}-J_{y_1})\omega_{y_1}\omega_{z_1}&=\sum M_{x_1};\\ J_{y_1}\dot{\omega}_{y_1}+(J_{x_1}-J_{z_1})\omega_{x_1}\omega_{z_1}&=\sum M_{y_1};\\ J_{z_1}\dot{\omega}_{z_1}+(J_{y_1}-J_{x_1})\omega_{x_1}\omega_{y_1}&=\sum M_{z_1}.\end{aligned}\right\} \qquad (1.28)$$

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In right side are written to the projection of the moments of all external forces on the appropriate coordinate axes.

In the examination of the motion of the projectile of the relatively moved in space system of coordinates $0x_iy_iz_i$ they use the vector equalities

$$\frac{d(m\bar{v}_i)}{dt} + \bar{\omega}\times m\bar{v}_i=\sum\bar{F} \qquad (1.29)$$

and

$$\frac{d\bar{K}}{dt}+\bar{\omega}\times\bar{K}=\sum\bar{M}. \qquad (1.30)$$

Kinetic energy of projectile in its rotary motion relative to the center of mass can be expressed through the moments of the inertia of projectile and projection of the instantaneous angular rate of rotation of projectile on the appropriate axes.

$$T=\frac{1}{2}[Ap^2+Bq^2+Cr^2], \qquad (1.31)$$

where $A=J_{z_1}$, $B=J_{y_1}$, $C=J_{x_1}$ - torque/moments of the inertia of projectile of its relatively principal axes of inertia x_1, y_1, z_1; p, q, r - projection of the vector of the instantaneous angular rate of rotation of projectile on the named axes.

Since the torque/moments of the inertia of projectile of relatively equatorial axes are identical (A=B), the last/latter equality can be rewritten in this form

$$T=\frac{1}{2}[A(p^2+q^2)+Cr^2]. \qquad (1.32)$$

During the compilation of the differential equations of the rotary motion of projectile, frequently is utilized the equation of Lagrange in generalized coordinates (equation of Lagrange of 2 kinds), which in this case takes the form

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right)-\frac{\partial T}{\partial q_i}=Q_i. \qquad (1.33)$$

Is significantly simplified also dependence (1.25) for calculating the change in the kinetic energy of missile is record/written in the form

$$dT_n=\delta A_F. \qquad (1.34)$$

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Chapter II.

FORCES AND MOMENTS, WHICH ACT ON ROCKET AND PROJECTILE IN FLIGHT.

The acting on rocket forces and moments can be conditionally divided into external and internal. To external relate forces and the torque/moments, produced by the effect of environment - aerodynamic forces and torque/moments and forces, determined by the effect of the Earth. Under the effect of the Earth subsequently, let us understand combined action on the rocket (in the examination of its relative

action) of the gravity force, the centrifugal inertial force and Coriolis's force, determined by the rotation of the Earth. To internal can be attributed forces and the torque/moments, determined by the separation of mass from rocket (i.e. reaction forces and their torque/moments) and caused by the displacement of work substance (fuel/propellant and gases) within body (Coriolis forces, which appear during the oscillation/vibrations of rocket, and the variation forces, which are exhibited during the unsteady motion of masses within body, and their torque/moments). Last/latter three factors are usually related to secondary (supplementary) factors.

Control forces and torque/moments, depending on the principle of operation and construction of control devices, can be referred also to external and to internal, and also they can be exhibited during the combined action of internal and environmental factors. The unguided rockets and projectiles, it is logical, are not tested the effect of control forces. On projectile and mine of barrel artillery piece, act only aerodynamic forces and torque/moments and the forces, determined by the effect of the Earth.

The combined action of all forces in the process of flight on rocket or projectile does not make it possible to isolate and to determine experimentally the value of each of them. Therefore in engineering practice during the calculated and experimental

determination of each of the acting factors, is accepted the superposition principle of forces (principle of superpositions). In the majority of the cases, the thrust, aerodynamic and variation forces are calculated and they are experimentally determined independently of each other. So, thrust is determined under bench conditions with fixed engine, i.e., in the absence of aerodynamic forces; aerodynamic forces are determined experimentally in wind tunnels or with the firing on ballistic routes projectiles or the models of rockets with the shut-down engine.

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Application/use of this method for determining the forces gives the acceptable for practice accuracy of results.

The study of the mutual effect of reactive and aerodynamic forces is the object/subject of special investigations. As examples can serve testings of the rocket or its model power-on in wind tunnel on aerodynamic rail car or with firing on ballistic route. The setting of similar experiments is very complex, it requires long time and considerable material expenditures.

The senses of the vector of forces and torque/moments, which act on rocket or projectile in flight, are different: the thrust of main

engine acts in the X direction of rocket or close to it; the direction of aerodynamic force depends on the angle between vectors of the velocity of the center of mass and the axis of rocket or projectile; the gravity direction, as a rule, does not coincide with two preceding/previous and so forth. Therefore, in the general case, for performance calculation of the motion of the rockets and projectiles, solution of aerodynamic tasks, questions of strength, stabilization and control, determining experimental motion characteristics with firing and for other target/purposes it is necessary to apply different coordinate systems. During the solution of more complex problems, are utilized together several systems, during the solution of simple - one or two.

§1. SYSTEMS OF COORDINATES AND ANGLES FOR DETERMINING THE POSITION OF THE FLYING BODY IN SPACE.

The position of rocket as solid body is determined by three linear coordinates and three angles. As a rule, the system of differential equations of motion they are written in right-handed coordinate system. For calculations most frequently are utilized rectangular, cylindrical and spherical coordinates. In experimental ballistics of the coordinate system for determining the three-dimensional/space aircraft attitude, they are selected depending on the method of the instrument realization of

measurements.

For the solution of questions of the theory of the flight of the flight vehicles, driving/moving in the field of gravity, most frequently are utilized the following coordinate systems: terrestrial, connected, half-connected, high-speed/velocity (flow) and half-speed.

They usually utilize several varieties of earth-based coordinates system.

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For the beginning of earth-based coordinates system, can be accepted the center of mass of the Earth, the launching point or another fixed relative to the Earth point. Let us designate terrestrial rectangular coordinate system $Ox_3y_3z_3$.

In the following presentation this system let us use more frequent than others, but the index "Z" in a series of the cases let us omit. The axis of ordinates Oy_3 is headed on a radius of the Earth, other two axes, comprising right system, can be directed convenient for conducting of concrete/specific/actual investigation by form. Axis Ox_3 most frequently is headed for target/purpose or in

the assigned initial direction of motion.

For the study of the absolute motion of the ballistic missiles of long-range action, frequently is applied the inertial coordinate system. In the general case under inertial coordinate system, is understood the system, which occupies in space the positive seat or it moves evenly and rectilinearly. In the theory of flight for inertial coordinate system, conditionally is accepted the coordinate system whose beginning is combined with the center of mass of the Earth, but axes do not change their direction in space. Inertial coordinate system participates only in the forward motion of the Earth around the sun, and the position of its axes does not depend on the diurnal rotation) (unlike Earth-fixed coordinate systems and rotating together with it, utilized during the study relative motion rockets Earth.

The rectangular Earth-fixed coordinate system whose beginning is combined with the center of mass of the Earth, and one of the axes is directed toward north along the axis of the rotation of the Earth, it is called the geocentric coordinate system.

If the center of mass of the Earth is selected as the beginning of spherical coordinates, then it they call geocentric spherical coordinates.

Communication/connection between geocentric rectangular and geocentric spherical coordinates (Fig. 2.1) is expressed by the simple correlations

$$\left.\begin{aligned} x_3 &= r\cos\varphi_{ru}\sin\lambda;\\ y_3 &= r\sin\varphi_{ru};\\ z_3 &= r\cos\varphi_{ru}\cos\lambda, \end{aligned}\right\} \quad (2.1)$$

where the angles φ_{ru} and λ - geocentric latitude and longitude of the position of rocket.

The coordinate systems with the origin on the surface of the Earth are called topocentric.

For determining the position of rocket or aerial target relative to the surface of the Earth frequently is utilized topocentric spherical coordinates (Fig. 2.2).

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The position of the center of mass of rocket (target/purpose) R is determined by the value of radius-vector r, called sometimes slant range, and two vectorial angles: azimuth A, calculated clockwise in local horizontal plane off direction in north, and by the angle of elevation ε, calculated in vertical plane. During the compilation of equations of motion in spherical coordinates in a series of the

cases, is more convenient for the preservation/retention/maintaining of the unity of the reference directions of angles in horizontal plane instead of the azimuth A to introduce angle $A_* = -A$. In this case, in Fig. 2.2 by coordinate surfaces of spherical coordinates they will be: sphere with a radius of r, the vertical plane, passing through radius $\bar{r}$, and a cone with apex/vertex at point 0 and the apex angle, equal to 180° - 2ε. The coordinate lines: (r) - are direct/straight of radius-vector; (ε) - circumference of the great circle of sphere, passing through the given point R; (A) - circumference from the cross section of sphere by the plane, parallel to the plane $0x_3z_3$, passing through the given point. The coordinate axes [r], $[\varepsilon]$ and [A] the curvilinear coordinate system are tangential to coordinate lines.

Transfer/transition from the axes of curvilinear spherical coordinates to the axes of terrestrial system $0x_3y_3z_3$ is realized with the aid of Table 2.1.

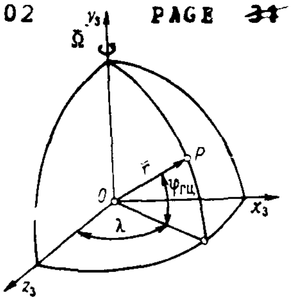

FIG. 2.1

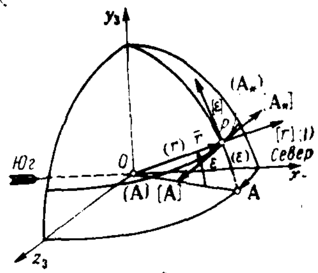

FIG. 2.2

Fig. 2.1. The relative position of geocentric rectangular and spherical coordinates.

Fig. 2.2. Topocentric spherical coordinates.

Key: (1). North.

Table 2.1. Cosines of the angles between the axes of curvilinear and earth-based coordinates system.

Координат-(1) ные оси	$0x_3$	$0y_3$	$0z_3$
$[r]$	$\cos\varepsilon\cos A$	$\sin\varepsilon$	$\cos\varepsilon\sin A$
$[\varepsilon]$	$-\sin\varepsilon\cos A$	$\cos\varepsilon$	$-\sin\varepsilon\sin A$
$[A]$	$-\sin A$	0	$\cos A$

Key: (1). Coordinate axes.

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On flat/plane drawings and the schematics, which depict missile

trajectories, the earth's axis Oy_3 is arrange/located vertically, but axis Ox - it is horizontal. Frequently with the joint flat/plane image of the missile trajectories and the earth's surface the axis Oy_3, passing through the center of the Earth and the place of start, they conditionally depict vertically without depending on real geographic latitude of the place of start (φ_r).

During the solution of two-dimensional problems, connected with performance calculation of the motion of the long range ballistic missiles and Earth satellites, they use terrestrial polar coordinate system (Fig. 2.3). The position of the center of mass of rocket in this case is determined by radius r and by the vectorial angle Φ, calculated in the plane of drawing.

In some two-dimensional problems of the theory of flight, can be used also curvilinear coordinates. For them coordinate lines they will be: the circumference x_q on the surface of the Earth, which passes in range plane, and the part of the radius-vector, carried out from the center of the Earth, between the center of mass of rocket R and the conditional surface of the terrestrial sphere, determined by a radius R_3 (Fig. 2.4).

Communication/connection between the coordinates of curvilinear and rectangular earth-based coordinates system, with beginning at

launching point is visible from Fig. 2.4:

$$(R_3+y_q)^2=(R_3+y_3)^2+x_3^2.$$

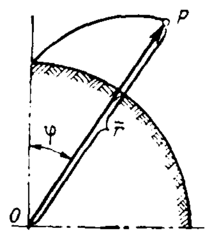
hence

$$y_q=\sqrt{(R_3+y_3)^2+x_3^2}-R_3 \qquad (2.2)$$

and

$$x_q=\frac{\pi R_3\varphi}{180}, \qquad (2.3)$$

where φ - vectorial angle in degrees.



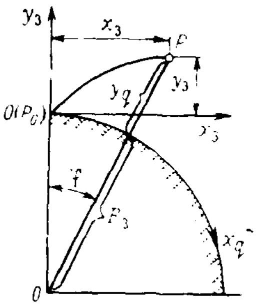


Fig. 2.4.

Fig. 2.3. Terrestrial polar coordinate system.

Fig. 2.4. Flat/plane curvilinear coordinate system.

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From Fig. 2.4.

$$\operatorname{tg}\varphi = \frac{x_3}{R_3 + y_3}.$$

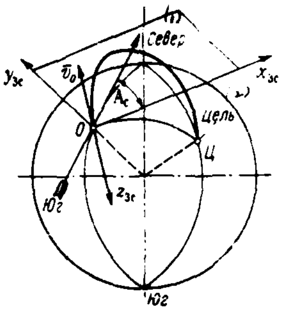
If vectorial angle is given in radians, then

$$\varphi = \varphi_{\text{рад}} \frac{180}{\pi} \text{ and } x_q = R_3 \varphi_{\text{рад}}.$$

Frequently is utilized the terrestrial topocentric rectangular starting system of coordinates $0x_{3.c}\, y_{3.c}\, z_{3.c}$ (Fig. 2.5). The beginning of the starting coordinate system is determined by the position of launcher and coincides with the center of mass of the rocket, flight-ready. In this case, coordinate axis $0y_{3.c}$ is directed vertically upward, and axes $0x_{3.c}$ and $0z_{3.c}$ lie/rest in the plane of starting horizon/level, the axis $0x_{3.c}$ indicating the line of fire. Vertical plane $y_{3.c}\, 0x_{3.c}$, passing through the vector of the initial

velocity, is called the plane of launching/starting or firing and sometimes - by the plane of casting. The position of the plane of launching/starting relative to the Earth is determined by launch azimuth or by the azimuth of firing.

The connected (mobile) coordinate system, designated $Ox_1y_1z_1$, is rigidly connected with rocket and is moved together with it. The origin of coordinates is usually arrange/located in the center of mass of rocket. One Of the axes of coordinates (Ox_1) it is directed along axis of rocket, remaining two - it is perpendicular to axis Ox_1 and to each other so that they comprise right-handed coordinate system.



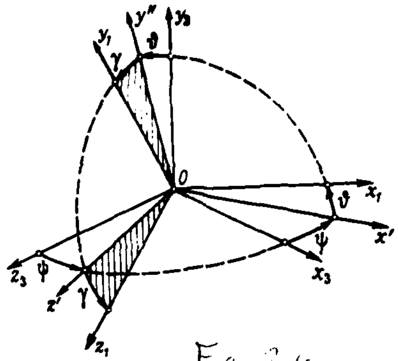


Fig. 2.6.

Fig. 2.5. Topocentric starting coordinate system.

Fig. 2.6. Diagram of mutual layout of connected and earth-based coordinates system with combined origin of coordinates.

Key: (1). North. (2). Target/purpose.

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If rocket is carried out according to aircraft configuration, then one of the axes of body-fixed system (Ox_1) it is directed along the airfoil chord of wing, and another (Oy_1) - perpendicular to it in the plane of the symmetry of rocket.

In the half-connected coordinate system, one of the axes coincides with the projection of velocity vector on the plane of the

symmetry of flight vehicle, two others lie/rest at the plane, perpendicular to the projection of velocity vector on the plane of the symmetry (one it is directed along the intersection of planes, another supplements system to right).

Figures 2.6 shows the diagram of mutual layout of connected and earth-based coordinates system with the combined origin of coordinates. Axis Ox_1 is directed along axis of rocket, axis Oy_1 lie/rests at its plane of symmetry, axis Oz_1 is directed perpendicularly to plane x_1Oy_1. The angle between the axis of rocket and its projection (Ox') on horizontal plane calls pitch angle ϑ. Are distinguished the pitch angle with respect to starting (horizontal) plane and the local pitch angle, measured with respect to horizontal plane, which is located at given torque/moment under rocket. This separation it is expedient to consider during determining of the motion characteristics of the rockets, intended for a firing to long range. The angle between the projection of axis of rocket (Ox') on horizontal plane and the terrestrial coordinate axis Ox_3 is called the yaw angle ψ. For some rockets, for example, ballistic, the yaw angle is determined depending on the instrument realization of measurements or in the plane, perpendicular to range plane and passing through the axis of rocket or the velocity vector its center of mass. If the yaw angle in the inclined plane, passing through longitudinal axis Ox_1 of axisymmetric rocket, we designate through

ψ_H, then of Fig. 2.7.

$$\sin\psi_H = \sin\psi\cos\theta.$$

It is obvious, with $\theta=0$ we will obtain that $\psi_H=\psi$. With low ψ the pitch angle, measured in the vertical plane, passing through the axis of rocket, is close to the appropriate angle, measured in range plane. The rotation of the rocket of relatively longitudinal axis is determined by attitude of roll γ, i.e., by the angle between the connected coordinate axis Oy_1 and the axis Oy'' in the vertical plane, passing through the axis of rocket (see Fig. 2.6).

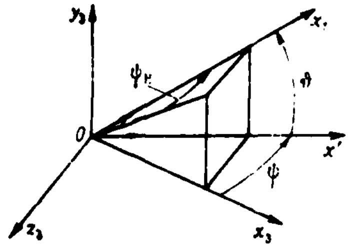

Fig. 2.7. Yaw angles, measured in horizontal and inclined planes.

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At the cruise missiles, made according to aircraft configuration, roll attitude can be defined as angle between the plane of symmetry and the vertical plane, passing through axis Ox_1, connected with the rocket of the coordinate system.

For the rotating rockets and projectiles (for example, turbojet) the position of the body coordinate system of relatively fixed system is determined with the aid of the Eulerian angles whose name corresponds to the names, accepted in the theory of gyroscopes. Let us accept for fixed coordinate system system Oxyz (Fig. 2.8). Let y axes, y, y_3, x, x_3 lie/rest at one (vertical) plane, moreover axes Oy and Ox with respect to axes Ox_3 and Oy_3 are turned to angle θ. For passage to the system of coordinates $Ox_1y_1z_1$ the first rotation is realized relative to axis Ox to the angle ν, called in the case in

question precession angle, in this case, axis Oy will occupy position Oy', and axis Oz - position Oz_1. The second rotation is realized relative to os Oz' to the angle θ, called nutation angle, in this case, axis Oy' will occupy position Oy'', and axis Ox - position Ox_1. The third rotation is realized relative to os Ox_1 to the angle φ, called the angle of spin, as a result of which axis Oy'' will occupy position Oy_1, and axis Oz' - position Oz_1. With respect to earth-based coordinate system, the position of the axis of projectile is determined by the pitch angle ϑ and by the yaw angle ψ. Angle φ corresponds to attitude of roll γ.

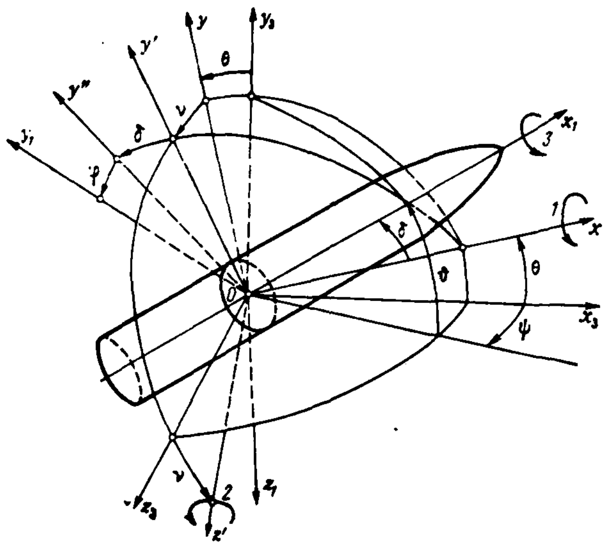

Fig. 2.8. Schematic of the angles, which determine the position of the rotating projectile.

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In the high-speed/velocity (flow) system of coordinates Oxyz, one of the axes coincides with the sense of the vector of the flight speed of the center of mass of rocket, another, to it perpendicular, lie/rests at the plane of the symmetry of flight vehicle. As preceding/previous, the high-speed/velocity coordinate system is right rectangular system and is utilized usually during the study of the phenomena of the flow around bodies. High-speed/velocity system is connected with velocity vector and is moved together with it

during the motion of rocket. During the study of the phenomena of flow in the turned motion (model is fixed, and flow moves) this coordinate system is fixed and one of its axes it is directed along the velocity vector of the undisturbed flow.

In the half-speed coordinate system one of the axes, just as in high-speed/velocity, it coincides with velocity vector, another is directed perpendicularly to it and lie/rests at vertical plane, the third axis is directed horizontally.

Figures 2.9 shows the diagram of the mutual layout of high-speed/velocity, half-speed and earth-based coordinates system with the combined origin of coordinates. Axis Ox is directed along velocity vector, axis Oy lie/rests at the plane of the symmetry of flight vehicle, axis Oz is perpendicular to plane yOx.

The position of the high-speed/velocity system of coordinates of the relatively combined terrestrial system is determined by three angles θ, ψ and γ_c: θ - slope angle to the horizon the tangent to trajectory (angle between vectors of speed and local horizontal plane); ψ - angle of rotation of trajectory; γ_c - attitude of roll of the high-speed/velocity coordinate system.

Axis Oy* of the half-speed coordinate system lie/rests at the

vertical plane, passing through the velocity vector $\bar{v}$, the auxiliary axis Ox*, which lies at horizontal plane, and the vertical axis Oy_3 of combined earth-based coordinate system. Axis Oz* of the half-speed coordinate system lie/rests at the horizontal plane, perpendicular to plane x*Oy* . Passage to half-speed system from combined earth-based coordinate system is realized via the consecutive rotation of the latter at angles ψ and θ. Half-speed system coincides with high-speed/velocity when $\gamma_c=0$.

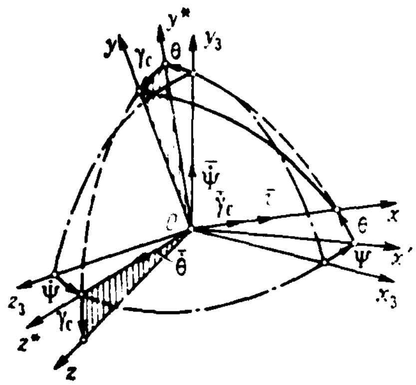

Fig. 2.9. Diagram of the mutual layout of high-speed/velocity, half-speed and earth-based coordinates system with the combined origin of coordinates.

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The projections of velocity vector on the axis of earth-based coordinate system are obvious from Fig. 2.9

$$\left.\begin{aligned} v_{x_3} &= v\cos\theta\cos\Psi; \quad v_{y_3} = v\sin\theta; \\ v_{z_3} &= -v\cos\theta\sin\Psi. \end{aligned}\right\} \qquad (2.4)$$

In the examination of two-dimensional problems when $\Psi=0$, the horizontal and elevations of speed are respectively equal to:

$$v_{x_3}=u=v\cos\theta; \quad v_{y_3}=w=v\sin\theta. \qquad (2.5)$$

After unit on the launching pad of the vertically starting rocket, body axis Ox_1 coincides with axis $Oy_{3.c}$. After separation from table, during vertical flight, coincide axes Ox_1, Oy_{3c} and Ox (high-speed/velocity system).

Communication/connection between mobile (connected) and by continuous operations of coordinates (Fig. 2.10 and 2.11) is realized with the aid of the angle of attack and slip angle. It is known two versions of the determination of the angles of attack and slip. In connection with the axisymmetric rockets, which do not have the clearly expressed lifting wings, angle of attack is called the angle between vectors of speed and the axis of rocket Ox_1 (see Fig. 2.10). In this case the plane of angle of attack is called the plane of resistance.

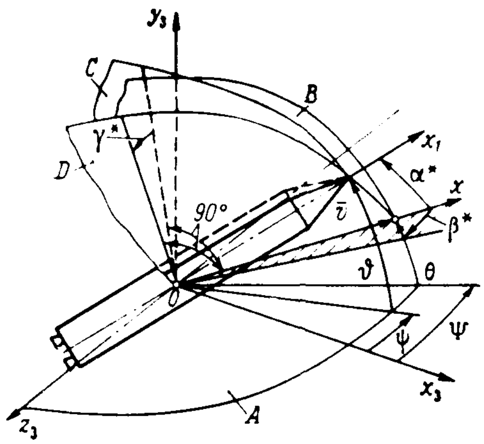

Fig. 2.10. The diagram of the mutual layout of the flow, connected and combined earth-based coordinate system for the axisymmetric wingless rockets: A - horizontal coordinate plane; B and C - vertical planes, carried out respectively through the velocity vector and the axis of rocket; D - inclined plane, carried out through the velocity vector and the axis of rocket (plane of resistance).

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Let us designate the indicated angle of attack α^*, and the angle between the plane of resistance and the vertical plane $x0y_3$ - γ^*. The angle between $\bar{v}$ and its projection on the vertical plane, passing through the axis of rocket Ox_1, calls angle of slip β^*.

Of the rockets of aircraft configuration, which have the clearly

expressed wings, and of aircraft by angle of attack α it is accepted to call the angle between the projection of velocity vector on the plane of the symmetry of aircraft and the airfoil chord of wing or the longitudinal axis Ox_1 (see Fig. 2.11). Angle of slip β is called the angle between vectors of speed and the plane of the symmetry of aircraft. If $\beta=\psi=\Psi=0$, then we will obtain the motion in vertical plane, during which $\vartheta=\theta+\alpha$.

Passage from the axes of one system of rectilinear coordinates to another is realized on the formulas, known from analytical geometry, with the aid of the tables of direction cosines.

The tables of direction cosines are utilized not only for the conversion of the coordinates, which determine the position of point in space, but also for the conversion of the comprising any forces, torque/moments and motion characteristics which can be expressed in vector form (for example, force component of weight, aerodynamic forces, etc.). The common/general/total formulas of conversion take the following form:

$$\left.\begin{aligned} x&=a_1x'+a_2y'+a_3z'; & x'&=a_1x+b_1y+c_1z;\\ y&=b_1x'+b_2y'+b_3z'; & y'&=a_2x+b_2y+c_2z;\\ z&=c_1x'+c_2y'+c_3z'; & z'&=a_3x+b_3y+c_3z.\end{aligned}\right\} \quad (2.6)$$

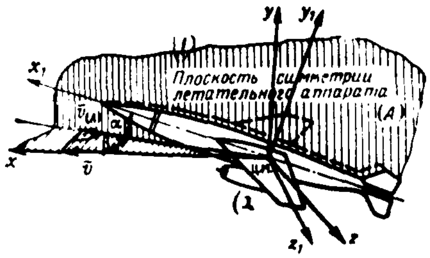

Fig. 2.11. Diagram of the mutual layout of flow and body coordinate systems.

Key: (1). Plane of the symmetry of flight vehicle. (2) Center of mass.

Table 2.2. The general view of the table of direction cosines.

Проекции вектора (1)	x'	y'	z'
x	a_1	a_2	a_3
y	b_1	b_2	b_3
z	c_1	c_2	c_3

Key: (1). Projections of vector.

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Are given below the tables of direction cosines for different pairs of rectangular coordinate systems.

Table 2.3a. Cosines of the angles between the axes of the high-speed/velocity and half-speed coordinate systems.

(1) Оси координат	$0x$	$0y$	$0z$
$0x$	1	0	0
$0y^*$	0	$\cos\gamma_c$	$-\sin\gamma_c$
$0z^*$	0	$\sin\gamma_c$	$\cos\gamma_c$

Key: (1). Coordinate axes.

Table 2.3b. Cosines of the angles between axes of the connected and half-speed coordinate systems.

(1) Оси координат	$0x_1$	$0y_1$	$0z_1$
$0x$	$\cos\alpha\cos\beta$	$-\sin\alpha\cos\beta$	$\sin\beta$
$0y^*$	$\sin\alpha\cos\gamma_c + \sin\gamma_c\cos\alpha\sin\beta$	$\cos\gamma_c\cos\alpha - \sin\gamma_c\sin\alpha\sin\beta$	$-\sin\gamma_c\cos\beta$
$0z^*$	$\sin\gamma_c\sin\alpha - \cos\gamma_c\cos\alpha\sin\beta$	$\sin\gamma_c\cos\alpha + \cos\gamma_c\sin\alpha\sin\beta$	$\cos\gamma_c\cos\beta$

Key: (1). Coordinate axes.

Table 2.3c. Cosines of the angles between the axes of half-speed and earth-based coordinates system.

(1) Оси координат	$0x$	$0y^*$	$0z^*$
$0x_3$	$\cos\theta\cos\Psi$	$-\sin\theta\cos\Psi$	$\sin\Psi$
$0y_3$	$\sin\theta$	$\cos\theta$	0
$0z_3$	$-\cos\theta\sin\Psi$	$\sin\theta\sin\Psi$	$\cos\Psi$

Key: (1). Coordinate axes.

Table 2.3d. Cosines of the angles between axes of connected and earth-based coordinates system.

Table 2.3d. Cosines of the angles between axes of connected and earth-based coordinates system.

(1) Оси координат	$0x_1$	$0y_1$	$0z_1$
$0x_3$	$\cos\vartheta\cos\psi$	$-\cos\psi\sin\vartheta\cos\gamma+$ $+\sin\psi\sin\gamma$	$\cos\psi\sin\vartheta\sin\gamma+$ $+\sin\psi\cos\gamma$
$0y_3$	$\sin\vartheta$	$\cos\vartheta\cos\gamma$	$-\cos\vartheta\sin\gamma$
$0z_3$	$-\sin\psi\cos\vartheta$	$\cos\psi\sin\gamma+$ $+\sin\psi\sin\vartheta\cos\gamma$	$\cos\psi\cos\gamma-$ $-\sin\psi\sin\vartheta\sin\gamma$

Key: (1). Coordinate axes.

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The selection of the control system frequently sets limitations on the instrument realization of the coordinate systems. Is known separation in this sense into the Cartesian control, which uses of different form of the system of rectilinear coordinates, and polar control, which uses a polar coordinate system.

During the design of the control system, is provided for the measuring coordinate system, in which are measured the disturbance/perturbations of action, and the executive coordinate system, connected with the rocket in which is conducted the final adjustment of command/crews.

In the process of missile targetings to the driving/moving target/purposes along complex trajectories, is possible the phenomenon of the "torsion" ("fracture") of the executive coordinate

system of relatively measuring system. Let the surface-to-air missile be control/guided on radio beam. The measuring coordinate system is connected with lead beam so that one of the axes Ox_H coincides with the axis of beam central line, and axes Oy_H and Oz_H supplement system to right and they are constant/invariably connected with the antenna, which creates lead beam. The executive system of coordinates $Ox_py_pz_p$ is arrange/located on rocket. Axis Ox_p is connected with axis of rocket, and axes Oy_p and Oz_p are stabilized by on-board gyrostabilizer from rotation around axis Ox_p. Let us assume that prior to the start of the axis of the executive and measuring coordinate systems they coincide in the direction. In the process of missile targeting to target/purpose, both coordinate systems will be moved, the axis Ox_H of measuring system will track a target or aimed into set forward point.

Table 2.3e. Cosines of the angles between vectors angular velocities $\bar{\vartheta}, \bar{\xi}, \bar{\gamma}$ and the axes of body coordinate system.

(1) Оси координат	$\bar{\vartheta}$	$\bar{\xi}$	$\bar{\gamma}$
$0x_1$	0	$\sin\vartheta$	1
$0y_1$	$\sin\gamma$	$\cos\vartheta\cos\gamma$	0
$0z_1$	$\cos\gamma$	$-\cos\vartheta\sin\gamma$	0

Key: (1). Coordinate axes.

Table 2.3f. Cosines of the angles between axes of the connected and high-speed/velocity coordinate systems.

(1) Оси координат	$0x_1$	$0y_1$	$0z_1$
$0x$	$\cos\alpha\cos\beta$	$-\sin\alpha\cos\beta$	$\sin\beta$
$0y$	$\sin\alpha$	$\cos\alpha$	0
$0z$	$-\cos\alpha\sin\beta$	$\sin\alpha\sin\beta$	$\cos\beta$

Key: (1). Coordinate axes.

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The axis $0x_p$ of the executive coordinate system while maneuvering of rocket will coincide with axis of rocket, but axes $0y_p$ and $0z_p$ cannot be turned relative to axis $0x_p$ as a result of the action of gyrostabilizer. Each coordinate system, being moved in space, by special form differs from initial direction and the collinearity of their axes, assumed by us in the beginning, is lost, i.e., occurs the "torsion" of the coordinate axes. In the general case the "torsion" can be three-dimensional/space, with which in the process of induction occurs the angular separation of the executive and

measuring coordinate systems along all three axes. In the process of induction, the longitudinal axis of the well guided missile insignificantly oscillates relative to the axis beam central line and in the first approximation, they assume that the axes $0x_n$ and $0x_p$ of the measuring and executive coordinate systems coincide in the direction. In this case will occur the "torsion" of axes $0y_p, 0z_p$ and $0y_n, 0z_n$ in the plane, perpendicular to the overall direction conditionally coaxial $0x_n$ and $0x_p$.

§2. EFFECT OF THE GRAVITATIONAL FIELD OF EARTH AND ITS ROTATION ON ROCKET AND PROJECTILE FLIGHT.

2.1. Potential of the force of gravity, form and the size/dimensions of the Earth.

Potential function, or potential, is called the function $\Pi(x, y, z)$, whose total differential is equal to the elementary work, which acts on the point of the force:

$$d\Pi=\bar{F}d\bar{r}=\frac{\partial\Pi}{\partial x}dx+\frac{\partial\Pi}{\partial y}dy+\frac{\partial\Pi}{\partial z}dz.$$

The projections of the resultant of the applied to point forces are respectively equal to

$$F_x=\frac{\partial\Pi}{\partial x};\quad F_y=\frac{\partial\Pi}{\partial y}\quad \text{and}\quad F_z=\frac{\partial\Pi}{\partial z}.$$

Potential for a point unit mass, which is located out of the volume of the Earth at a distance l from the elementary mass dM Earth, according to the Newton law of gravity is determined by the expression

$$d\Pi_{\tau}=\frac{fdM}{l}, \qquad (2.7)$$

where f - constant of gravitational attraction; l - distance between points with unit mass (point P) and the elementary mass dM (Fig. 2.12). The potential of the Earth for a point unit mass obtains by integration for an entire mass of Earth.

$$\Pi_{\tau}=f\int\limits_{(M)}\frac{dM}{l}. \qquad (2.8)$$

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The written integral can be calculated only approximately. Are unknown accurately the form of the Earth and its size/dimensions, is unknown mass distribution within the Earth.

Elementary mass $dM=\mu_3 d\tau$, where μ_3 - mass density of substance, which is substantially changed by entire volume; $d\tau$ - volume element.

If distance from the conditional center of the Earth to elementary mass is designated ρ, and to external point P by r then, designating the angle between ρ and r through ψ (see Fig. 2.12), we will obtain

$$l=\sqrt{r^2+\rho^2-2r\rho\cos\psi}. \qquad (2.9)$$

In the process of rocket flight relative to the Earth will

change values r and ψ and, consequently, also values l. It is obvious that the potential will be changed in the process of moving the rocket relative to the Earth within some limits and it can be calculated approximately during the introduction of the various kinds of assumptions. The most essential assumptions concern the shape of the Earth, its size/dimensions and mass distribution.

For closer to the real form of the Earth, is accepted the figure, called geoid. Geoid - this figure, limited by level surface of the gravitational potential, which coincides with the surface of oceans, which are found in the undisturbed state, i.e., in the absence of the boss/inflows, ebb/discharges, atmospheric and any other disturbance/perturbations. Level surface of the gravitational potential is called such surface, at all points of which the value of the gravitational potential is equal. At present data for precise mathematical description geoid still insufficient. When conducting of the various kinds of calculating works (geodetic, astronomical and ballistic) as successive approximations to geoid they accept: the spherical model of the Earth - a sphere, spheroidal model - spheroid (ellipsoid of revolution), the general ellipsoid.

In Russia during long time when conducting the astrogeodetic works, was accepted Bessel's spheroid. In 1924 by international agreement the best spheroid considered Hayford's spheroid. In many

instances proves to be convenient to utilize the so-called normal spheroid, A. Klero's proposed.

The size/dimensions, which determine spheroid (its axes), were calculated on the basis of the degree measurements of the arc lengths of meridians.

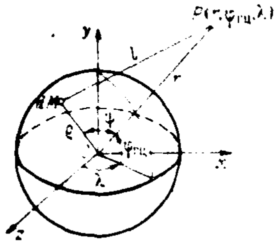

Fig. 2.12. Coordinates of point unit mass, which is located out of the volume of the Earth.

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Since the figure of the Earth is different from spheroid, then in different places on one and the same the latitude of the arc of meridians they have different curvature. Therefore the specific by degree measurements size/dimensions of ellipsoid depend on the place of measurement. To this are explained the available differences in the numerical values of the cell/elements of terrestrial spheroid, obtained by the separate authors.

Soviet geodesists under F. N. Krasovskiy's management/manual (1878-1948), utilizing degree measurements by the USSR, West Europe and USA, they determined the size/dimensions of biaxial ellipsoid. In accordance with the resolution of the Council of Ministers of the USSR of 7 April, 1946, in the geodetic works with the USSR is utilized the terrestrial ellipsoid of rotation, which in resolution

is named the Krasovskiy terrestrial ellipsoid. On the basis of the same works, were obtained the data for the general ellipsoid.

For the biaxial Krasovskiy ellipsoid, semimajor axis (the mean radius of equator) is accepted equal to a=6378245 m, semiminor axis b=6356863 m. The difference in the values of the axes of spheroid comprises ~42800 m. Compression of the spheroid

$$\alpha=\frac{a-b}{a}=\frac{1}{298.3}=0{,}003352.$$

Square of first eccentricity

$$l_1^2=\frac{a^2-b^2}{a^2}=0{,}006693.$$

Square of second eccentricity

$$l_2^2=\frac{a^2-b^2}{b^2}=0{,}006739.$$

For the spherical model of the Earth, the basic a constant value is a radius of terrestrial sphere. It can be determined differently. If we take arithmetic mean of three semi-axes of the Krasovskiy ellipsoid, then we will obtain 6371118 m. The radius of sphere, which has the same surface as the surface of the terrestrial ellipsoid, it is equal to 6371116 m. Radius of the sphere, which has the same volume, as the volume of ellipsoid, 6371110 m. All methods give close results.

The common/general/total expression for the potential of force of gravity, virtually suitable for different models of geoid, is obtained, after expanding expression for Π_T in a series of spherical

functions. Let us preliminarily examine a simpler conclusion/derivation, which well elucidates the physical sense of the first terms of expansion.

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From (2.9) we will obtain $\frac{1}{l}=\frac{1}{r\sqrt{1+\left(\frac{\varrho}{r}\right)^2-2\frac{\varrho}{r}\cos\psi}}=\mathcal{F}\left(\frac{\varrho}{r}\right).$

The expansion of the written function in a binomial series can be represented in the form of the sum of members, who include Legendre's polynomials $P_n(\cos\psi)$:

$$\mathcal{F}\left(\frac{\varrho}{r}\right)=\frac{1}{r}\sum_{n=0}^{\infty}\left(\frac{\varrho}{r}\right)^n P_n\cos\psi. \qquad (2.10)$$

The common/general/total expression for the polynomial of degree n takes the form

$$P_n(\cos\psi)=\frac{1}{2^n n!}\frac{d^n[(\cos\psi)^2-1]^n}{d(\cos\psi)^n}.$$

For the separate values of n, let us have:

$$\left.\begin{aligned}
&P_0(\cos\psi)=1;\\
&P_1(\cos\psi)=\cos\psi;\\
&P_2(\cos\psi)=\frac{3}{2}\cos^2\psi-\frac{1}{2};\\
&P_3(\cos\psi)=\frac{5}{2}\cos^3\psi-\frac{3}{2}\cos\psi;\\
&P_4(\cos\psi)=\frac{35}{8}\cos^4\psi-\frac{15}{4}\cos^2\psi+\frac{3}{8};\\
&\dots\dots\dots\dots\dots\dots
\end{aligned}\right| \qquad (2.11)$$

Utilizing (2.10), we will obtain from (2.8)

$$\Pi_r=\frac{f}{r}\int_{(M)}\sum_{n=0}^{\infty}\left(\frac{\varrho}{r}\right)^n P_n(\cos\psi)dM. \qquad (2.12)$$

We will be restricted to three terms of expansion and will

present potential in the sum of three integrals

$$\Pi_1 = \frac{f}{r}\left[\int_{(M)} dM + \frac{1}{r}\int_{(M)} \rho\cos\psi\, dM + \right.$$
$$\left. + \frac{1}{r^2}\int_{(M)} \rho^2\left(\frac{3}{2}\cos^2\psi - \frac{1}{2}\right) dM. \right] \quad (2.13)$$

Integrals can be undertaken in final form, if we apply some limitations on mass distribution and the form of the Earth. During uniform mass distribution according to the volume of the Earth for the coordinate system whose center is placed in the center of mass of the Earth,

$$\int_{(M)} dM = M.$$

present potential in the sum of three integrals

$$\Pi_1 = \frac{f}{r}\left[\int_{(M)} dM + \frac{1}{r}\int_{(M)} \rho\cos\psi\, dM + \right.$$
$$\left. + \frac{1}{r^2}\int_{(M)} \rho^2\left(\frac{3}{2}\cos^2\psi - \frac{1}{2}\right) dM. \right. \qquad (2.13)$$

Integrals can be undertaken in final form, if we apply some limitations on mass distribution and the form of the Earth. During uniform mass distribution according to the volume of the Earth for the coordinate system whose center is placed in the center of mass of the Earth,

$$\int_{(M)} dM = M.$$

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Under the same conditions, expressing cos ψ through the coordinates of particle dM and the coordinates of the external body of unit mass, we will obtain

$$\int\limits_{(M)} \varrho \cos \psi dM = 0.$$

The dependence of mass distribution or the asphericity of the Earth can be characterized by the moments of the inertia of the Earth, determined relative to the axes geocentric coordinate system. If we designate through B the torque/moment of inertia of relatively principal axis $(0y_3)$, that coincide with the rotational axis of the Earth, and through A and C - the torque/moments of inertia of the relatively principal axes, which lie at equatorial plane $(0x_3, 0z_3)$, and to use transfer equations (2.1), then we will obtain

$$\int\limits_{(M)} \varrho^2 \left(\frac{3}{2} \cos^2 \psi - \frac{1}{2}\right) dM = (A + C - 2B) \frac{1}{4} (3 \sin^2 \varphi_{\Gamma\text{ц}} - 1) + \\ + \frac{1}{4} (C - A) 3 \cos^2 \varphi_{\Gamma\text{ц}} \cos 2\lambda. \quad (2.14)$$

Designating the sum of the disregarded terms of the expansion through $\Pi_{\text{т.н.}}$ we will obtain formula for the potential of the earth's gravity

$$\Pi_{\text{т}}=\frac{fM}{r}+\frac{f}{4r^3}(A+C-2B)(3\sin^2\varphi_{\text{гц}}-1)+\\+\frac{3f}{4r^3}(C-A)\cos^2\varphi_{\text{гц}}\cos 2\lambda+\Pi_{\text{т.н}}. \quad (2.15)$$

In last/latter formula the first term represents by itself the potential of sphere with uniform or spherical mass distribution - the so-called potential of Newtonian attraction. The second term depends on latitude $\varphi_{\text{гц}}$ and considers the effect of the polar compression of the Earth, the third - considers the dependence of mass distribution of the Earth on longitude, i.e., reflects the effect of lateral compression. Subsequent members (designated $\Pi_{\text{т.н}}$) consider the dissymmetry of mass distribution in the northern and south parts of the Earth and other nonuniformities in the gravitational field of the Earth.

If one assumes that the Earth is correct ellipsoid of rotation with uniform mass distribution around rotational axis, we will obtain A = C and then

$$\Pi_{\text{т}}=\frac{fM}{r}+\frac{f}{2r^3}(A-B)(3\sin^2\varphi_{\text{гц}}-1). \quad (2.16)$$

For determination $\Pi_{T.H}$ it is necessary to consider the polynomials of Legendre of the higher orders, than the second. However, directly the use of subsequent members of expansion encounters the mathematical difficulties of practical order.

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Therefore in the more complete examination of the potential of the Earth, they prefer dependence for its determination to represent by a series of the spherical (spherical) functions which are expressed as the associated functions of Legendre. For trigonometric function ζ, the associated function of Legendre takes the form

$$P_{nm}(\zeta) = (1-\zeta^2)^{\frac{m}{2}} \frac{d^m}{d\zeta^m} P_n(\zeta), \qquad (2.17)$$

where the index of "n" determines the degree of function, the index of "m" - its order.

The gravitational potential of the Earth, expressed through spherical functions, takes the form

$$\Pi_T = \frac{fM}{r}\left\{1+\sum_{n=1}^{\infty}\sum_{m=0}^{N}\left(\frac{R_3}{r}\right)^n (C_{1nm}\cos m\lambda + C_{2nm}\sin m\lambda) P_{nm}(\sin\varphi_{Tn})\right\}. \qquad (2.18)$$

Here C_{1nm} and C_{2nm} — dimensionless numerical coefficients, $R_э$ — equatorial radius of the Earth.

If we accept mass distribution according to the volume of the Earth symmetrical relative to its rotational axis, then (2.18) it can be converted to the more convenient form

$$\Pi_т = \frac{fM}{r}\left\{1 + G_{20}\left(\frac{R_э}{r}\right)^2 P_{20}(\sin\varphi_{гц}) + \right.$$
$$\left. + G_{30}\left(\frac{R_э}{r}\right)^3 P_{30}(\sin\varphi_{гц}) + G_{40}\left(\frac{R_э}{r}\right)^4 P_{40}(\sin\varphi_{гц})\right\}. \quad (2.19)$$

The dimensionless parameters G_{n0} are determined by the so-called level surface and the angular rate of rotation of the Earth. Values $P_{n0}(\sin\varphi_{гц})$ are called zonal spherical functions.

For orientation let us bring data, that characterize the gravitational field of the Earth, obtained with the aid of geophysical satellites [57]:

$K = fM = 398603{,}2$ км^3/с^2 (1)
$\alpha = 1 : 298{,}2 \pm 0{,}2$
$G_{20} = -1082{,}645 \cdot 10^{-6}$
$G_{30} = 2{,}546 \cdot 10^{-6}$
$G_{40} = 1{,}649 \cdot 10^{-6}$
$R_3 = 6378{,}165$ км.

Key: (1). km^3/s^2.

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If the Earth to consider as sphere with mean radius R_3, in which the mass is distributed everly by volume, then of (2.19) follows that $\Pi_т = \frac{fM}{r} = \frac{K}{r}$. Consequently, in this case the potential (gravitational) field of the Earth will be central, and the acceleration from attracting force, which acts in it on the body of unit mass, can be found as

$$g_т = -\frac{d\Pi_т}{dr} = \frac{K}{r^2}.$$

Sign "minus" is here undertaken because vector $\bar{r}$ in the direction of which is taken the derivative $\frac{d\Pi_т}{dr}$, and vector $\bar{g}_т$ are directed to opposite sides.

Comparing values $g_т$ for radii r and R_3, we will obtain dependence $\frac{g_т}{g_{т0}} = \left(\frac{R_3}{r}\right)^2$, which it characterizes a change of

accelerating the attracting force in central gravitational field with removal/distance from its center.

2.2. Gravitational force and its potential.

To the body, which rotates together with the Earth, besides the attracting force acts the inertial force, caused by the rotation of the Earth. The combined action of the gravity force and centrifugal inertial force determines gravitational force. Gravitational force can be presented as sum

$$\bar{F}=\bar{F}_{т}+\bar{F}_{ц},$$

where $F_т$ — force vector of gravity;

$F_ц$ — vector of centrifugal inertial force.

In spherical geocentric coordinates the centrifugal inertial force, which acts on the body of mass m in the direction, perpendicular to the rotational axis of the Earth, is equal to

$$F_ц=mr\Omega^2\cos\varphi_{гц},$$

where Ω — the angular rate of rotation of the Earth.

The direction of the force of gravity coincides with the

direction of plumb line, i.e., with vertical line at the point of the earth's surface in question and normal to the surface of geoid. The angle between the standard (n) to surface and the equatorial plane of geoid is called geographical latitude φ_r unlike geocentric latitude φ_{ru} (Fig. 2.13). The longitude is counted off from the prime meridian as which is accepted the meridian of Greenwich.

Communication/connection between geocentric and geographic latitudes is establish/installed on the approximation formula

$$\mathrm{tg}\,\varphi_{ru}=\mathrm{tg}\,\varphi_r\,(1-l_1^2),$$

where l_1 — first eccentricity.

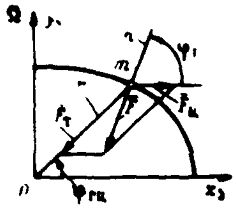

Fig. 2.13. Position of local vertical line, determined by geographic latitude

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The difference between angles $\varphi_r - \varphi_{ru}$ can be found also from the dependence

$$\varphi_r - \varphi_{ru} = a \sin 2\varphi_r.$$

Great value $(\varphi_r - \varphi_{ru})$ is equal to 11.5' at $\varphi_r = 45°$.

The gravitational potential also can be presented as sum of the potentials of the force of gravity and the centrifugal inertial force, determined by the diurnal rotation Earth

$$\Pi = \Pi_T + \Pi_u.$$

Taking into account that $d\Pi_{ц}=F_{ц}dr_{ц}$, where $r_{ц}=r\cos\varphi_{rц}$, we will obtain the potential of centrifugal inertial force, in reference to the unit of the mass

$$\Pi_{ц}=\frac{1}{2}\Omega^2r^2\cos^2\varphi_{rц}.$$

If we utilize for Π_{T} formula (2.16), then

$$\Pi=\frac{fM}{r}+\frac{f}{2r^3}(A-B)(3\sin^2\varphi_{rц}-1)+\frac{1}{2}\Omega^2r^2\cos^2\varphi_{rц}. \quad (2.20)$$

We will obtain formula for determining the acceleration of gravity. From the property of the potential, found for a unit mass,

$$g_S=\frac{\partial\Pi}{\partial S},$$

where g_S — projection of the acceleration of gravity on direction S;

$\frac{\partial\Pi}{\partial S}$ — derivative of the gravitational potential undertaken in the direction S.

Let us select as direction S normal to the surface of geoid to which acceleration $\bar{g}$ is projected by complete value. Designating the direction of normal to geoid through n (see Fig. 2.13), let us have:

$$g=g_n=\frac{\partial\Pi}{\partial n}.$$

Considering the low difference in angles $\varphi_{\text{гц}}$ and $\varphi_{\text{г}}$, derivative in the direction n replace to derivative in the direction r

$$\frac{\partial \Pi}{\partial r} = \frac{\partial \Pi}{\partial n} \cos(\widehat{\bar{n}\bar{r}}).$$

Taking into account contrast of reference directions for n and r and the smallness of the angle between $\bar{r}$ and $\bar{r}$, it is possible to write

$$-1 \leqslant \cos(\widehat{\bar{n}\bar{r}}) \leqslant -0{,}999995,$$

where numeral 0.999995 it corresponds to the greatest angle $\varphi_{\text{г}} - \varphi_{\text{гц}} = 11.5'$. Considering the low difference $\cos(\widehat{\bar{n}\bar{r}})$ from unit, we will obtain

$$g = \frac{\partial \Pi}{\partial n} = -\frac{\partial \Pi}{\partial r}.$$

It differentiated (2.20) on r, we will obtain

$$g = \frac{fM}{r^2} + \frac{3f}{2r^4}(A-B)(3\sin^2\varphi_{\text{гц}} - 1) - \Omega^2 r \cos^2\varphi_{\text{гц}}. \quad (2.21)$$

For the spherical model of the Earth, without the account of its

rotation, we will obtain from (2.21)

$$g=\frac{fM}{r^2}=g_r.$$

Into formula (2.21) frequently are introduced values

$$\mu=\frac{A-B}{R_e^2} \text{ и } q=\frac{\Omega^2 R_e}{fM/R_e^2}=\frac{\Omega^2 R_e^3}{fM}.$$

Key: (1). and.

Value μ has dimensionality of mass, and q - the dimensionless parameter of the figure of the Earth, equal to the ratio of the acceleration of centrifugal force to the value of the acceleration of gravity in equatorial plane. Calculations show that

$$\mu=0,0011\ M;\quad q=0,003468.$$

Introducing in (2.21) values μ and q, we will obtain

$$g=\frac{fM}{r^2}\left[1+\frac{3}{2}\frac{\mu}{M}\frac{R_e^2}{r^2}(1-3\sin^2\varphi_{ru})-q\frac{r^3}{R_e^3}\cos^2\varphi_{ru}\right]. \quad (2.22)$$

Converting last/latter equality omitting in the process of the transformation of the values the order of smallness of which is higher than the first, it is possible to obtain simple formula for the calculation of the surface gravity Earth

$$g_0=g_{0e}(1+\beta\sin^2\varphi_{ru}),$$

where g_{00} — acceleration of gravity at equator when $\varphi_{ru}=0$.

Value β is called Clairauts's coefficient. Is numerical $g_{00}=9,78034$ m/s^2 and β = 0.00528CC1 [57]. When it is possible to accept g = const, usually are taken it equal to 9.81 m/s². For the spherical model Earth

$$g=g_0\left(\frac{R_3}{r}\right)^2.$$

2.3. Rotational effect of the Earth on rocket and projectile flight.

The Earth completes in space complex motion - annual inversion around the sun and diurnal rotation of relatively its axis; the earth's axis completes nutational and precessional motions.

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However, during the study of the motion of the rockets and projectiles, in view of the short duration of their flight, they consider that the motion of the Earth along solar orbit can be

accepted as rectilinear uniform forward motion; the nutational oscillation/vibrations of the Earth and its precession are not considered, since these motions are characterized by very low angular velocities (period of precessional motion - 26000 years, and the period of nutational oscillations - 18.6 years, with the amplitude, which does not exceed 9.2"). The diurnal rotation of the Earth is virtually uniform; one revolution is completed after 23 h of 56 min and 4 s, angular rate of rotation is equal to

$$\Omega = \frac{2\pi}{(23\cdot 60 + 56)\, 60 + 4} = 7{,}292\cdot 10^{-5}\ 1/s.$$

The effect of the diurnal rotation of the Earth on rocket flight and projectiles is easy to trace, if we examine their moti .. in the inertial geocentric coordinate system. At the moment of shot (launching/starting) the initial velocity of rocket in relative motion - v_0, while in absolute motion $v_{a0} = v_0 + v_{e0}$, where v_{e0} — the linear speed of rocket, determined by the movable rotary motion of the Earth and which depends on geographic latitude of the location of the launching site (here and throughout the trajectory elements of the center of mass in absolute motion let us note by index "a").

It is obvious

$$v_{e0} = \Omega R \cos\varphi_{r0},$$

where R - distance of the launching site from the conditional center of the Earth;

φ_{r0} — geographic latitude of the location of the launching site.

At poles $v_{e0}=0$; at equator v_{e0} is approximately equal to 1674 km/h, or 465 m/s.

The Earth rotates relative to polar axis from west to the east; therefore during starting/launching eastwards $v_{a0}>v_0$, a during starting/launching westwards (against the rotation of the Earth) $v_{a0}<v_0$.

During free flight the trajectory of the absolute motion of the rocket of class "surface - surface" (or Earth satellite) in the first approximation, i.e., for the spherical model of the Earth with uniformly concentric location of masses, is plane curve and its plane will occupy the constant/invariable attitude. At the same time as a result of rotating the Earth, will change in inertial space target position. For total flying time, the target/purpose will change its position, counting from zero time, to value

$$\Delta L_{\text{ц}}=\Omega R t_{\text{п}} \cos \varphi_{\text{г. п}}, \qquad (2.23)$$

where $\Delta L_{\text{ц}}$ — movement of target/purpose on longitude;

$t_{п}$ — total flying time;

$\varphi_{т.ц}$ — geographic latitude of target/purpose which remains constant/invariable in the process of period of earths rotation on its axis.

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The place of start and the point of impact in the rocket (projectile) of class "surface, surface," is located on the Earth, observation after rocket flight is realized from the point/items, arrange/located on the earth's surface. Therefore usually the trajectory calculation of the motion of rockets and projectiles carries out in the system of coordinates, connected with the rotating Earth, i.e., in relative motion. Let us establish communication/connection of initial conditions of departure in absolute and in relative motions. Let us designate geocentric inertial coordinate system through $x_a y_a z_a$ and the geocentric, connected with the Earth, coordinate system through xyz.

The motion of rocket in the first coordinate system will be absolute, in the second - relative. For determining the position of

rocket in inertial coordinate system, we utilize usual spherical coordinates $r_a, \varphi_{r.a}, \lambda_a$, and in that connected with Earth - $r, \varphi_{r.a}, \lambda$. Communication/connection between the longitude in absolute and relative motions let us determine according to the formula

$$\lambda_a = \lambda + \Omega t.$$

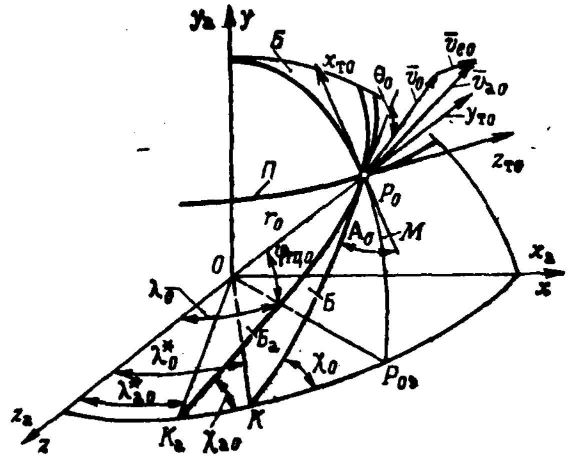

Fig. 2.14. Initial conditions of firing in absolute and relative motions.

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Initial conditions for trajectory of relative motion of the center of mass of rocket at the moment of time $t = t_0$ will be values r_0, $\varphi_{\text{гц}0}$, λ_0, initial velocity v_0 and angle of departure θ_0 (Fig. 2.14). For angle of departure, is accepted the angle between vectors of the initial velocity and the plane, tangential to sphere at release point P_0. The plane of casting is called the plane, passing through the vector of the initial velocity and the conditional center of the Earth under which let us understand the center of the spherical model of the Earth. Line OK of the intersection of the plane of departure (B) with equatorial plane is called nodal line.

Longitude of nodal line is determined in equatorial plane by angle λ* between the axis Oz and the nodal line. The inclination of the plane of casting to equatorial plane is determined by angle χ. Values of the named angles at zero time let us designate through λ_0^* and χ_0. From sector $P_0OP_{0\infty}$ and rectangular spherical triangle $P_0KP_{0\infty}$ let us have

$$\cos\chi_0=\cos\varphi_{\text{гц}0}\sin A_0;$$
$$\lambda_0^*=\lambda_0-\arcsin(\operatorname{ctg}\chi_0\operatorname{tg}\varphi_{\text{гц}0}),$$

where angle A_0 - azimuth of firing in relative motion, calculated from meridian M, passing through point R_0.

Let us find initial conditions in absolute motion. Let at zero time the inertial geocentric coordinates coincide with the coordinates, connected with the Earth, and then point R_0 will also render/show combined. The initial velocity in absolute motion is equal to

$$\bar{v}_{a0}=\bar{v}_0+\bar{v}_{e0}, \tag{2.25}$$

where the velocity of following of point R_0 is equal to $v_{e0}=\Omega r_0\cos\varphi_{\text{гц}0}$.

Is directed v_{e0} by collinearly tangent toward parallel P at point R_0.

Let us place into point R_0 the beginning of topocentric rectangular coordinate system axis $Oy_{\text{т}0}$ it is directed along

radius r_0, axis x_{T0} — to north, axis z_{T0} — tangentially to parallel at launch point. In the topocentric system of coordinates (Fig. 2.15) let us show the azimuth of firing A_0 and angle of departure θ_0 in relative motion and will construct the vector sum of velocities (2.25) (in Fig. 2.14 and 2.15 meridian and the parallel, passing through point P_0, are noted letters M and E). From Fig. 2.15 we find

$$v_0 \sin\theta_0 = v_{a0} \sin\theta_{a0};$$
$$v_0 \cos\theta_0 \cos A_0 = v_{a0} \cos\theta_{a0} \cos A_{a0};$$
$$v_0 \cos\theta_0 \sin A_0 + \Omega r_0 \cos\varphi_{rn0} = v_{a0} \cos\theta_{a0} \sin A_{a0}.$$

From the first equality we find angle of departure in the absolute motion

$$\theta_{a0} = \arcsin\left(\frac{v_0}{v_{a0}} \sin\theta_0\right). \qquad (2.26)$$

From the second - azimuth of the firing

$$A_{a0} = \arccos\left(\frac{v_0}{v_{a0}} \frac{\cos\theta_0 \cos A_0}{\cos\theta_{a0}}\right). \qquad (2.27)$$

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From the third equality, converting, we will obtain formula for the calculation of the module/modulus of the initial velocity

$$v_{a0} = \sqrt{v_0^2 + 2v_0\Omega r_0 \cos\varphi_{rn0} \cos\theta_0 \sin A_0 + \Omega^2 r_0^2 \cos^2\varphi_{rn0}}. \qquad (2.28)$$

Returning to Fig. 2.14, from spherical rectangle $K_a P_0 P_{\omega 0}$ we find the inclination of the plane of casting in absolute motion ($Б_a$) to equatorial plane

$$\cos \chi_{a0} = \cos \varphi_{ru0} \sin A_{a0}$$

and the longitude of nodal line

$$\lambda_{a0}^{*} = \lambda_0 - \arcsin(\mathrm{ctg}\, \chi_{a0} \, \mathrm{tg}\, \varphi_{ru0}). \tag{2.29}$$

The equation of motion of rocket in inertial coordinate system, written in vector form, takes the form

$$m\bar{a}_a = \sum \bar{F}, \tag{2.30}$$

where $\bar{a}_a$ — acceleration in absolute motion, ΣF — is resulting of the aerodynamic, control, gravitational and reaction forces, applied to rocket.

Sweeping last/latter equation and solving it under initial conditions (2.26), (2.27) and (2.28), it is possible to obtain all the characteristics of trajectory in absolute motion including the

coordinates of collision point with the earth's surface Ts' (Fig. 2.16). Plot/depositing in accordance with formula (2.23) correction for the rotation ΔL_{n} Earth we will obtain the position of the place of target/purpose at zero time - point Ts on parallel Π_{n}. In Fig. 2.16 longitudes of launch point (point R_0) - λ_0, the longitude of target position at the moment of launching/starting in the connected with the Earth geocentric system of coordinates $-\lambda_{\text{u}}$. The longitude of real target position at the moment of an incidence/drop in the nose cone in the inertial system of coordinates on formula (2.24) is equal to

$$\lambda_{u.s}=\lambda_u+2t_n.$$

The central angle 2ψ, which corresponds to complete flying range in inertial space, can be determined in the process of trajectory calculation. In Fig. 2.16 missile trajectory in absolute motion depicts the curve R_0Ts', in relative, motion - three-dimensional curve R_0Ts. Thus, in the examination of trajectory in relative motion target range along the earth's surface is equal to R_0Ts.

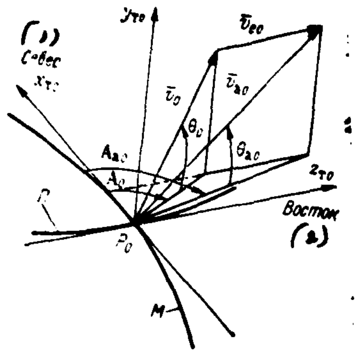

Fig. 2.15. The initial velocities, the azimuths and angles of departure in absolute and relative motions.

Key: (1). North. (2). East.

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Let us determine the acceleration of rocket in relative motion $\bar{a}_{отн} = \frac{d\bar{v}_{отн}}{dt}$.

Since

$$\bar{a}_a = \bar{a}_{отн} + \bar{a}_{пер} + \bar{a}_{кор},$$

the fundamental dynamic equation for relative motion will take the form

$$m\bar{a}_{отн} = \sum \bar{F} - m\bar{a}_{пер} - m\bar{a}_{кор}. \qquad (2.31)$$

The value of translaticnal acceleraticr is located (Fig. 2.71) through the formula

$$a_{пер}=r\Omega^2\cos\varphi_{гц}. \qquad (2.32)$$

The vector of movable (centripetal) acceleration is directed from the center of mass of rocket toward the rotational axis of the Earth along the shortest distance. Bearing in mind that subsequently we will comprise the equation of relative action of rocket in geocentric coordinates, let us write the projection of translational acceleration on these axes of the coordinates

$$a_{пер\,x}=-x\Omega^2;\quad a_{пер\,y}=0;\quad a_{пер\,z}=-z\Omega^2. \qquad (2.33)$$

Value of the Coriolis acceleration of determination from the known formula

$$a_{кор}=2v_{отн}\Omega\sin(\widehat{\bar{v}_{отн}\bar{\Omega}}). \qquad (2.34)$$

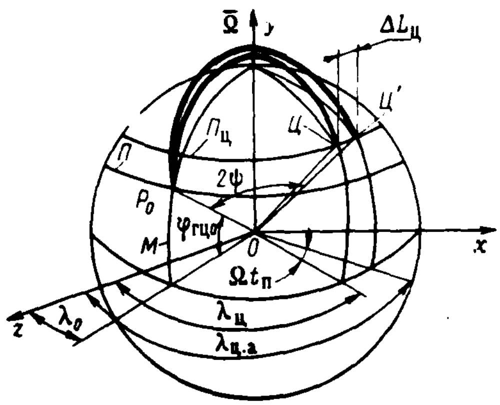

Fig. 2.16. Trajectory of the motion of rocket relative to rotating Earth.

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The vector of Coriolis acceleration is perpendicular the plane of vectors $\bar{\Omega}$ and $\bar{v}_{отн}$, if we them lead to common/general/total beginning. Direction $\bar{a}_{кор}$, in accordance with the rule of vector algebra, is taken similar so that the vectors $\bar{a}_{кор}$, $\bar{\Omega}$ and $\bar{v}_{отн}$ would comprise the right-handed triad. In this case, it is necessary that the observer, viewer from the terminus of the vector $\bar{a}_{кор}$ would see rotation $\bar{\Omega}$ to $\bar{v}_{отн}$ those occur on the low part of the circumference counterclockwise. Since the velocity $\bar{v}_{отн}$ in the process of moving the rocket will change direction, then also $\bar{a}_{кор}$ can be directed differently. Assuming that the directions known of those comprise

$\bar{v}_{отн}$ correspond to positive reference direction, then it is possible to establish value and direction of these comprise $\bar{a}_{кор}$ in the same geocentric system of coordinates (see Fig. 2.17). Utilizing (2.34) and the rule of the determination of direction $\bar{a}_{кор}$, we will obtain:

$$a_{кор\,x} = 2\Omega v_{отн\,z}; \quad a_{кор\,z} = -2\Omega v_{отн\,x}. \qquad (2.35)$$

Since $\bar{\Omega}$ and $\bar{v}_{отн\,y}$ are collinear, then $a_{кор\,y}=0$.

During the determination of direction $\bar{a}_{кор}$ in space, it is possible to also use Joukowski's rule, in accordance with whom the direction $\bar{a}_{кор}$ will be established by the rotation of projection $\bar{v}_{отн}$ on equatorial plane on angle $\frac{\pi}{2}$ to the side of movable rotation. For example, with firing along meridian from the Northern Hemisphere into south vector $\bar{a}_{кор}$ will be directed to the east of collinearly tangent toward parallel, while the projection of vector $\bar{v}_{отн}$ on equatorial plane will be directed from the center of the Earth.

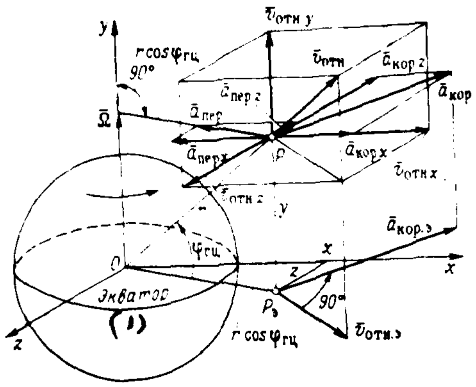

Fig. 2.17. Resolving of vector of relative velocity, movable and Coriolis accelerations along the axes of the rectangular geocentric coordinate system.

Key: (1). Equator.

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As soon as projection $v_{\text{отн}}$ on equatorial plane will be obtained direction to center. The Earth, the vector $a_{\text{кор}}$ will turn itself to west (Fig. 2.18). During the calculation of trajectories it is necessary to keep in mind that the force of inertia of translational action and Coriolis's force enter in the right side of the equation of relative motion with minus sign (2.31). Obviously that missile

trajectory in relative motion due to the action of the movable and Coriolis forces of inertia (and also still some factors) will be three-dimensional curve, i.e., the rocket during motion will be displaced sideways relative to range plane.

If movable (centrifugal) acceleration is united with the acceleration of the force of gravity into the overall acceleration of gravity (see §2, Section 2.2), then Coriolis acceleration remains only factor, considering the rotational effect of the Earth on relative motion of rocket. For the model of normal spheroid of the Earth, the maximum value of Coriolis acceleration is numerically equal to approximately 1.5o/o of the acceleration of gravity on each of thousand meters of the equatorial projection $v_{отн.э}$ of the relative speed of flight vehicle. The calculations conducted show that the rotational effect of the Earth on the flight of projectiles it is expedient to consider beginning from firing distance approximately 30,000 m.

3. The earth's atmosphere and of its property.

Atmosphere, which surrounds the Earth, calls the atmosphere. At height/altitude to 5000 m, is contained about 50o/o of entire mass of air, at height/altitude to 20,000 m ~90o/o. The basic physical atmospheric parameters - air density, weight or is mass, the

temperature of air, barometric pressure, speed of sound and wind - very significantly they affect the action characteristics of rockets and projectiles. The numerical values of the named meteorological characteristics depend on the physical state, the chemical composition and the structure of the atmosphere. For the study of the state of the atmosphere, is created the wide grid/network of the weather stations, scattered on entire terrestial globe. Investigations are carried out by meteorological instrumentation with the aid of pilots's spheres, it is radioprobing, the specially equipped aircraft, meteorological rockets and Earth satellites; the results of measurements undergo scientific processing and are generalized.

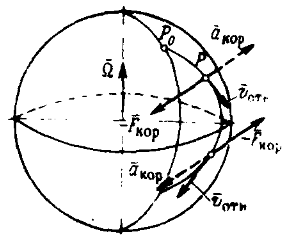

Fig. 2.18. Direction of Coriolis accelerations and forces with firing along meridian.

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The Earth's atmosphere by the chemical composition is conventionally designated as nitric-oxygen, it contains ~ 76o/o nitrogen, ~21% oxygen, ~3o/o water vapor, hydrogen, of carbon dioxide and series of other gases. Are known several principles of the construction of the schematics of the atmosphere. In composition of air the atmosphere is subdivided into homosphere and to heterosphere. In the homosphere, which stretches to height/altitudes ~95,000 m, the air composition with height/altitude barely changes. In heterosphere nitrogen, oxygen and other gases under the effect of the ultraviolet radiation of the sun dissociate and are located in atomic state.

Since the temperature of air is the basic parameter, which determines the physical constants of the atmosphere, the greatest interest for ballistics represents the schematic of the structure of the atmosphere according to the character of the temperature distribution with height/altitude. In this schematic the Earth's atmosphere is subdivided into five base layers, by the named spheres.

Lower layer - troposphere, extending in middle latitudes to height/altitudes ~11000 m, and in equatorial fields - to height/altitudes ~16000 m. The height/altitude of the troposphere depends on season, increasing by summer and decreasing by winter. In the troposphere is contained by ~75o/o of entire mass of the atmosphere and basic part of water vapor. In the troposphere are form/shaped all weather phenomena. Distinctive features of the troposphere - temperature decrease of air with height/altitude. However, and in summer after clear cold nights can in summer be observed the temperature inversions, at which the temperature on height/altitude first grow/rises, and then it begins to decrease. In the troposphere occur the considerable horizontal and vertical flows of air masses - winds. Horizontal winds are caused by a difference in the barometric pressure in the different places of the earth's surface, vertical - by a difference in the temperature on height/altitude.

Following layer - stratosphere, which stretches in middle latitudes from ~11,000 to ~50,000 m. The stratosphere to height/altitudes ~30,000-35,000 m is characterized by temperature constancy, but at larger height/altitude, with approach/approximation to upper boundary of the stratosphere, the temperature grow/rises; in this case, occur the considerable diurnal and day-to-day oscillations of temperature. A change in the temperature gradient between the troposphere and the stratosphere occurs in the relatively narrow layer, called tropopause. The thickness of the layer of tropopause varies from several hundred meters to ÷2000 m. In the relatively narrow layer, which covers tropopause, are observed the powerful displacements of air masses from west to the east, the so-called jet streams. Lower than tropopause one kilometer are approximately to observed flows (winds) with the maximum velocities, places those reach ~110 m/s (400 km/h). The range of jet streams is characterized by high-speed gradients in vertical and horizontal directions.

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Above the stratosphere is arrange/located the mesosphere, which stretches from height/altitude ~50,000 to ~90,000 m. It is characterized by temperature decrease to upper bound of layer and by increased turbulence.

Thermosphere - this a layer are atmospheric from ~90,000 to ~500,000 m, which is characterized by the continuous increase of temperature. In the upper part of the thermosphere at height/altitudes 400,000-500,000 m the temperature of air reaches ~1500°K.

A layer arranged/located of height/altitudes ~500,000 m to the external boundary of the atmosphere, i.e., approximately to 2,000,000-3,000,000 m, is called exosphere. In exosphere the atmosphere is very rarefied. The transition layers between the named spheres are called respectively of stratopause, mesopause and thermo-pause.

The important characteristics of the state of the atmosphere are also pressure and air density which unlike temperature with an increase in the height/altitude monotonically decrease.

The power effect of the atmosphere on flight vehicle is characterized by velocity head $q=\frac{\rho v^2}{2}$ (ρ - mass air density), which has vital importance at height/altitudes to 25,000-30,000 m. On height/altitudes, high 30,000 m, the effect of the atmosphere it is expedient to consider only during the calculation a or of Earth satellite.

3.1. Standard atmosphere.

Investigations showed that the physical atmospheric parameters considerably change depending on climatic conditions, the season and height/altitude. Ballistic calculations are performed for the normal meteorological conditions, which correspond to average/mean statistical experimental data or to the so-called standard atmosphere. The deviation of meteorological conditions from their normal values is considered separately in the theory of the corrections (see Chapter XI).

To 1920 in our country, they used conditional atmospheric, obtained by processing surface meteorological conditions in Moscow area. In 1920 was accepted international standard atmosphere (MSA). For artillery practice into 1927, they introduced the normal artillery atmosphere (NAA). In 1949 were published the detailed tables of standard atmosphere (GOST [ГОСТ - All-union State Standard] 4401-48). With the development of high-altitude aircraft and rocket engineering, appeared the need and the possibility of the upper atmosphere research. Toward the end 50-*th* year , was accumulated large experimental material, which allowed the coordination committee of the AS USSR to project/emerge with the

project of new standard atmosphere. Was first accepted time/temporary standard atmosphere VSA-60, but later - standard atmosphere SA-64 or simply SA.

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The tables of standard atmosphere (GOST 4401-64) are intended for bringing the results of calculations and measurements of the aircraft characteristics and engines and to identical atmospheric conditions, for the graduation of instruments, for use during processing of the results of geophysical and meteorological measurements, etc.

In the SA-64 in function from altitude H, are given the following characteristics of the atmosphere: kinetic temperature T (in °C and °K); the barometric pressure p (kgf/m² and mm Hg); density ρ (in kg/m³ and kg•s²/m⁴); molecular weight M; the speed of sound a (in m/s and km/h); dynamic viscosity μ (in poise and kg•s/m²); the free-fall acceleration in body ν (in m/s²); mean free path of molecules l (m). It is assumed that wind at all height/altitudes is absent.

Depending on the character of a change in the temperature, entire/all atmosphere to height/altitude 200,000 m is broken into 11

layers; for each layer it is accepted that the molecular-scale temperature T_M in the function of geopotential height Φ changes according to linear law with gradient $a_M = \frac{\Delta T_M}{\Delta \Phi}$. Geopotential height is determined from the equation

$$\frac{d\Phi}{dH} = \frac{g_T}{g_{T0}}. \qquad (2.36)$$

Substituting in (2.36) for the spherical model Earth $\frac{g_T}{g_{T0}} = \frac{R_3^2}{(R_3 + H)^2}$, let us have

$$\Phi = \int_0^H \frac{R_3^2}{(R_3 + H)^2} dH.$$

After integration we will obtain the formula, which connects geopotential height with geometric:

$$\Phi = \frac{R_3 H}{R_3 + H}.$$

The values of the gradient of the temperature in the appropriate layers are equal to:

№ слоя (1)	1	2	3	4	5
Интервал высот, м (2)	(3) До 11000	(4) От 11000 до 25000	(4) От 25000 до 46000	(4) От 46000 до 54000	(4) От 54000 до 80000
$a_{м} \cdot 10^{8}$ (град/гп. м) (5)	−651122	0	+276098	0	−349544

Key: (1). No layer. (2). Interval cf height/altitudes, m. (3). To. (4). Frcm to. (5). (deg/geopotential m).

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№ слоя (1)	6	7	8	9	10	11
(2) Интервал высот. м	(3) От 80000 до 95000	(5) От 95000 до 110000	(4) От 110000 до 120000	(5) От 120000 до 150000	(5) От 150000 до 160000	(5) От 160000 до 200000
(4) $a_м 10^6$ (град гп. м)	0	+500000	+801741	+2346357	+1987408	+3084[illegible]

Key: (1). No layer. (2). Interval of height/altitudes, m. (3). From to. (4). (deg/geopotential m).

Kinetic temperature is connected with molecular-scale temperature by the dependence

$$T = T_M \frac{M}{M_0},$$

where M_0 and M - molecular weight cf air at the level of sea and at height/altitude H. For H = 0-95,000 m, molecular weight is constant and equal to $M = M_0$ = 28,966. At height/altitudes it is more than 95,000 m, where continues the process cf the gaseous dissociation of the atmosphere, it decreases. For the determination of molecular weight of air in the interval of heights H = 95,000-110,000 m in SA-64, is accepted the dependence

$$M = 23 + \frac{5,966}{145\,000}\sqrt{145\,000^2 - (H - 95\,000)^2}.$$

while for heights H = 110,000-200,000 m, is given special table (with H = 200,000 m H = 27,000).

Formulas, which determine pressure change with height/altitude, instituted on hypothesis about the vertical equilibrium of the atmosphere. On this hypothesis the weight of the horizontal layer of air of the elementary thickness dH and of unit area is balanced by the elementary difference dp in the pressures, which act on upper and lower the base/root of layer dp = -gρdH.

Utilizing an equation of state

$$p=g\varrho RT, \qquad (2.37)$$

where R - gas constant, equal R = of 29.27 m/deg, we will obtain

$$\frac{dp}{p}=-\frac{1}{R}\frac{dH}{T}.$$

Hence common/general/total formula for p will take the form

$$p=p_*e^{-\frac{1}{R}\int_0^H\frac{dH}{T}} \qquad (2.38)$$

(in formula (2.38) and the subsequent dependences by index * are noted the values of the parameters, which correspond to lower boundary of a layer in question).

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Integral can be taken, if is known dependence T = f (H). We convert (2.38) for geopotential height, after substituting from (2.36)

$$dH = \frac{g_{T0}}{g_T} d\Phi.$$

After this we will obtain

$$p = p_* e^{-\frac{g_{T0}}{R} \int\limits_{\Phi_*}^{\Phi} \frac{d\Phi}{g_T T}}. \qquad (2.39)$$

For isothermal layers (2nd, 4th and the 6th) with $a_M = 0$ and $T = T_M = \text{const}$ after integration (2.39) we will obtain

$$p = p_* e^{-\frac{G_0}{R_{y\text{д}} T}(\Phi - \Phi_*)}, \qquad (2.40)$$

where into exponent is introduced $R_{y\text{д}}$ — the specific gas constant of dry air ($R_{y\text{д}} = 287{,}039$ m²/deg•s) and G_0 - coefficient, numerically equal to the free-fall acceleration in the body at the level of sea and which has dimensionality m²/s•geopotential m.

By analogy with formula for the pressure

$$\varrho = \varrho_* e^{-\frac{G_0}{R_{y\text{д}} T}(\Phi - \Phi_*)}. \qquad (2.41)$$

For layers with the linearly changing temperature (1st, 3rd, 5th, 7th and from the eighth on the 11th)

$$T_M = T_{M*} + a_M(\Phi - \Phi_*);$$

$$p = p_* e^{-\frac{G_0}{a_M R_{y\partial}} \ln \frac{T_M}{T_{M*}}}; \quad (2.42)$$

$$\varrho = \varrho_* e^{-\left(1 + \frac{G_0}{a_M R_{y\partial}}\right) \ln \frac{T_M}{T_{M*}}}. \quad (2.43)$$

The given in SA-64 values of dynamic and kinematic moduli of viscosity were determined by the formulas

$$\mu = \mu_{0,0} \left(\frac{T}{T_0}\right)^{3/2} \frac{T_0 + 110,4}{T + 110,4} \text{ и } \nu = \frac{\mu}{\varrho}.$$

Key: (1). and.

The speed of sound was located from the expression

$$a = \sqrt{kgRT} = 20,0463 \sqrt{T} \text{ м/с},$$

Key: (1). m/s.

and the average/mean length of the free of molecules l, was determined by the formula

$$l = 1,2550 \nu \sqrt{\frac{\varrho}{p}} \text{ м}. \quad (2.44)$$

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In the dependence for μ $\mu_{0 \bullet 0}$ = $1.75 \bullet 10^{-6}$ kg•s/m² with T_0 = 273.16°K (unlike μ_0 with T_0 = 288.15°K).

The initial values of the characteristics of the atmosphere (at the level of sea and geographic latitude 45°32'45") were determined on the basis of experiments and theoretical of dependence and in SA-64 were accepted equal: barometric pressure - p_0 = 10,332.3 kgf/m² = 760 mm Hg; the density ρ_0 = 1.2250 kg/m³ = 0.12492 kg•s²/m⁴; temperature T_0 = 288.15°K (15°C); the speed of sound a_0 = 340.28 m/s; the molecular weight of air M_0 = 28.966; coefficient of dynamic viscosity μ_0 = $1.8247 \bullet 10^{-6}$ kg•s/m²; average radius of the earth R_3 = 6,371,210 m; the free-fall acceleration g_0 = 9.80665 m/s².

These initial conditions was in connection with designed the table of standard atmosphere, containing the average values of the main parameters for height/altitudes from -2000 to 200,000 m. The space of table ΔH (20-5000 m) was selected so that it would be

conveniently used during practical calculations.

Data given in SA-64 for height/altitudes from 200,000 to 300,000 m should examine only as recommended, since at the high altitudes (especially more than 100 km) of the characteristic of the atmosphere (temperature, density, molecular weight, etc.) are subjected to the very strong changes, caused by the oscillations of solar radiation. Tabulated data correspond to the average level of solar activity. Curve/graphs, which show change T, p and ρ in function from height/altitude H (on SA-64), given to Fig. 2.19 and 2.20.

For the small height/altitudes when $H \ll R_3$, it is possible to accept $\Phi \approx H$ and $g_т \approx g_{т.0}$=const. These assumptions make it possible to obtain formulas for the calculation of the values of weather factors directly according to altitude of flight, usually determined during the solution of the problems of external ballistics for rockets and the projectiles, intended for a firing for relatively small distances ($x_c <$ 50,000 м).

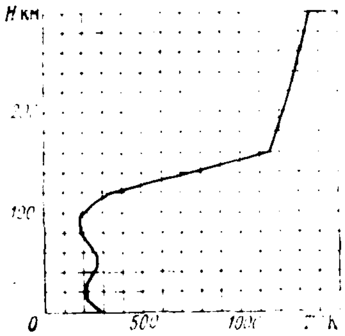

Fig. 2.19. Dependence of temperature on height/altitude for standard atmosphere.

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In artillery practice finds a use the normal artillery atmosphere (NAA). Unlike SA-64 in NAA is considered the air humidity by introduction instead of actual temperature T of conditional temperature - τ. We will obtain the simplified dependences, which were being utilized in NAA for the calculation of weight density and pressure. For weight density $P = \rho g$ dry air from (2.37) we obtain

$$\Pi = \frac{p}{RT}.$$

In NAA instead of the barometric pressure p kgf/m² is introduced

pressure, measured in mm Hg and designated h. Since p = 10,333/760 h = 13.6 h, then respectively

$$\Pi = 13{,}6\ \frac{h}{RT}. \qquad (2.45)$$

Pressure humid air h can be defined as sum of partial pressures dry air h_c and the water vapor e

$$h = h_c + e. \qquad (2.46)$$

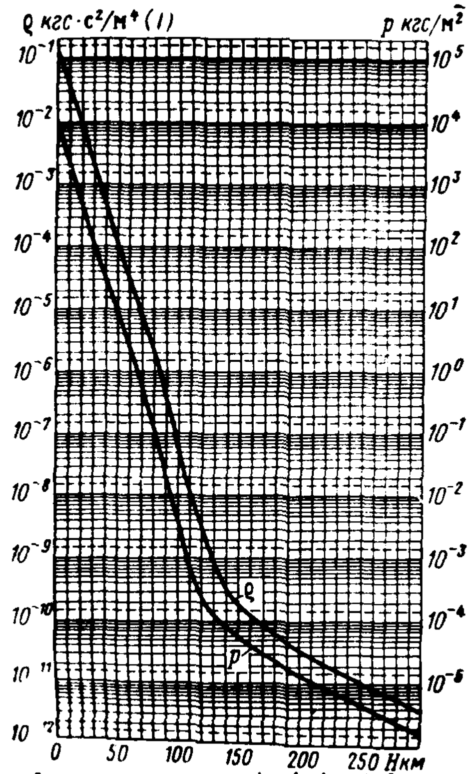

Fig. 2.20. Dependence of density and pressure on height/altitude for standard atmosphere.

Key: (1). $kg \cdot s^2/m^4$. (2). kg/m^2.

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The weight density of humid air per unit of volume (m^3) is located through the similar dependence

$$\Pi = \Pi_c + \Pi_e. \quad (2.47)$$

Counting that with sufficient for practice accuracy, that at identical pressure and the temperature water vapor density it is equal to 5/8 from the density of dry air, we will obtain

$$\Pi_e = \frac{5}{8}\,13{,}6\,\frac{e}{RT}$$

and finally

$$\Pi = 13{,}6\,\frac{h}{RT}\left(1 - \frac{3}{8}\,\frac{e}{h}\right). \qquad (2.48)$$

If necessary to consider air humidity into ballistic calculations is introduced so-called conditional or virtual temperature

$$\tau = \frac{T}{1 - \frac{3}{8}\,\frac{e}{h}}, \qquad (2.49)$$

and subsequently instead of the real humid air, which is characterized by values h, P, T and e, is examined conditional dry air with characteristics h, P and τ; this conditional air exerts the same resistance to the driving/moving in it projectile as actual air. Taking into account τ formula for weight and mass densities, they take the form

$$\Pi = 13{,}6\,\frac{h}{R\tau}, \qquad (2.50)$$

$$\varrho = 13{,}6\,\frac{h}{gR\tau}. \qquad (2.51)$$

Correction into temperature for air humidity is insignificant

$$\delta T \approx 0.3\%.$$

For standard conditions at the level of sea in NAA are accepted: $h_{0N} = 750$ mm Hg; $e_{0N} = 6.35$ mm Hg, that corresponds to 50o/o of relative humidity $\tau_{0N} = 288.8°$ K ($t_{0N} = +15°$ C); Π_{0N} 1,206 kg/m³ $\varrho_{0N} = 0.1229$ kg•s²/m⁴ $a_{0N} = 340.8$ m/s. Just as in other standard atmosphere, it is accepted that the atmosphere on all height/altitudes is located in calm state, i.e., wind is absent. During the determination of a change of the pressure and density with height/altitude in NAA, is accepted the dependence $\tau = f(y)$, obtained D. A. Venttselem on the basis of processing the results of the repeated sourcing of the atmosphere for average summer conditions [9].

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To heights $y \leqslant$ 9300 m, it is accepted

$$\tau = \tau_{0N} - G_1 y. \quad (2.52)$$

where G_1 - the temperature gradient, equal to 0.006328 deg/m = const.

For height/altitudes from 9300 to 12,000 m, it is accepted that G_2 linearly decreases from 0.006328 to zero, i.e.,

$$G_2 = 0.006328 - 0.006328 \frac{y - 9300}{12000 - 9300}. \quad (2.53)$$

A change in the temperature with height/altitude will be determined by the formula

$$\tau=\tau_{9300}-\int_{9300}^{y} G_2 d\tau.$$

The substitution of formula (2.53) and integration makes it possible to obtain formula for changing the temperature with the height/altitude

$$\tau=230,0-0,006328(y-9300)+1,172\cdot 10^{-6}(y-9300)^2.$$

For height/altitudes 12,000, m < y < 31,000 m is accepted τ = as 221.5°K = const.

The values of pressure and density, led to NAA, were calculated from the obtained from (2.38) formula $h=h_0 e^{-\frac{1}{R}\int_0^y \frac{dy}{\tau}}$ and following from (2.45) to the formula

$$\Pi=13,6\frac{h_0}{R\tau}e^{-\frac{1}{R}\int_0^y \frac{dy}{\tau}}. \qquad (2.54)$$

If we relate this density to air density on the surface of the

Earth, then it is possible to obtain the dimensionless function of density change with height/altitude. On the surface Earth

$$\Pi_0 = 13.6\,\frac{h_0}{R\tau_0},$$

then

$$\frac{\Pi}{\Pi_0} = \frac{\tau_0}{\tau}\, e^{-\frac{1}{R}\int_0^y \frac{dy}{\tau}}.$$

Respectively it is possible to write, also, for the pressure

$$\frac{h}{h_0} = e^{-\frac{1}{R}\int_0^y \frac{dy}{\tau}}.$$

For height/altitudes to 9300 m and normal meteorological conditions, i.e., when $h_0 = h_{0N}$ and $\Pi_0 = \Pi_{0N}$, utilizing formula (2.52), we will obtain:

$$\frac{h}{h_{0N}} = \pi(y) = \left(\frac{\tau}{\tau_{0N}}\right)^{\frac{1}{R G_1}} \quad (1) \quad \frac{\Pi}{\Pi_{0N}} = H(y) = \pi(y)\,\frac{\tau_{0N}}{\tau}. \qquad (2.55)$$

Key: (1). and.

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For complex temperature dependences $\tau = f(y)$ the integral in the right side of formulas (2.54) is taken by one of the numerical methods.

According to the given above formulas is calculated a series of useful auxiliary tables for the dimensionless functions

$H(y)$; $\tau_{0N}\tau$; $\sqrt{\tau_{0N}\tau}$; $\pi(y)$ и других (1)

Key: (1). and others.

Along with NAA was utilized also international standard atmosphere MSA, but mainly during the solution of different aviation problems. Parameters of MSA for a zero level are the following:

$p_0 = 760$ мм рт. ст. (1), $T_0 = 288°$ K, $\varrho_0 = 0{,}125$ кгс·с²/м⁴ (2)

Key: (1). mm Hg. (2). kg·s²/m⁴.

To height/altitudes 11,000 m is accepted the linear dependence of a change in the temperature with the height/altitude

$$T = T_0 - 0{,}00654\, y.$$

On the basis of common/general/total formulas (2.42 and 2.43) after a series of transformations it is possible to obtain

$$p = p_0 \left(1 - \frac{y}{44\,300}\right)^{5{,}256} \quad (2.55)$$

and

$$\varrho = \varrho_0 \left(1 - \frac{y}{44\,300}\right)^{4{,}256} \quad (2.56)$$

For height/altitudes from 11000 to 20,000 m, the temperature of air on MSA is accepted by constant and equal T = 216.5°K.

Air humidity in MSA is not considered.

During the solution of the practical tasks, for example, during ballistic design, for calculating the dimensionless function of density change with height/altitude it is convenient to use empirical formulas. Are known the formulas: professor V. P. Vetchinkin

$$H(y) = \frac{20\,000 - y}{20\,000 + y}, \qquad (2.57)$$

linear

$$H(y) = 1 - ky, \qquad (2.58)$$

hyperbolic

$$H(y) = \frac{1}{1 + ky} \qquad (2.59)$$

and exponential

$$H(y) = e^{-ky}. \qquad (2.60)$$

In all formulas y should be taken in m; in last three formulas coefficient k = 0.0001.

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§4. AERODYNAMIC FORCES AND THEIR MOMENTS.

In flight of rocket or projectile in the atmosphere on them, acts air resistance, called aerodynamic.

Aerodynamic drag $\bar{R}$ is composed of the forces of pressure air, directed along normals to the surface of flight vehicle, and the frictional forces of air, tangential to it. Resultant force $\bar{R}$ is applied to flight vehicle at the point which calls center of pressure (Fig. 2.21). Usually center of pressure does not coincide with the center of mass of flight vehicle and during the displacement of force of $\bar{R} = \bar{R}'$ to the center of mass appears torque/moment $\bar{M}_{q}$, formed by force couple $\bar{R}$ and $\bar{R}''$. Force $\bar{R}'$, applied in the center of mass of rocket, calls the main vector of aerodynamic forces, and torque/moment $\bar{M}_{a}$, - by the main moment of aerodynamic forces. In essence the action of aerodynamic force leads to a decrease in the velocity of rocket flight. The action of torque/moment M_{a} causes the rotary motion of rocket around the center of mass. By the study of the phenomena, accompanying interaction of flight vehicle with the

incident airflow, is occupied aerodynamics [35, 36]. Let us here examine only the questions, direct-connected with practical application/use in external ballistics of formulas for the calculation of aerodynamic forces and torque/moments.

It is establish/installed that if we do not consider unsteady flow condition, then the air resistance depends in the first approximation, on form and the size/dimensions of rocket, velocity of its flight v, on the air density ρ and of its ductility/toughness/viscosity μ, on the speed of sound in air a, determined by its temperature and which affects the disturbance propagation in air, and also on the position, occupied by the rocket of relative direction of its motion along trajectory, characterized by the angles of attack α and of slip β, i.e.,

$$R=f(d, v, \varrho, \mu, a, \alpha, \beta), \qquad (2.61)$$

where d - a significant dimension of rocket (for example, the diameter of the greatest cross section of body which frequently calls maximum cross section or simply by midsection).

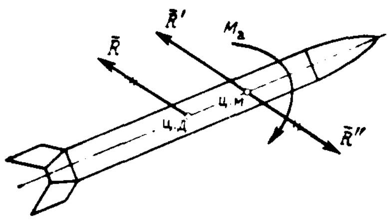

Fig. 2.21. Schematic of bringing the main vector of aerodynamic forces to the center of mass of rocket for two-dimensional problem.

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With the aid of the theory of similitude and dimensionality, functional dependence (2.61) is converted so that the independent variables become dimensionless and their number decreases. As a result s obtained the formula

$$R=\frac{\varrho v^2}{2}\frac{\pi d^2}{4}\varphi\left(\frac{vd\varrho}{\mu};\ \frac{v}{a};\ \alpha,\ \beta\right), \qquad (2.62)$$

where φ - a sign of certain functional dependence.

More frequent instead of (2.62) is utilized the formula

$$R=qSc_R(M,\ \mathrm{Re},\ \alpha,\ \beta), \qquad (2.63)$$

where $q=\frac{\varrho v^2}{2}$ — velocity head of the incident undisturbed flow;

S - area of maximum cross section of rocket;

c_R — dimensionless aerodynamic coefficient, depending on Mach number $M=\frac{v}{a}$, Reynolds number $\frac{v l \rho}{\mu}$, angles α and β.

If during calculation R render/shows necessary to consider a change in the angles α and β in time or the angular velocities of the rotation of rocket ω_i, i.e. the unsteady flow condition of missile body of air, then it is necessary under the sign of function in formulas (2.61)-(2.63) to introduce values $\dot{\alpha}$, $\dot{\beta}$, ω_i and time t.

Investigation with the aid of the theory of similitude and dimensionality made it possible to also obtain following formula for the value of the main moment of aerodynamic forces M_a:

$$M_a = qSlm, \qquad (2.64)$$

where, besides previously named values, l — characteristic length of rocket (for example, the length of rocket from its bottom to the apex of the cone); m - dimensionless aerodynamic coefficient, depending on the form of rocket, its position on the trajectory, the velocity of rotary motion, time, etc.

For separate comprising $\bar{R}$ and $\bar{M}_a$ also are applied the dependences, similar (2.63) and (2.64).

Is decomposed the main vector of aerodynamic forces to components along the axes of flow $(0xyz)$ and connected $(0x_1y_1z_1)$ the systems of coordinates (Fig. 2.22). These comprising have following designations and the names:

a) in the continuous operation of the coordinates: X - drag; Y - lift; Z - lateral force;

b) in body coordinate system: X_1 - longitudinal or tangential resisting force; Y_1 - normal force; Z_1 - lateral or transverse force.

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Those comprise of the main moment of aerodynamic forces, undertaken along the axes of flow and body-fixed systems, have following designations and the names: M_x and M_{x_1} — moments of roll respectively in flow and body coordinate systems; M_y and M_{y_1} — yawing moments; M_z and M_{z_1} — pitching moments (sometimes them they call pitching moments).

Knowing comprising $\bar{R}$ or $\bar{M}_a$ in any coordinate system, we easily

find their complete values. For example,

$$R=\sqrt{X^2+Y^2+Z^2}.$$

Directions $\overline{R}$ and $\overline{M}_a$ are determined each by three angles which can be determined by the values of their cosines, with respect equal (see Fig. 2.22):

$$\cos(\widehat{\overline{x};\overline{R}})=\frac{X}{R};\quad \cos(\widehat{\overline{y};\overline{R}})=\frac{Y}{R};$$

$$\cos(\widehat{\overline{z};\overline{R}})=\frac{Z}{R},$$

or

$$\cos(\widehat{\overline{x}_1;\overline{M}_a})=\frac{M_{x1}}{M_a};$$

$$\cos(\widehat{\overline{y}_1;\overline{M}_a})=\frac{M_{y_1}}{M_a};\quad \cos(\widehat{\overline{z}_1;\overline{M}_a})=\frac{M_{z_1}}{M_a}.$$

Since of majority comprising R and $\overline{M}_a$ the names coincide, it is compulsorily necessary to give not only values these of those comprise, but also to specify, to which coordinate system they are related.

In aerodynamic designs conveniently to deal not with force component and torque/moments, but with their coefficients. In

accordance with formulas (2.63) and (2.64). it is possible to write:

$$\left.\begin{aligned} X&=qSc_x; & M_x&=qSlm_x;\\ Y&=qSc_y; & M_y&=qSlm_y;\\ Z&=qSc_z; & M_z&=qSlm_z, \end{aligned}\right\} \qquad (2.65)$$

where $c_x, c_y, c_z, m_x, m_y, m_z$ — the corresponding dimensionless force coefficients and torque/moments in the continuous operation of coordinates.

Comprising $\overline{R}$ and $\overline{M}_a$ in that connected of coordinates are determined from formulas, similar formulas (2.65), but by that having index "1" in the designations force component and torque/moments and their coefficients. The names coefficients connect the designations of those force component and torque/moments which they determine. For example: c_x — drag coefficient; c_{x_1} — coefficient of longitudinal force; m_z — pitching-moment coefficient in the continuous operation of coordinates; m_{z_1} — pitching-moment coefficient in body coordinate system.

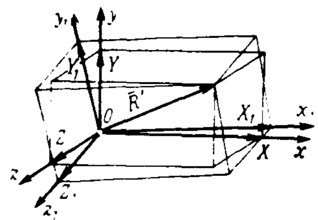

Fig. 2.22. Expansion of the main vector of aerodynamic forces into those comprise in the connected and continuous operations of coordinates.

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When conducting of theoretical calculations and experiments in wind tunnels, aerodynamic coefficients are determined in any coordinate system.

Passage from one coordinate system to another is realized on formulas (2.6). From Fig. 2.22 are obvious the dependences between aerodynamic force components in the connected and continuous operations of the coordinates

$$\left.\begin{aligned}
X &= X_1\cos(\widehat{\bar{x}_1;\bar{x}}) + Y_1\cos(\widehat{\bar{y}_1;\bar{x}}) + Z_1\cos(\widehat{\bar{z}_1;\bar{x}});\\
Y &= X_1\cos(\widehat{\bar{x}_1;\bar{y}}) + Y_1\cos(\widehat{\bar{y}_1;\bar{y}}) + Z_1\cos(\widehat{\bar{z}_1;\bar{y}});\\
Z &= X_1\cos(\widehat{\bar{x}_1;\bar{z}}) + Y_1\cos(\widehat{\bar{y}_1;\bar{z}}) + Z_1\cos(\widehat{\bar{z}_1;\bar{z}})
\end{aligned}\right| \qquad (2.66)$$

The values of aerodynamic forces differ from the aerodynamic coefficients of these forces to constant value qS, while the values of torque/moments from moment coefficients - to a constant value qSl, therefore formula, of one coordinate system into another, they will be the same as for the conversion of forces themselves and torque/moments. For example, for the conversion of the aerodynamic coefficients, found in body coordinate system, in connection with continuous operation, on the basis of formulas (2.66) it is possible to write:

$$\left.\begin{aligned} c_x &= c_{x_1}\cos(\widehat{x_1;x}) + c_{y_1}\cos(\widehat{y_1;x}) + c_{z_1}\cos(\widehat{z_1;x}); \\ c_y &= c_{x_1}\cos(\widehat{x_1;y}) + c_{y_1}\cos(\widehat{y_1;y}) + c_{z_1}\cos(\widehat{z_1;y}); \\ c_z &= c_{x_1}\cos(\widehat{x_1;z}) + c_{y_1}\cos(\widehat{y_1;z}) + c_{z_1}\cos(\widehat{z_1;z}). \end{aligned}\right| \quad (2.67)$$

The motion of flight vehicle in air, strictly speaking, it is not possible to consider being steady; therefore the aerodynamic

force coefficients and torque/moments must depend, besides the values, given in formula (2.61) even from the characteristics of unsteady flow condition.

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For example, for the coefficients of lift and lateral forces taking into account control forces it is possible to write these functional dependences:

$$\left.\begin{array}{l} c_y = f(\alpha, \beta, M, Re, \delta_{y_1}, \delta_{z_1}, \bar{\dot{\alpha}}, \bar{\dot{\beta}}, \bar{\dot{\delta}}_{y_1}, \bar{\dot{\delta}}_{z_1}, \bar{\omega}_x, \bar{\omega}_y, \bar{\omega}_z, t \ldots); \\ c_z = f(\alpha, \beta, M, Re, \delta_{y_1}, \delta_{z_1}, \bar{\dot{\alpha}}, \bar{\dot{\beta}}, \bar{\dot{\delta}}_{y_1}, \bar{\dot{\delta}}_{z_1}, \bar{\omega}_x, \bar{\omega}_y, \bar{\omega}_z, t \ldots), \end{array}\right\} \quad (2.68)$$

where $\delta_{y_1}, \delta_{z_1}$ — angles of rotation of the controls, governing change lateral and lift, respectively around axes oy_1 and oz_1, body coordinate system;

$\bar{\omega}_x, \bar{\omega}_y, \bar{\omega}_z$ — given angular rates of rotation of flight vehicle of the relatively fixed earth's axis;

$\bar{\dot{\alpha}}, \bar{\dot{\beta}}$ - given angular rates of rotation of flight vehicle of relatively drag axes;

$\bar{\dot{\delta}}_{y_1}, \bar{\dot{\delta}}_{z_1}$ given angular rates of rotation of controls.

Given angular velocities indicated are values dimensionless,

equal to

$$\bar{\alpha}=\alpha\frac{l}{v};\quad \bar{\beta}=\dot{\beta}\frac{l}{v};\quad \bar{\delta}_{y_1}=\dot{\delta}_{y_1}\frac{l}{v};\quad \bar{\delta}_{z_1}=\dot{\delta}_{z_1}\frac{l}{v};$$
$$\bar{\omega}_x=\omega_x\frac{l}{v};\quad \bar{\omega}_y=\omega_y\frac{l}{v};\quad \bar{\omega}_z=\omega_z\frac{l}{v}\ \text{и т. п. (1)}\qquad (2.69)$$

to Key: (1). and the like.

Aerodynamic moments can be presented as products

$$M_{z_1}=Y_1\Delta l;\quad M_{y_1}=Z_1\Delta l,\qquad (2.70)$$

where Δl - distance between centers of masses and the center of pressure.

Accepting for Y_1 and Z_1 the dependences, similar (2.65) and (2.68), we will obtain expressions for torque/moments through the coefficients of the corresponding aerodynamic forces

$$M_{z_1}=qSl\frac{\Delta l}{l}c_{y_1}(\alpha, \beta, M, Re, \delta_{y_1}, \delta_{z_1}, \bar{\alpha}, \bar{\beta}\ldots);$$
$$M_{y_1}=qSl\frac{\Delta l}{l}c_{z_1}(\alpha, \beta, M, Re, \delta_{y_1}, \delta_{z_1}, \bar{\alpha}, \bar{\beta}\ldots).\qquad (2.71)$$

Only in the simplest cases (motion with constant $\bar{V}$, α, β or flow around bodies by steady flow) value Δl is constant and can easily be determined by experimental and calculations. With various kinds oscillations and the sharp maneuvering of flight vehicle the flow, which flows around it, will become unsteady, the center-of-pressure

location will be changed and respectively it will be be changed value Δl, formula for determining which will be converted the complex functional dependence. Therefore during the determination of the torque/moments, which act on flight vehicle in flight, it is accepted to find for them the aerodynamic coefficients which already connect relation $\Delta l/l$ in implicit form. In this case, expression for torque/moments it is analogous (2.64), is obtained the form:

$$\left.\begin{aligned} M_{z_1} &= qSlm_{z_1}(\alpha, \beta, M, Re, \delta_{y_1}, \delta_{z_1}, \bar{\dot{\alpha}}, \bar{\dot{\beta}}, \bar{\dot{\delta}}_{y_1}, \bar{\dot{\delta}}_{z_1}, \bar{\omega}_{y_1}, \ldots); \\ M_{y_1} &= qSlm_{y_1}(\alpha, \beta, M, Re, \delta_{y_1}, \delta_{z_1}, \bar{\dot{\alpha}}, \bar{\dot{\beta}}, \bar{\dot{\delta}}_{y_1}, \bar{\dot{\delta}}_{z_1}, \bar{\omega}_{y_1}, \ldots) \ldots \end{aligned}\right\} \quad (2.72)$$

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Obtaining for the coefficients of the aerodynamic moments of the formulas which would reflect all the determining factors, and their use represents great difficulties; therefore usually are considered the factors, which have on torque/moments only essential effect.

The conducted in aerodynamics investigations showed that for the pitching-moment coefficient in body coordinate system it is possible to be restricted to the dependence

$$m_{z_1} = f(\alpha, \delta_{z_1}, \bar{\omega}_{z_1}, \bar{\dot{\alpha}}, \bar{\dot{\delta}}_{z_1}). \quad (2.73)$$

Formula (2.73) is valid during small changes in the Mach number; however, it is necessary to bear in mind, that during considerable change of M the form curved $m_{z_1}(\alpha)$ substantially changes and this must be considered.

We convert the common/general/total expression for m_{z_1} to more convenient for calculations form.

Equalizing total differential for the sum of the partial differentials replacing approximately $dm_{z_1}\approx\Delta m_{z_1}=m_{z_1}-m_{z_{1,0}}$, we will obtain

$$m_{z_1}=m_{z_{1,0}}+\frac{\partial m_{z_1}}{\partial\alpha}\alpha+\frac{\partial m_{z_1}}{\partial\delta_{z_1}}\delta_{z_1}+$$
$$+\frac{\partial m_{z_1}}{\partial\bar{\omega}_{z_1}}\bar{\omega}_{z_1}+\frac{\partial m_{z_1}}{\partial\dot{\bar{\alpha}}}\dot{\bar{\alpha}}+\frac{\partial m_{z_1}}{\partial\dot{\bar{\delta}}_{z_1}}\dot{\bar{\delta}}_{z_1}. \qquad (2.74)$$

Here $m_{z_{1,0}}$ -- value of the aerodynamic coefficient of zero values α, δ_{z_1}, ω_{z_1}, $\dot{\alpha}$ and $\dot{\delta}_{z_1}$.

Partial derivatives on the angles

$$m^{\alpha}_{z_1}=\frac{\partial m_{z_1}}{\partial\alpha},\quad m^{\delta_{z_1}}_{z_1}=\frac{\partial m_{z_1}}{\partial\delta_{z_1}}$$

are called by static derivative.

Derivatives on angular velocities $m_{z_1}^{\bar{\omega}_{z_1}} = \frac{\partial m_{z_1}}{\partial \bar{\omega}_{z_1}}$; $m_{z_1}^{\bar{\dot{\alpha}}} = \frac{\partial m_{z_1}}{\partial \bar{\dot{\alpha}}}$; $m_{z_1}^{\bar{\dot{\delta}}_{z_1}} = \frac{\partial m_{z_1}}{\partial \bar{\dot{\delta}}_{z_1}}$ are called the rotary derivatives.

In simplified recording we will obtain

$$m_{z_1} = m_{z_{1,0}} + m_{z_1}^{\alpha}\alpha + m_{z_1}^{\delta_{z_1}}\delta_{z_1} + m_{z_1}^{\bar{\omega}_{z_1}}\bar{\omega}_{z_1} + m_{z_1}^{\bar{\dot{\alpha}}}\bar{\dot{\alpha}} + m_{z_1}^{\bar{\dot{\delta}}_{z_1}}\bar{\dot{\delta}}_{z_1}. \quad (2.75)$$

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For the yawing-moment coefficient is applied the formula, similar written for the pitching-moment coefficient

$$m_{y_1}=m_{y_1}^{\beta}\beta+m_{y_1}^{\delta_{y_1}}\delta_{y_1}+m_{y_1}^{\bar{\omega}_{x_1}}\bar{\omega}_{x_1}+$$
$$+m_{y_1}^{\bar{\omega}_{y_1}}\bar{\omega}_{y_1}+m_{y_1}^{\dot{\bar{\beta}}}\dot{\bar{\beta}}+m_{y_1}^{\dot{\bar{\delta}}_{y_1}}\dot{\bar{\delta}}_{y_1}. \quad (2.76)$$

Here the static derivatives are equal to

$$m_{y_1}^{\beta}=\frac{\partial m_{y_1}}{\partial\beta};\quad m_{y_1}^{\delta_{y_1}}=\frac{\partial m_{y_1}}{\partial\delta_{y_1}},$$

the rotary derivatives -

$$m_{y_1}^{\bar{\omega}_{x_1}}=\frac{\partial m_{y_1}}{\partial\bar{\omega}_{x_1}};\quad m_{y_1}^{\bar{\omega}_{y_1}}=\frac{\partial m_{y_1}}{\partial\bar{\omega}_{y_1}};\quad m_{y_1}^{\dot{\bar{\beta}}}=\frac{\partial m_{y_1}}{\partial\dot{\bar{\beta}}};\quad m_{y_1}^{\dot{\bar{\delta}}_{y_1}}=\frac{\partial m_{y_1}}{\partial\dot{\bar{\delta}}_{y_1}}.$$

At zero values $\beta;\ \delta_{y_1};\ \omega_{x_1};\ \omega_{y_1};\ \dot{\beta};\ \dot{\delta}_{y_1}$ aerodynamic coefficient $m_{y_1 0}=0$, therefore it is not included in formula (2.76).

In formulas (2.75) and (2.76) the terms, which contain the angular rates of rotation $\dot{\alpha}$, $\dot{\beta}$ and ω_i, characterize air resistance to oscillations and the rotation of flight vehicle. This resistance causes the attenuation (damping) of rotary displacement/movements. Therefore the rotary derivatives can be named also damping

derivatives.

Numerical values of named derivatives depend on aerodynamic layout, design concept, the size/dimensions of flight vehicle and from Mach number. Of unguided rockets $\delta_{z_1}=\delta_{y_1}=\dot{\delta}_{z_1}=\dot{\delta}_{y_1}=0$ and then

$$\left.\begin{aligned} m_{z_1} &= m_{z_{1,0}} + m_{z_1}^{\alpha}\alpha + m_{z_1}^{\bar{\dot{\alpha}}}\bar{\dot{\alpha}} + m_{z_1}^{\bar{\omega}_{z_1}}\bar{\omega}_{z_1}; \\ m_{y_1} &= m_{y_1}^{\beta}\beta + m_{y_1}^{\bar{\dot{\beta}}}\bar{\dot{\beta}} + m_{y_1}^{\bar{\omega}_{x_1}}\bar{\omega}_{x_1} + m_{y_1}^{\bar{\omega}_{y_1}}\bar{\omega}_{y_1}. \end{aligned}\right\} \quad (2.77)$$

The moment of roll is equal to

$$M_{x_1} = qSlm_{x_1}.$$

Of the work of controls and the fixed Mach number, dimensionless coefficient m_{x_1} is determined by the following functional dependence:

$$m_{x_1} = f(\alpha, \beta, \delta_{z_1}, \delta_{y_1}, \delta_{x_1}, \bar{\omega}_{x_1}, \bar{\omega}_{y_1}, \bar{\omega}_{z_1}). \quad (2.78)$$

Here, besides known values, δ_{x_1} - the angle of deflection of controls of bank from their free positions.

The expansion/disclosure of functional dependence upon consideration of the mutual effect of the determining parameters leads to complex common/general/total expression for the rolling-moment coefficient [36]:

$$\begin{aligned} m_{x_1} = m_{x_{1,0}} + m_{x_1}^{\beta}\beta + m_{x_1}^{\delta_{y_1}}\delta_{y_1} + m_{x_1}^{\delta_{x_1}}\delta_{x_1} + \\ + \frac{\partial^2 m_{x_1}}{\partial\alpha\partial\beta}\alpha\beta + \frac{\partial^2 m_{x_1}}{\partial\alpha\partial\delta_{y_1}}\alpha\delta_{y_1} + \frac{\partial^2 m_{x_1}}{\partial\beta\partial\delta_{z_1}}\beta\delta_{z_1} + \\ + m_{x_1}^{\bar{\omega}_{x_1}}\bar{\omega}_{x_1} + m_{x_1}^{\bar{\omega}_{y_1}}\bar{\omega}_{y_1} + \frac{\partial^2 m_{x_1}}{\partial\alpha\partial\bar{\omega}_{y_1}}\alpha\bar{\omega}_{y_1} + \frac{\partial^2 m_{x_1}}{\partial\beta\partial\bar{\omega}_{z_1}}\beta\bar{\omega}_{z_1}. \end{aligned} \quad (2.79)$$

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Here $m_{x_1 0}$ - comprising the rolling-moment coefficient, determined by aerodynamic asymmetry of flight vehicle; $m_{x_1}^{\beta}\beta$, $m_{x_1}^{\delta}\delta$ - comprising coefficient, determined by slip and the deflection of rudder; $m_{x_1}^{\delta_{x_1}}\delta_{x_1}$ - comprising coefficient, determined by the deflection of the controls, governing the bank of flight vehicle; $\frac{\partial^2 m_{x_1}}{\partial\alpha\,\partial\beta}\alpha\beta$, $\frac{\partial^2 m_{x_1}}{\partial\alpha\partial\delta_{y_1}}\alpha\delta_{y_1}$, $\frac{\partial^2 m_{x_1}}{\partial\beta\partial\delta_{z_1}}\beta\delta_{z_1}$ - composing the rolling-moment coefficient, the determined by mutual wing influence and tail assemblies; $m_{x_1}^{\bar{\omega}_{x_1}}\bar{\omega}_{x_1}$ - coefficient of the damping moment of roll, created by wings and tail assembly; $m_{x_1}^{\bar{\omega}_{y_1}}\bar{\omega}_{y_1}$, $\frac{\partial^2 m_{x_1}}{\partial\alpha\partial\bar{\omega}_{y_1}}\alpha\bar{\omega}_{y_1}$, $\frac{\partial^2 m_{x_1}}{\partial\beta\partial\bar{\omega}_{z_1}}\beta\bar{\omega}_{z_1}$ - comprising of the spiral moment of roll, which appears during the rotation of flight vehicle around axes Oy_1 and Oz_1.

As can be seen from common formula (2.79), the rolling-moment coefficient to a considerable degree is determined by so-called cross aerodynamic communication/connections. The mutual effect of longitudinal and yawing motions is exhibited especially strongly with bank and rotation of relatively longitudinal axis of flight vehicle

with the developed lifting surfaces.

In certain cases the vector of the main moment of aerodynamic forces it is convenient to represent by the sum

$$\overline{M}_a = \overline{M}_{ст} + M_д, \qquad (2.80)$$

where $\overline{M}_{ст}$ - stabilizing or tilting moment; $\overline{M}_д$ - damping moment.

The name of torque/moment $M_{ст}$ depends on the direction of its action in the plane of the angle of attack α or sideslip β. If torque/moment increases these angles, it is called inverting, if it decreases - that stabilize. The torque/moment of damping $M_д$ always acts against direction of rotation during oscillations relative to the center of mass. It attempts to inhibit (to extinguish) the oscillations of rocket and its rotation.

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The separately undertaken torque/moment of damping on the basis of theory of similitude can be represented by the expression

$$M_{д i} = qSl\, m_i^{\bar{\omega}_i} \frac{\omega_i l}{v} = qSl\, m_i^{\bar{\omega}_i} \bar{\omega}_i, \qquad (2.81)$$

where $m_i^{\bar{\omega}_i}$ - the corresponding rotary derivative.

In certain cases the torque/moments of damping represent these depend on the first degree of flight speed:

$$M_{x_i} = qSl\,m_i^{\bar{\omega}_i}\,\bar{\omega}_i = S\varrho v l^2\,m_{x_i}\,\omega_i. \qquad (2.82)$$

Here m_{x_i} they call the damping factor; it is equal to

$$m_{x_i} = \frac{1}{2}\,m_i^{\bar{\omega}_i}.$$

During the solution of the problems of spatial motion, the damping factor must be determined by all three axes $m_{x x_1}, m_{x y_1}, m_{x z_1}$. The damping factors and the torque/moments of damping can be represented as sums of the corresponding coefficients of the basic structural/design assemblies of flight vehicle. For example, during the determination of the damping moment of the rocket or aircraft configuration relative to axis Oz_1 common/general/total damping moment is defined as sum of the damping moments of tail assembly, wings and body. On rockets and projectiles, not having sharply protruding surfaces, i.e., wingless and finless, the factor of damping body will be differing by equatorial axes, but the difference in the damping moments will be determined by a difference in values ω_{z_1} and ω_{y_1}.

In the process of undisturbed balanced rocket flight (or fin-stabilized projectile) occur smooth charge of the angle attack and slip. In this case, as showed many examinations,

angular velocities of rotary motion barely affect the value of the main vector of aerodynamic forces and its components. Therefore aerodynamic coefficients for hoisting and lateral forces are frequently determined without the account of the angular rates of rotation, which were being included by us more complete functional dependences (2.68), but compulsorily are considered them during the determination of moment coefficients. It is accepted also for simplification to count that with low α and β the lifting and lateral forces do not depend on each other. Taking into account the noted simplifications they obtain

$$c_y = c_{y_0} + c_y^{\alpha}\alpha + c_y^{\delta_{z_1}}\delta_{z_1}; \quad (2.83)$$

$$c_z = c_z^{\beta}\beta + c_z^{\delta_{y_1}}\delta_{y_1}. \quad (2.84)$$

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In axisymmetric aerodynamic configurations $c_{y_0}=0$. Furthermore, during performance calculation of the action of axisymmetric flight vehicles in the case when the control devices of the relatively body axes of coordinates occupy the fixed/recorded position, governing aerodynamic force it is possible not to separately consider, but to include them in those components of total aerodynamic force. In this case the guided missile will be equivalent unguided, and then

$$Y = \frac{\rho v^2}{2} S c_y^{\alpha}\alpha; \qquad Z = \frac{\rho v^2}{2} S c_z^{\beta}\beta; \quad (2.85)$$

$$M_{z_1} = \frac{\rho v^2}{2} S l m_{z_1}^{\alpha}\alpha; \qquad M_{y_1} = \frac{\rho v^2}{2} S l m_{y_1}^{\beta}\beta. \quad (2.86)$$

The drag coefficient in general form can be presented thus:

$$c_x = f(M, Re, \alpha, \beta).$$

Dependence c_x on Re begins noticeably to be exhibited during considerable change in altitude of flight; for the account to this dependence, is necessary conversion c_x to different height/altitudes. The account of effect α and β is carried out through the complex coefficient of inductive resistance

$$c_{x_i}=c_{x_i}(\beta)+c_{x_i}(\alpha). \qquad (2.87)$$

Complete drag coefficient

$$c_x=c_{x_0}+c_{x_i}, \qquad (2.88)$$

where c_{x_0} - drag coefficient during the coincidence of longitudinal axis and velocity vector of the center of mass of flight vehicle, i.e., of condition $\alpha=\beta=0$.

For the flight vehicles, which have the relatively planar trajectories, for example, for surface-to-surface missiles, when it is possible to count $\beta\approx 0$, they are most frequently limited pain simple dependence

$$c_x=c_{x_0}+c_{x_i}(\alpha). \qquad (2.89)$$

In flight with relatively small angles of attack

$$X=qSc_x(M). \qquad (2.90)$$

For the flight vehicles, close in aercsynasic shape, the graph/diagrams of dependences $c_x(M)$ prove tc be also close that it makes it possible in a series cf the cases tc use in the initial stage cf design the aerodynamic coefficients, determined for thcse exist, it is good themselves reccssended, cbjects.

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Since identical similarity not always can be had, then into calculation is introduced factor cf proporticnality i, called factcr cf the form

$$i=\frac{c_x\left(\frac{v}{a}\right)}{c_{x\,\text{эт}}\left(\frac{v}{a}\right)}, \qquad (2.91)$$

where $c_x\left(\frac{v}{a}\right)$ - an unknown aerodynamic coefficient for the newly design/projected object; $c_{x\,\text{эт}}\left(\frac{v}{a}\right)$ - aerodynamic ceefficient known, aerodynamically similar to that design/projected it is good itself the reccmmended object (stardard).

If it is known $c_{x\,\text{эт}}\left(\frac{v}{a}\right)$ for any rocket (cr projectile), then for the rocket, close in aercdyramic shape, it is possible to accept

$$X=\frac{\rho v^2}{2}Sic_{x\,\text{эт}}\left(\frac{v}{a}\right). \qquad (2.92)$$

If the forms of rockets (projectiles) and of flight condition not are identically similar, then, obviously, i≠1 depends on ratio v/a. To accurately establish this dependence is possible only either experimentally or, after obtaining theoretically value $c_x\left(\frac{v}{a}\right)$ for the newly design/projected object, but in this case, drops off the advantage of introduction i. Therefore it is accepted to take the value of factor of form i for the calculation of this trajectory by a constant value, determining it approximately.

The numerical value i depends on the form of the newly design/projected object and values $c_x\left(\frac{v}{a}\right)$ for known standard objects (rockets, the projectiles), with which new object is compared. Therefore it is always necessary to indicate that in connection with which standard or as it occasionally referred to as, to the standard law of air resistance $c_{x_{\text{эт}}}\left(\frac{v}{a}\right)$ is determined coefficient i. It is convenient to utilize i as the coefficient of the agreement of calculation regarding firing distance with experiment. In this case i will consider not only form of projectile, but also all factors, not reflected in this calculation, for example, the motion of projectile relative to the center of mass. By this it is possible to explain certain difference in the numerical values of form factor, determined on the basis of experimental data, obtained by the firing one and the

same projectile, but at different speeds and angles of departure.

In equation (2.92) enters mass density ρ kg.s²/m. If we pass to weight density Π kG/m³, we will obtain

$$X=\frac{\pi d^2}{8}\,\frac{\Pi}{g}\,v^2 i\,c_{x\,\text{эт}}\left(\frac{v}{a}\right). \qquad (2.93)$$

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Let us multiply and let us divide the right side of equation (2.93) by product $Q\Pi_{0N}\cdot 10^3$. As a result let us have

$$X=\frac{Q}{g}\,\frac{id^2}{Q}\,10^3\,\frac{\Pi}{\Pi_{0N}}\cdot\frac{\pi}{8\cdot 10^3}\,\Pi_{0N}v^2 c_{x\,\text{эт}}\left(\frac{v}{a}\right).$$

In the obtained dependence factor $(id^2/Q)\,10^3=c$ is called of ballistic coefficient, and the term

$$F(v)=\frac{\pi}{8\cdot 10^3}\,\Pi_{0N}\,v^2 c_{x\,\text{эт}}\left(\frac{v}{a}\right)=4{,}74\cdot 10^{-4}v^2 c_{x\,\text{эт}}\left(\frac{v}{a}\right) \qquad (2.94)$$

is function from the air resistance force.

After the appropriate replacement we will obtain the convenient formula

$$X=mcH(y)F(v), \qquad (2.95)$$

in which product $cH(y)F(v)$ has the physical sense of the acceleration of projectile from the action of the air resistance force and is

designated through J.

During the use of formula (2.95) for rockets on the section of the operation of engine (powered flight) it is necessary to keep in mind that m and c are alternating/variable and equal to

$$m=\frac{Q-\int_0^t Q_{\text{сек}}\,dt}{g};\qquad (2.96)$$

$$c=\frac{id^2\,10^3}{Q-\int_0^t Q_{\text{сек}}\,dt},\qquad (2.97)$$

where $Q_{\text{сек}}$ - weight gas flow rate per second from the engine of rocket. After replacing in (2.95) and reducing, we will obtain for the flight vehicle whose mass is alternating/variable,

$$X=\frac{id^2}{g}\,10^3 H(y)F(v).\qquad (2.98)$$

Sometimes instead of function F(v) are utilized the functions

$$G(v)=\frac{F(v)}{v}=4{,}74\cdot 10^{-4}vc_{x\,\text{эт}}\frac{v}{a}\qquad (2.99)$$

or

$$K\left(\frac{v}{a}\right)=\frac{F(v)}{v^2}=4{,}74\cdot 10^{-4}c_{x\,\text{эт}}\left(\frac{v}{a}\right).\qquad (2.100)$$

For the rockets (projectiles), different in form, obviously, are different functions F(v) and G(v). When conducting of ballistic calculations, the numerical value of factor of form i must be selected in connection with the utilized function F(v) or G(v).

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For the projectiles of barrel artillery pieces, are known empirically established/installed standard functions $F(v)$, occasionally referred to as, not it is entirely correct, by the laws of air resistance.

1. Euler during the solution of the problem of the flight of projectile uses function $F(v)=Bv^2$, establish/installed by I. N'yuton and used in essence for subsonic speeds.

For the projectiles, driving/moving with high speeds (to 1000 m/s), Russian artillerymen N. V. Mayevskiy and N. A. Zabudskiy will establish the law of form $F(v) = Bv^n$, in which $n=f(v)$, and will determine values of B and n for the accepted by them intervals of velocities. Are known also the law of the Italian artilleryman Siacci, the law, obtained by the French artillerymen of Garnier and Dupui (1921-1923), the Soviet law of 1930 and the law of the artillery academy im. Dzerzhinskii (law of 1943). Empirical dependence for the standard functions $F(v)$ and $G(v)$ most frequently are assigned in the form of special tables.

During use in calculations of the already available tables for

F(v) and G(v) and during the calculation of tables for new standard functions one should consider that the speed of sound a depends on the temperature of air and, therefore, is changed with height/altitude. As is known,

$$a = \sqrt{kgR\tau},$$

where k - index of polytropic process.

If we for relatively short trajectories ($x_c \leqslant 50$ km) accept g = const, and k - being independent of temperature, then it will seem that the speed of sound is proportional $\sqrt{\tau}$. In order to have tables of functions F(v) or G(v) with one entry, it is necessary to count them for one speed of sound, but to preserve in this case the equality

$$c_x\left(\frac{v}{a}\right) = c_x\left(\frac{v_\tau}{a_{0N}}\right),$$

or

$$\frac{v}{a} = \frac{v_\tau}{a_{0N}}, \qquad (2.101)$$

where a_{0N} - speed of sound under standard conditions, taken to constant during the calculation of tables; v_τ - conditional tabular speed.

It is obvious that

$$v_\tau = v\,\frac{a_{0N}}{a} = v\sqrt{\frac{\tau_{0N}}{\tau}}. \qquad (2.102)$$

Consequently, utilizing formula (2.94), we will obtain

$$F(v)=4{,}74\cdot 10^{-4}v_\tau^2\frac{\tau}{\tau_{0N}}c_{x\,\text{эт}}\left(\frac{v_\tau}{a_{0N}}\right).$$

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Designating

$$F(v_\tau)=4{,}74\cdot 10^{-4}v_\tau^2 c_{x\,\text{эт}}\left(\frac{v_\tau}{a_{0N}}\right),$$

we will obtain

$$F(v)=\frac{\tau}{\tau_{0N}}F(v_\tau). \qquad (2.103)$$

Respectively

$$G(v)=\sqrt{\frac{\tau}{\tau_{0N}}}G(v_\tau) \qquad (2.104)$$

and, after comparison (2.103) and (2.104),

$$G(v_\tau)=\frac{F(v_\tau)}{v_\tau}. \qquad (2.105)$$

Upon consideration of air humidity by means of the introduction of virtual temperature its weight density is determined by formula (2.45).

With this function of density the change with height/altitude can be represented thus:

$$H(y)=\frac{\Pi}{\Pi_{0N}}=\frac{h}{h_{0N}}\,\frac{\tau_{0N}}{\tau},$$

i.e. it depends on the relation of virtual temperatures and pressures. If we now return to formula (2.95) and to replace in it F(v) and H(y), then we will obtain

$$X=mc\frac{h}{h_{0N}}F(v_\tau), \qquad (2.106)$$

or

$$X=mc\pi(y)F(v_\tau). \qquad (2.107)$$

If we in (2.95) with the aid of (2.104) introduce function $G(v_\tau)$, then we will obtain

$$X=mcH_\tau(y)vG(v_\tau), \qquad (2.108)$$

where $H_\tau(y)=H(y)\sqrt{\frac{\tau}{\tau_{0N}}}$. For this function are also comprised the tables with input value y.

during performance calculation of the action of the rotating artillery shells, they usually consider that these projectiles are stable on trajectory and they move with the low angles of deflection of the longitudinal axis of projectile from velocity vector (angle δ in Fig. 2.8). In this case the drag (tangential component of the main

vector of aerodynamic forces R_τ) considers independent variable of δ and is determined with δ=0, and normal component - R_N and the tilting moment M accepts proportional to angle δ (Fig. 2.23).

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Furthermore, it is convenient during calculations into formulas for the named aerodynamic characteristics to introduce the bore of projectile - d m, function H(y) and corresponding aerodynamic coefficients $K_i\left(\frac{v}{a}\right)$. Taking into account this formulas (2.65) take the form:

$$X = R_\tau = R_{\tau 0} = \frac{id^2}{g} 10^3 H(y) v^2 K_D\left(\frac{v}{a}\right); \qquad (2.109)$$

$$Y = Z = R_N = \frac{dl}{g} 10^3 H(y) v^2 K_N\left(\frac{v}{a}\right)\delta; \qquad (2.110)$$

$$M_y = M_z = M = \frac{d^2h}{g} 10^3 H(y) v^2 K_M\left(\frac{v}{a}\right)\delta. \qquad (2.111)$$

Aerodynamic coefficients $K_N\left(\frac{v}{a}\right)$ and $K_M\left(\frac{v}{a}\right)$ are determined from the data of trial firings. Value h=Δl corresponds to conditional distance between centers of pressure and the center of mass of projectile and is determined usually from empirical formulas. For an example it is possible to give the known formula of Gobar

$$h = h_1 + 0.57h_r - 0.16d,$$

where h_1 and h_r (see Fig. 2.23) - the size/dimensions, which characterize the position of the center of mass of projectile and its

length head - ogival, or conical, part.

Characteristic, that are inherent only in fast-turning projectiles, are aerodynamic characteristics - Magnus' force R_{Ma}, the moment of Magnus' force M_{Ma} and the torque/moment of surface friction Γ. Magnus' force is that component of the main vector of aerodynamic forces, which is proportional to product $\omega_{x_1}\delta$. The mechanism of the emergence of this force is visible from Fig. with 2.24.

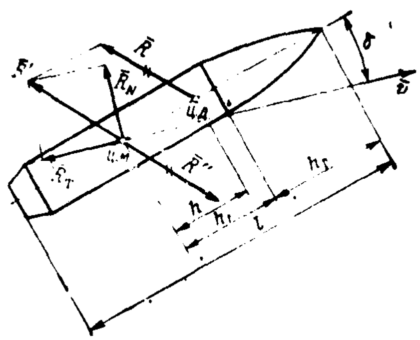

Fig. 2.23. Circuit of bringing the main vector of aerodynamic forces to the center of mass of the rotating projectile.

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The particles of the incident to projectile with a a velocity of of v airflow, meeting the particles of air, rotated by the surface of projectile, create a difference in the pressure near the wall. As a result of the redistribution of pressure over the surface of projectile, appears the secondary force, perpendicular to the plane of angle δ and directed to the side, reverse/inverse the mixing zones of counterflows. For determining this force, is accepted the formula

$$R_{M_n} = \frac{d^2 l}{g} 10^3 H(y) v \times$$
$$\times K_{M_n}\left(\frac{v}{a}\right) \omega_{x_1} \delta, \qquad (2.112)$$

where $K_{M_a}\left(\frac{v}{a}\right)$ - an experimental aerodynamic coefficient.

Since in the general case force vector of Magnus does not pass through the center of mass, appears the moment of this force. Effect R_{Ma} and its torque/moment on the flight of projectiles is studied comparatively little.

Skin-friction force decreases the angular velocity of the spin of projectile of relatively longitudinal axis. The moment of skin-friction force can be isolated from the main moment of aerodynamic forces and it is determined by the formula

$$\Gamma = \frac{d^3 l}{g} 10^3 H(y) v K_{\Gamma}\left(\frac{v}{a}\right) \omega_{x_1}, \qquad (2.113)$$

where $K_{\Gamma}\left(\frac{v}{a}\right)$ - an aerodynamic coefficient. As show calculations and experiment, torque/moment Γ significantly decreases the angular rate of rotation of projectile in the process of flight.

§ 5. THRUST.

Thrust - this one of basic forces, which act on rocket in flight, caused by the work of its engine plant. To determine the thrust of jet engine in flight is possible only on indirect experimental-design path. Therefore, as a rule, thrust is determined under the static conditions on special stands. The combined action of

the forces, considered by the right side of equation (1.12), including the Coriolis forces, determined by the oscillation of rocket, by the motion of gases and by the displacement/movement of the center of mass during burnout, it can be experimental determined in wind tunnel, if rocket power-on to fasten in it is hinged then so that the axis of rocket could complete oscillations.

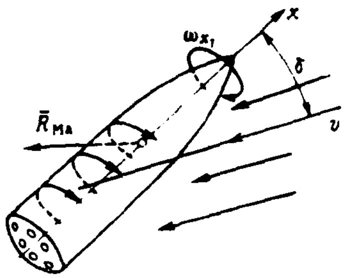

Fig. 2.24. Circuit of the formation/education of Magnus' force.

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Hinge fitting introduces considerable distortions into that flow around rocket external flow how is decreased the accuracy of results. Therefore Coriolis forces and torque/moments it is expedient to determine separately in the absence of external flow.

Combined action of the thrust, aerodynamic and variation forces is determined in wind tunnel with the securely fastened rocket. Arranging the longitudinal axis of rocket along flow, so that from the aerodynamic forces acts only drag, it is possible to measure on the supports of rocket the total acting force, called the effective thrust of engine

$$P_{э\phi} = P - X - \frac{\delta q_{вар}}{dt}, \qquad (2.114)$$

where P - bench thrust.

If we do not consider variation forces and to accept the speed of the external flow of the equal to zero, then on the supports of rocket will be determined the bench engine thrust.

5.1. BENCH ENGINE THRUST.

Under bench thrust is understood resultant force of air pressure and the escaping gases, applied to fixed rocket, which is located in the fixed undisturbed atmosphere.

Separately measure jet force (second term in formula 1.8) it is not represented possible, and it are determined together with the forces of static pressure, which act in the X direction of rocket.

On the external surface of rocket act the forces, determined by the atmospheric pressure p, which correspond to the height/altitude on which is arrange/located the rocket. They in value are equal to the product of pressure and area and are directed perpendicular to that area/site on which they act. All forces, which act on the lateral surface of rocket, balance each other. Since while the motor is running atmospheric pressure does not act on nozzle exit section through which occur/flow/last the gases, then will appear unbalanced

force pS_a, directed to the side of the cutflow of gases (S_a - a nozzle exit area). In nozzle exit section acts opposed force p_aS_a, where p_a - pressure the escaping behind nozzle gases in this cross section.

Thus, in connection with bench conditions in equation (1.8) one should replace $\sum F$ by $S_a(p_a-p)$ and then, after accepting dv/dt=C, we will obtain for thrust the equation

$$P=\left|\frac{dm}{dt}\right|w_{отн}+S_a(p_a-p). \qquad (2.115)$$

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After replacing in equation (2.115) the mass flow rate of the gas per second through weight flow rate $\left|\frac{dm}{dt}\right|=\frac{Q_{сек}}{g}$, we will obtain formula for thrust in other writing:

$$P=\frac{Q_{сек}}{g}w_{отн}+S_a(p_a-p). \qquad (2.116)$$

In the case when it is possible to accept $p\approx0$,

$$P=\frac{Q_{сек}}{g}w_{отн}+S_ap_a. \qquad (2.117)$$

If rocket is arrange/located on the surface of the Earth on zero level (y=0), then its thrust is equal to

$$P_0=\frac{Q_{сек}}{g}w_{отн}+S_a(p_a-p_0). \qquad (2.118)$$

Comparing formulas (2.116) and (2.118), we will obtain

$$P = P_0 + S_a p_{0N}\left(1 - \frac{p}{p_{0N}}\right) \quad (2.119)$$

Since $\pi(y) = \frac{p}{p_{0N}}$, we will obtain finally

$$P = P_0 + S_a p_{0N}[1 - \pi(y)]. \quad (2.120)$$

After removing from the right side of formula (2.116) for brackets $\frac{Q_{сек}}{g}$, we will obtain the simplified formula for thrust

$$P = \frac{Q_{сек}}{g} w_e, \quad (2.121)$$

where

$$w_e = w_{отн} + \frac{S_a g}{Q_{сек}}(p_a - p); \quad (2.122)$$

w_e - the value, named French scholar P. Langevin by the effective exhaust gas velocity.

Calculations show that in formula (2.122) second term in comparison with the first little comprises usually not more than 10-15¢/o; therefore effective discharge velocity is determined in by base speed of gas in nozzle exit section $w_{отн}$. If we relate thrust to flow rate per second, then we will obtain the formula, which determines specific thrust, or the so-called unit/single momentum/impulse/pulse:

$$P_{уд} = J_{ед} = \frac{P}{Q_{сек}} \; \frac{\text{KG}\cdot\text{s}}{\text{KG}} \; \text{кгс}\cdot\text{с}/\text{кгс}. \quad (2.123)$$

From (2.118) we will obtain

$$P_{y\text{д}} = \frac{w_{\text{отн}}}{g} + \frac{S}{Q_{\text{сек}}}(p_a - p_h). \qquad (2.124)$$

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Hence it is apparent that with decompression in the which surrounds rocket air the specific thrust increases. The specific thrust in space is greater than specific thrust on the Earth to 10-15ϙ/o.

Expression for unit/single momentum/impulse/pulse $J_{e\text{д}}$ can be also obtained from the common/general/total expression, which determines the power impulse of thrust, if we take w_e=const and to relate the total impulse of thrust to the fuel load Q_T, which burned down for the operating time of engine t_K.

$$J_{e\text{д}} = \frac{1}{Q_T}\int_0^{t_K} P\,dt = \frac{w_e}{Q_T g}\int_0^{t_K} Q_{\text{сек}}\,dt.$$

When $Q_{\text{сек}} = \text{const}$ we will obtain $\int_0^{t_K} Q_{\text{сек}}\,dt = Q_{\text{сек}} \cdot t_K = Q_T$ and then

$$J_{e\text{д}} = \frac{w_e}{g}. \qquad (2.125)$$

From comparison (2.124) and (2.125) it follows

$$J_{e\text{д}} = P_{y\text{д}} = \frac{w_e}{g}. \qquad (2.126)$$

Let us designate through $\Sigma \overline{P}$ resultant force of the thrust of all rocket engines, led to the center of mass of rocket. Utilizing (1.8), we will obtain for a rocket the equation of the forward motion of the center of mass in the form

$$m\frac{d\bar{v}}{dt}=\sum\bar{F}+\sum\bar{P}. \qquad (2.127)$$

5.2. SECONDARY FORCES AND MOMENTS.

Secondary forces and moments can be divided into two large groups: external (aerodynamic) and internal. Both internal and external secondary forces and torque/moments appear during the three-dimensional/space curvilinear flight also of various kinds the oscillations of flight vehicle.

The external secondary forces and the torque/moments usually include various kinds the damping aerodynamic forces and torque/moments, the aerodynamic forces and the torque/moments, caused by downwash and by the delay of downwash, cross aerodynamic forces and torque/moments. To internal supplementary factors usually are related internal torque of damping, determined by Coriolis acceleration, and other forces and the torque/moments, caused by the

displacement/movement of fuel/propellant and working gases within missile body. The majorities of secondary forces and torque/moments are determined by the angular rates of rotation of flight vehicle relative to the center of mass.

The damping and cross aerodynamic forces and torque/moments are examined above in § 4.

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Let us enumerate here internal secondary forces and their moments with respect to the center of mass: $\bar{P}_{газ}$, $\bar{M}_{газ}$ - force and torque/moment, determined by the flow of gases within missile body; $\bar{P}_{газ.Кор}$, $\bar{M}_{газ.Кор}$ - force of Coriolis and the moment of this force, determined by the flow of gases within oscillating missile body.

Rockets with engine on liquid propellant test the effect of secondary forces and torque/moments, determined with the motion of propellant components within missile body; $\bar{P}_{ж.т}$, $\bar{M}_{ж.т}$ - force and torque/moment, determined by the motion of propellant components along conduit/manifolds; $\bar{P}_{ж.б}$, $\bar{M}_{ж.б}$ - force and torque/moment, determined by the motion of propellant components in tanks; $\bar{P}_{ж.т.Кор}$, $\bar{P}_{ж.б.Кор}$, $\bar{M}_{ж.т.Кор}$, $\bar{M}_{ж.б.Кор}$ - force and Coriolis's torque/moments, determined by the oscillation of rocket and by the motion of

propellant components along conduit/manifolds and in tanks. Separately can be isolated the forces and the torque/moments, determined by the elastic deformations of missile body and by displacement within the body of different solid driving/moving masses (for example, the rotor of turbojet engine), etc. The part of the named forces and torque/moments is caused by the transiency of process and can be referred to the variation forces, not considered on the smallness of effect.

Let us examine in more detail force and the torque/moments, considered during the ballistic calculations, connected with the evaluation of stability of motion, control and accuracy of firing.

The Coriolis force, acting on elementary mass dm_{ix}, driving/moving along the i channel within missile body (Fig. 2.25), is equal to

$$dP_{Kop\,i}=2U_{ix}\omega dm_{ix},$$

where

$$dm_{ix}=\varrho_{ix}S_{ix}dx;$$

U_{ix}, ϱ_{ix} and S_{ix} - a rate of relative motion of working medium/propellant, its mass density and transverse area in cross section x of the i channel.

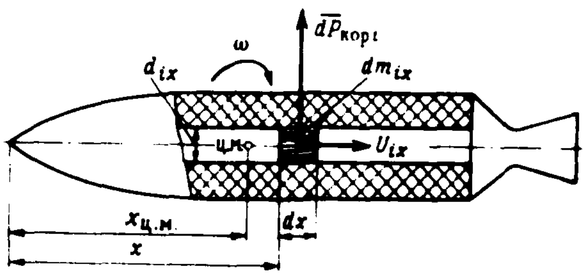

Fig. 2.25. Diagram of the formation of internal Coriolis force during the displacement of working medium/propellant along the housing of the oscillating rocket.

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According to continuity condition, the flow rate per second of mass along the i channel at the given instant is constant and equal to

$$|\dot{m}|_i = \varrho_{ix} S_{ix} U_{ix}.$$

then $dP_{\text{кор}\,i} = 2|\dot{m}|_i \omega\, dx.$

the torque/moment of Coriolis force will be equal to

$$dM_{\text{кор}\,i} = -2\omega|\dot{m}|_i (x_i - x_{\text{ц.м}})\, dx.$$

Total torque/moment of Coriolis force will be equal to the sum

cf the integrals, calculated according to all channels of the motion cf working medium/propellant,

$$M_{\kappa,p} = -2\omega \sum_{1}^{i} \int_{x_0}^{x_k} |\dot{m}|_i (x_i - x_{\text{ц.м}}) dx, \qquad (2.128)$$

where x_0 and x_k - value x, which correspond tc beginning and the end/lead of the motion of work substance cr the section of the i channel in question.

Minus sign shows that the moment cf Ccriclis force will act against direction of rotaticn, i.e., it will be damping in the case cf flow from the center of mass. The flcw, driving/moving to the center of mass, will give the moment of the Coriolis force, directed tc the side cf the rotation of flight vehicle.

To supplementary factcrs usually are related the force and the torque/moment, caused by the displacement ci the center of mass cf the rocket during burnout during the engine operation. In equations (1.11) and (1.12) this force is determined ty expression $2m[\bar{\omega}\times\bar{r}_r]$. It just as Coriolis force, is directed perpendicular to axis of rccket.

For simplicity let us take rocket with the grain of end burning (Fig. 2.26). let us designate the position of the center of mass of rccket at the moment of time t through $x_{\text{ц.м}}$, and at the moment cf time t+dt through $x_{\text{ц.м1}} = x_{\text{ц.м}} - dx_{\text{ц.м}}$. Let for time dt from grain be

separate/liberated dm mass of rocket. Static moment with respect to the forepart/nose point of rocket before the isolation/evolution of mass dm will be equal to

$$M_Q = m x_{ц.м}. \quad (2.129)$$

Static torque after the isolation/evolution of mass dm

$$M_{Q1} = M_Q - |\dot{m}| x_{отд} dt = (m - dm)(x_{ц.м} - dx_{ц.м}).$$

Omitting on smallness terms $dmdx_{ц.м}$, let us have:

$$m x_{ц.м} - m dx_{ц.м} - x_{ц.м} dm = M_Q - |\dot{m}| x_{отд} dt. \quad (2.130)$$

Deducting (2.129) from last/latter equality, we will obtain

$$m dx_{ц.м} + x_{ц.м} dm = |\dot{m}| x_{отд} dt.$$

Hence

$$m v_r = m \frac{dx_{ц.м}}{dt} = |\dot{m}| (x_{отд} - x_{ц.м}),$$

or

$$v_r = \frac{|\dot{m}| (x_{отд} - x_{ц.м})}{m}. \quad (2.131)$$

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Thus, during the rotation of housing with angular velocity ω the secondary force, caused by the displacement of the center of mass,

will be equal to

$$2m\omega v_r = 2\omega|\dot{m}|(x_{\text{о.т.д}} - x_{\text{ц.м}}).$$

Supplementary torque/moment from this force is approximately equal to

$$\Delta M_{\text{Кор}} = 2\omega|\dot{m}|(x_{\text{о.т.д}} - x_{\text{ц.м}})^2. \qquad (2.132)$$

Internal secondary forces and torque/moments have noticeable effect on the motion of flight vehicles with jet engines in the rarefied layers of the atmosphere, and one should consider them during the stability analysis and control. During motion in the dense layers of the atmosphere, the effect of internal secondary forces and torque/moments is insignificant in comparison with the action basic and supplementary external aerodynamic forces and torque/moments, and they in calculations are not considered. During designed performance calculations of the motion of the center of mass of flight vehicle, the secondary forces and torque/moments usually are not considered.

§ 6. CONTROL FORCES AND MOMENTS.

Rocket control and projectiles in flight is realized by the control system, integral part of which are actuating elements or controls. Actuating elements or controls, as them are frequently called, create control forces and torque/moments. According to the principle of the creation of control forces and torque/moments controls is accepted to divide into three type: aerodynamic, gas-dynamic and those mixed.

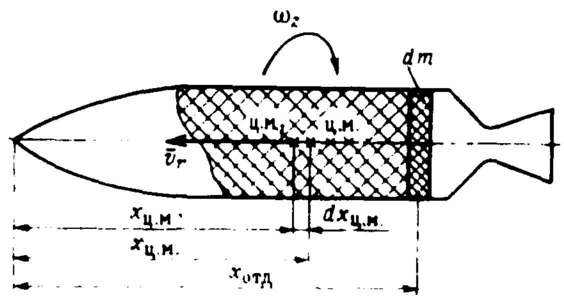

Fig. 2.26. Circuit of the displacement of the center of mass of the rocket during burnout.

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Aerodynamic (air) or as them occasionally referred to as, aircraft controls work only in the atmosphere during interaction with the airflow, which flows around flight vehicle. Gas-dynamic controls can work also in the atmosphere, and in the rarefied layers of the atmosphere (outer space), since they work only because of the energy, which is isolated with the combustion of fuel/propellant.

Many of the controlled objects, for example, rockets, winged missiles, guided torpedoes and aircraft bombs have only aerodynamic controls. Some rockets have the combined controls, which consist of the various kinds of aerodynamic and gas-dynamic devices. Aerodynamic controls usually are divided into steering surfaces (controls),

pivoting wings and the interrupters of air flow (interceptor/spoilers).

The operating principle of controls and pivoting wings lies in the fact that they, being turned relative to free position and body axes of flight vehicle, change in the process of flight their angle of attack and, correspondingly, the angle of attack (or slip) of flight vehicle as a whole.

Functional dependences for aerodynamic coefficients c_y and c_z (2.68) and for torque/moments M_{y_1} and M_{z_1} (2.71) determine communication/connection between the angles of rotation of controls, the angles of rotation of the housing of flight vehicle and the corresponding angular velocities. For an independent account in the equations of motion of control forces and torque/moments, it is necessary to isolate composing of aerodynamic coefficients, determined by the rotation of control devices. For example, for the elevators (pitch) and of yaw longitudinal and normal control forces are respectively equal to

$$\left.\begin{aligned} X_{p1} &= S_p q\left(c_{x\,p1_0} + c_{x\,p1}^{\delta_{z_1}}\delta_{z_1}\right); \\ Y_{p1} &= S_p q c_{y\,p1}^{\delta_{z_1}}\delta_{z_1}; \\ Z_{p1} &= S_p q c_{z\,p1}^{\delta_{y_1}}\delta_{y_1}, \end{aligned}\right\} \qquad (2.133)$$

where S_p - a characteristic area of controls; q - velocity head;

$c_{x\,pl_0}, c_{xp_1}^{\delta_{z_1}}, c_{y\,pl}^{\delta_{z_1}}, c_{z\,pl}^{\delta_{y_1}}$ - aerodynamic coefficients of controls.

Moment characteristics are determined usually more precise taking into account the angular velocities of the rotation of controls

$$\left.\begin{aligned} m_{py_1} &= m_{py_1}^{\delta_{y_1}}\delta_{y_1} + m_{py_1}^{\dot{\delta}_{y_1}}\dot{\delta}_{y_1}; \\ m_{pz_1} &= m_{pz_1}^{\delta_{z_1}}\delta_{z_1} + m_{pz_1}^{\dot{\delta}_{z_1}}\dot{\delta}_{z_1}. \end{aligned}\right\} \qquad (2.134)$$

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The moments of control forces are equal to

$$M_{py_1} = Sqlm_{py_1}; \quad M_{pz_1} = Sqlm_{pz_1}. \qquad (2.135)$$

Depending on the designation/purpose of rocket or missile and their aerodynamic layout, air vanes can be placed in different places of body (Fig. 2.27). For clarity in figure, the controls are shaded.

Paired air vanes, placed in wing tip and which are deflect/diverted to different sides, are called ailerons. The paired end controls, which are deflect/diverted independently of each other to any side, are called elevons. The deflection of ailerons and elevons to different sides leads to the rotation of flight vehicle of relatively longitudinal axis.

The aerodynamic characteristics of pivcting wing are determined just as usual wing.

Of miniature/small rockets with the limited maneuver, is applied the control with the aid of the interrupters of air flow or interceptor/spoilers. Interceptor/spoilers represent a thin plate, arrange/located in middle or on the trailing wing edge perpendicular to the incident flow which is given into oscillatory motion in the direction, perpendicular to wing chord.

Normal force, created by one interceptor/spoiler and directed to the side, reverse/inverse to the direction of the output of plate for the wing plane, is determined on the formula

$$Y_{\text{н1}}=c_{y\,\text{н1}}\,q h_{\text{н}} l_{\text{н}} k. \qquad (2.136)$$

Here $h_{\text{н}}, l_{\text{н}}$ - the height/altitude of the output of plate and the length of interceptor/spoiler. Coefficient of the command/crew

$$k=(t_1-t_2)(t_1+t_2),$$

where t_1 and t_2 - retention time of interceptor/spoiler in upper and lower positions.

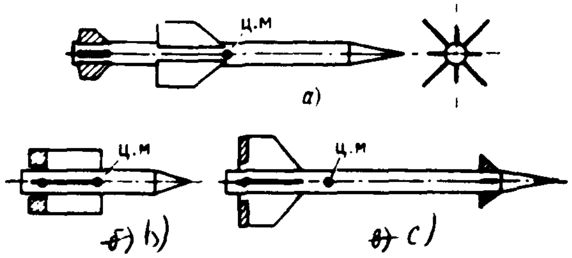

Fig. 2.27. Diagram of the layout of controls on missile body: a) normal aerodynamic configuration; b) diagram "tailless aircraft"; c) diagram "canard".

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Interceptor/spoiler causes the appearance of longitudinal force, which increases drag of flight vehicle. longitudinal force from one interceptor/spoiler can be determined by the formula

$$X_{и1}=c_{x\,и1}qh_{и}l_{и}. \qquad (2.137)$$

The named above controls are brought into action by steering drives. To the aerodynamic controls, which do not have special drive, one should relate rollerons.

Rolleron - this is the aileron, within which is arrange/located the massive disk, which rotates with high angular velocity and which

has the properties of gyroscope. The rotational axis of disk $0y_{p1}$ is perpendicular to the plane of the aileron which in turn can pivot around axis $0z_{p1}$. Before the launching of rocket, the disk untwists to velocity Ω_{yp1}, which during rocket flight is supported by the effect of the incident airflow, which flows around the teeth of disk, which protrude for the flank edge of aileron. If as a result of any reasons appears the perturbation moment of roll $\bar{M}_{x1}$ and the rocket together with rollerons will start up with angular velocity $\bar{\Omega}_{x_1}$, then will appear acting on rolleron gyroscopic torque/moment $\bar{M}_{r}$, directed along the axis $0z_{p1}$.

Under the action of the gyroscopic torque/moment the rolleron will turn itself around axis $0z_{p1}$ to angle δ_{r}, moreover the direction of rotation will be similar so that the angular velocity vector $\bar{\Omega}_{yp1}$ would attempt on the shortest path to be combined with angular velocity vector $\bar{\Omega}_{x_1}$ (Joukowski's rule). Arranged/located in symmetrical stabilizer second rolleron whose disk is twisted in opposite direction to the first, in turn, will be deflected from its free position, but already by angle $-\delta_{r}$. As a result for each of the rollerons will arise control forces $\pm Y_{p1}$, which will create torque/moment M_{px_1}, which parries the moment of roll M_{x_1}. The examined organ/control whose action is instituted on the use of a gyroscopic effect, can be described as "the passive" automatic machine of stabilization, capable of only stabilizing the angular

velocity of the rotation of rocket of its relatively longitudinal axis, it not to control the movement of rocket along bank.

At present one of the most widely used forms of gas-dynamic control devices are the jet vanes.

The jet vanes - this four plates of special shape from heat-resistant material (for example, graphite), which are arrange/located at the nozzle outlet or nozzle unit in two mutually perpendicular planes, passing through the axis of rocket. The jet vanes can be turned relative to their rotational axes to angle δ to some side or other.

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In the engine operation, the escaping behind nozzle gas flow will flow controls, also, for each of them arise the longitudinal $X_{\text{газ.р}}$ and normal $Y_{\text{газ.рl}}(Z_{\text{газ.рl}})$ forces which can be calculated from the dependences:

$$X_{\text{газ.рl}}=\frac{1}{2}S_{\text{газ.р}}\varrho_{\text{газ}}w^2_{\text{газ}}c_{x\,\text{газ.рl}}; \quad (2.138)$$

$$Y_{\text{газ.рl}}=\frac{1}{2}S_{\text{газ.р}}\varrho_{\text{газ}}w^2_{\text{газ}}c^{\delta_{z_1}}_{y\,\text{газ.рl}}\delta_{z_1}; \quad (2.139)$$

$$Z_{\text{газ.рl}}=\frac{1}{2}S_{\text{газ.р}}\varrho_{\text{газ}}w^2_{\text{газ}}c^{\delta_{y_1}}_{z\,\text{газ.рl}}\delta_{y_1}, \quad (2.140)$$

where $S_{\text{газ.р}}$ - a characteristic area of the jet vane; $\varrho_{\text{газ}}$ - density

of gas flow; w_{ras} - rate of flow of gases in the location of controls.

Force $\Sigma X_{\text{ras.pl}}$ in the majority of the cases is not manager and is considered during the calculation of trajectories by the decrease of the complete thrust of engines; the normal and lateral forces $\Sigma Y_{\text{ras.pl}}$ and $\Sigma Z_{\text{ras.pl}}$ are utilized for the flight control of rocket. Expressions for the governing torque/moments, created by the jet vanes, have a structure, similar (2.135); therefore here they are not brought.

By analogy with air vanes, one pair of the jet vanes, which has separate drives, during the motion of rocket is arrange/located in vertical plane and fulfills the functions of rudders (yaw) and of bank; the second pair of the controls, connected synchronously, elevators (pitch).

Well known also methods of rocket control by rotation relative to the housing entire engine and by the rotation only of nozzle or special nozzle (deflector), placed immediately after nozzle and covering the flow of the escaping gases. Control can be reached also by the overlap of nozzles. For this purpose the engine must have not one, but somewhat it puffs, that were being arrange/located relative to the axis of rocket along circumference. In this case the overlap of one or several nozzles will ensure obtaining the control force of

the necessary value and direction. They are utilized for control of force vector of thrust and trim tabs - the flaps, moved into nozzle it is perpendicular to gas flow. This method of control in the foreign press acknowledges by very reliable, although it differs in terms of the greatest thrust loss.

Recently considerable attention is given to the method of control of force vector of the thrust by the injection (injection) of liquid or gas of propulsion nozzle.

The detailed calculation of rollerons and complex gas-dynamic control devices is developed by V. N. Koshev is placed in [33].

The mines of artillery pieces have stabilizers (tail assembly), rigidly fastened with housing and, as a rule, do not have control devices.

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Chapter III.

EQUATIONS OF MOTION OF THE ROCKETS OF "SURFACE-TO-SURFACE" CLASS IN THE DENSE LAYERS OF THE ATMOSPHERE.

§ 1. Three-dimensional equations of motion of rockets.

1.1. Complete system of equations.

The motion of rockets let us examine in the system of coordinates $0x_3y_3z_3$, rigidly connected with the Earth. In this case, for simplification in the problem the effect of the Earth let us consider by the introduction of constant in value and direction of acceleration from gravitational force, disregarding Coriolis acceleration; the curvature of the Earth also consider we will not. With these assumptions the system of equations can be written on the basis of equations (1.16) and (1.21).

It is simpler anything to write system of equations for determining the velocity v of the forward action of the center of mass of rocket relative to the Earth in projections on the axis of

the half-speed coordinate system, since in this system $v_x = v$, $v_{y^*} = v_{z^*} = 0$. Then equations (1.16) are simplified and they will take the form

$$v = \frac{\sum F_x}{m}; \quad v\omega_{z^*} = \frac{\sum F_{y^*}}{m}; \quad v\omega_{y^*} = -\frac{\sum F_{z^*}}{m}. \qquad (3.1)$$

For determining the projections of the angular rate of rotation of the half-speed system of coordinates Oxy*z* relative to fixed system $(0x_3y_3z_3)$ in the axis half-speed coordinate system we will use Fig. 2.9, from which it follows

$$\omega_x = \dot{\Psi} \sin\theta; \quad \omega_{y^*} = \dot{\Psi} \cos\theta; \quad \omega_{z^*} = \dot{\theta}. \qquad (3.2)$$

Substituting ω_{y^*} and ω_{z^*} in (3.1), we will obtain the system

$$\dot{v} = \frac{\sum F_x}{m}; \quad v\dot{\theta} = \frac{\sum F_{y^*}}{m}; \quad v\dot{\Psi}\cos\theta = -\frac{\sum F_{z^*}}{m}. \qquad (3.3)$$

During the writing of the right sides of equations (3.3) let us consider thrust component, force of gravity, aerodynamic and control forces.

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Let the thrust be assigned by projections on the body axes of coordinates P_{x_1}, P_{y_1}, P_{z_1}. Assigning the direction of thrust similarly, it is convenient subsequently during calculations of the action

characteristics of the rocket to consider the gas-dynamic control forces which appear during the deviation of force vector of thrust from axis of rocket. The control forces, created by separate gas-dynamic controls, can be included in thrust component along body axes. Furthermore, it will be possible to also consider the effect of eccentricity of thrust on the action characteristics of rocket.

Aerodynamic force let us assign components X; Y; Z in the high-speed/velocity coordinate system. Then, after using the table of direction cosines (2.3a) and transfer equations (2.6), we obtain the projections of aerodynamic force on the half-speed axes of the coordinates

$$\left.\begin{aligned} &R_x=-X;\\ &R_{y^*}=Y^*=Y\cos\gamma_c-Z\sin\gamma_c;\\ &R_{z^*}=Z^*=Y\sin\gamma_c+Z\cos\gamma_c.\end{aligned}\right| \qquad (3.4$$

Values X, Y and Z in the first approximation, can be determined by formulas (2.85) and (2.90).

In the body axes of coordinates, let us write control aerodynamic forces, created by one-type aerodynamic controls - $\sum X_{p1}$; $\sum Y_{p1}$; $\sum Z_{p1}$.

Using the table of direction cosines (2.35) and transfer equations (2.6), we will obtain the projections of the thrust and

control forces on the half-speed coordinate axes:

$$\left.\begin{aligned}
&P_x-\sum X_p=\left(P_{x_1}-\sum X_{p1}\right)\cos\alpha\cos\beta-\\
&\quad-\left(P_{y_1}+\sum Y_{p1}\right)\sin\alpha\cos\beta+\left(P_{z_1}+\sum Z_{p1}\right)\sin\beta;\\
&P_{y^*}+\sum Y_p^*=\left(P_{x_1}-\sum X_{p1}\right)(\sin\alpha\cos\gamma_c+\\
&\quad+\sin\gamma_c\cos\alpha\sin\beta)+\left(P_{y_1}+\sum Y_{p1}\right)(\cos\gamma_c\cos\alpha-\\
&\quad-\sin\gamma_c\sin\alpha\sin\beta)-\left(P_{z_1}+\sum Z_{p1}\right)\sin\gamma_c\cos\beta;\\
&P_{z^*}+\sum Z_p^*=\left(P_{x_1}-\sum X_{p1}\right)(\sin\gamma_c\sin\alpha-\\
&\quad-\cos\gamma_c\cos\alpha\sin\beta)+\left(P_{y_1}+\sum Y_{p1}\right)(\sin\gamma_c\cos\alpha+\\
&+\cos\gamma_c\sin\alpha\sin\beta)+\left(P_{z_1}+\sum Z_{p1}\right)\cos\gamma_c\cos\beta.
\end{aligned}\right| \qquad (3.5)$$

Since we is not considered the variability of gravitational force and the surface curvature of the Earth, then the projections of gravitational force on the half-speed coordinate axes are obvious:

$$Q_x=-Q\sin\theta;\quad Q_{y^*}=-Q\cos\theta;\quad Q_{z^*}=0. \qquad (3.6)$$

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Substituting the obtained above components of all those considered by us forces (3.5) and (3.6) and formulas for angular velocities (3.2) into fundamental equations (3.1), we will obtain the system of equations, which describes the motion of the center of mass of rocket in the half-speed system of coordinates

$$\dot{v}=\frac{1}{m}\left(P_x-\sum X_p-X+Q_x\right); \qquad (3.7)$$

$$v\dot{\theta}=\frac{1}{m}\left(P_{y^*}+\sum Y_p^*+Y^*+Q_{y^*}\right); \qquad (3.8)$$

$$v\dot{\Psi}\cos\theta=-\frac{1}{m}\left(P_{z^*}+\sum Z_p^*+Z^*\right). \qquad (3.9)$$

The equations of the rotary motion of rockets and aircraft usually write in projections on the body axes of coordinates. Any another, the not connected with rocket, coordinate system is moved relative to rocket, but this leads to the need for using during the study of the motion of the rocket by the variable values of the moments of inertia even with m = const, which introduces complications. Let us write the equations of rotary motion in projections on the body axes of coordinates, assuming that the body axes of coordinates coincide with principal central inertia axes:

$$J_{x_1}\dot{\omega}_{x_1}+(J_{z_1}-J_{y_1})\omega_{y_1}\omega_{z_1}=\sum M_{x_1}+\sum M_{p\,x_1}; \qquad (3.10)$$

$$J_{y_1}\dot{\omega}_{y_1}+(J_{x_1}-J_{z_1})\omega_{x_1}\omega_{z_1}=\sum M_{y_1}+\sum M_{p\,y_1}; \qquad (3.11)$$

$$J_{z_1}\dot{\omega}_{z_1}+(J_{y_1}-J_{x_1})\omega_{x_1}\omega_{y_1}=\sum M_{z_1}+\sum M_{p\,z_1}, \qquad (3.12)$$

where $\sum M_{x_1}$; $\sum M_{y_1}$; $\sum M_{z_1}$ - sum of the projections of the moments of external forces and thrust on the body axes of coordinates (without the account of control forces); $\sum M_{p\,x_1}$; $\sum M_{p\,y_1}$; $\sum M_{p\,z_1}$ - sum of the projections of the moments of control forces on body axes of coordinates.

While the motor is running the moments of inertia will be values alternating/variable as a result of a change in the mass of rocket because of the fuel consumption. Numerical values $J_{x_1}, J_{y_1}, J_{z_1}$ for a rocket are defined just as for any compound, i.e., for calculating the moments of inertia it is necessary to have the detailed drawings of rocket and to know the law of a change of its masses in flight. Work according to the calculated determination of the moments of inertia and with respect to their change is very tedious and laborious. Therefore during their approximate ballistic calculations they take by constants. During the more precise calculations, connected with stability and controllability, it is necessary to consider the variability of the moments of inertia.

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The experimental methods of determining the moments of the inertia of compounds, the giving more accurate results in comparison with calculated, are developed in sufficient detail in mechanics, and we concern them will not.

For setting of communication/connections between derived $\dot\theta$, $\dot\psi$, $\dot\gamma$ and angular velocities ω_{x_1}, ω_{y_1} and ω_{z_1} we will use the table of direction cosines 2.3a and by transfer equations (2.6) with the aid of which we will obtain

$$\left.\begin{aligned}\omega_{x_1} &= \dot\psi\sin\theta + \dot\gamma;\\ \omega_{y_1} &= \dot\psi\cos\theta\cos\gamma + \dot\theta\sin\gamma;\\ \omega_{z_1} &= -\dot\psi\cos\theta\sin\gamma + \dot\theta\cos\gamma.\end{aligned}\right\} \qquad (3.13)$$

Solving these equations together, we will obtain

$$\dot{\vartheta}=\omega_{y_1}\sin\gamma+\omega_{z_1}\cos\gamma; \quad (3.14)$$

$$\dot{\psi}=\frac{1}{\cos\vartheta}(\omega_{y_1}\cos\gamma-\omega_{z_1}\sin\gamma); \quad (3.15)$$

$$\dot{\gamma}=\omega_{x_1}-\operatorname{tg}\vartheta(\omega_{y_1}\cos\gamma-\omega_{z_1}\sin\gamma). \quad (3.16)$$

During the determination of amounts of aerodynamic forces in the process of the solution of the spatial problem of moving the rocket, it is necessary to know the values of angles α, β and γ_c.

Determining the direction cosines of consecutive passage from body axes to high-speed/velocity, from the high-speed/velocity to half-speed and from the half-speed to their terrestrial and by equalizing direction cosines of direct passage from body axes to terrestrial, we will obtain the following relationship/ratios between the angles:

$$\sin\theta=\sin\vartheta\cos\alpha\cos\beta+\cos\vartheta(\sin\alpha\cos\gamma_c+\cos\alpha\sin\beta\sin\gamma_c); \quad (3.17)$$

$$\sin\psi\cos\gamma=\sin\Psi\cos\beta\cos\gamma_c+\cos\Psi(\sin\beta\cos\theta+\sin\gamma_c\sin\theta\cos\beta)-\cos\psi\sin\vartheta\sin\gamma; \quad (3.18)$$

$$\cos\vartheta\sin\gamma=\sin\gamma_c\cos\beta\cos\theta-\sin\beta\sin\theta. \quad (3.19)$$

Hence

$$\gamma_c=\arcsin\frac{\cos\vartheta\sin\gamma+\sin\beta\sin\theta}{\cos\beta\cos\theta},$$

a angles α and β are determined during the solution of complex quadratic trigonometric equations.

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If we now utilize expressions for the projections of the velocity vector of the center of mass of rocket on the axis of earth-based coordinate system (see Fig. 2.9), then we will obtain

$$\frac{dx_3}{dt}=v\cos\theta\cos\Psi; \qquad (3.20)$$

$$\frac{dy_3}{dt}=v\sin\theta; \qquad (3.21)$$

$$\frac{dz_3}{dt}=-v\cos\theta\sin\Psi. \qquad (3.22)$$

Respectively distance from the origin of coordinates to the center of mass of rocket (slant range will be equal

$$r=\sqrt{x_3^2+y_3^2+z_3^2}. \qquad (3.23)$$

Equations (3.7)-(3.12) and (3.14)-(3.23) together with the equation

$$m=m_0-\int_0^t|\dot{m}|\,dt, \qquad (3.24)$$

by that determining mass change, will comprise the system from 17 equations, which describes the spatial motion of rocket. This system of equations can be applied for different target/purposes. Most frequently it are utilized for the solution of direct problem of

external ballistics. In this case, if will be known all the geometric, weight, inertia, aerodynamic characteristics of rocket; the characteristics, which determine the operation of its engine; the laws of a change of the control forces $\sum X_p(t)$, $\sum Y^*_p(t)$, $\sum Z^*_p(t)$ and of the torque/moments $\sum M_{px_1}(t)$, $\sum M_{py_1}(t)$, $\sum M_{pz_1}(t)$ and the initial flight conditions, equation of system can be by one or the other method integrated.

As a result of solution, will be found all the motion characteristics of the rocket

$$v(t);\quad \theta(t);\quad \Psi(t);\quad x_3(t);\quad y_3(t);\quad z_3(t);$$
$$r(t);\quad \vartheta(t);\quad \psi(t);\quad \gamma(t);\quad \alpha(t);\quad \beta(t);$$
$$\gamma_c(t);\quad \omega_{x_1}(t);\quad \omega_{y_1}(t);\quad \omega_{z_1}(t);\quad m(t).$$

In a series of the cases, can be assigned the part of the motion characteristics: the value of coordinates, velocities or angles. The motion characteristics of rocket are assigned as a function of time or other values, for example, velocity change in time v(t), pitch angle in time $\vartheta(t)$, the angles of deflection of control devices with respect to time $\delta_{z_1}(t)$, $\delta_{y_1}(t)$, $\delta_{x_1}(t)$ or $y_3=f(x_3)$ and so forth.

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These functions are called programmed equations

During the assignment of programmed equations, the part or the

fundamental equations of system (3.7)-(3.12) and (3.14)-(3.23) will render/show excess. For example, if is assigned $\vartheta(t)$, then value $\dot{\vartheta}$ can be determined directly by differentiation of function $\vartheta(t)$ and equation (3.14) will be excess. In this case, it is logical, the solution of direct problem is simplified.

For the more complete study of controlled flight, stability of motion and stabilization of flight vehicle apparatus to the written equations, it is necessary to add the equations of control, which describe the operation of the control system. During the flight control from different parameters, the rocket must have controls with the help of which can change the corresponding parameter of the motion of rocket. For example, during control from the angular parameters ϑ, ψ and γ rocket has the appropriate control devices - controls of pitch, yaw and bank. During control on velocity modulus, the rocket must have the actuating elements, which give possibility to affect v by the way of a change in the thrust or drag (for example, nozzle with inner body or of various kinds aerodynamic speed brakes and flaps). In the general case the position of executive control device in the process of rocket flight depends on many parameters. They include changes in the named above angles, angular velocities and accelerations, coordinates of the center of mass, velocities and accelerations in coordinates. For providing the programmed change in the determining parameter, actuating element

must be set in the position, which corresponds to the difference between the measured and programmed values of the parameters. For example, during programming with respect to pitch angle, angular velocity and angular acceleration the equation of control for determining the deflection of the control of pitch takes the following form:

$$\delta_{z_1} = k_{0\vartheta}\Delta\vartheta + k_{1\vartheta}\dot{\vartheta} + k_{2\vartheta}\ddot{\vartheta}, \qquad (3.25)$$

where $k_{0\vartheta}; k_{1\vartheta}; k_{2\vartheta}$ - coefficients of the equation of control along the channel of pitch.

Analogously can be written the equations of control, also, for other parameters of the action of rocket.

During speed control of the center of mass, the displacement of actuating element will be determined by the formula:

$$l_v = k_{0v}\Delta v + k_{1v}\dot{v}, \qquad (3.26)$$

where $\Delta v = v - v_{\text{пр}}$ ($v_{\text{пр}}$ - programmed velocity).

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In the general case of the equation of control, it is possible to write thus:

$$\delta_1 = \sum_{0}^{j} F_1 (\xi_1 - \xi_{1\text{пр}})^j; \qquad (3.27)$$

$$\left.\begin{array}{l}\delta_2=\sum_{0}^{j} F_2(\xi_2-\xi_{2\,np})^{j};\\ \cdots\cdots\cdots\cdots\\ \delta_i=\sum_{0}^{j} F_i(\xi_i-\xi_{i\,np})^{j};\end{array}\right| \quad (3.27)$$

where ξ_i - the parameter of the motion of rocket on which is conducted the control; j - last/latter derivative considered along the appropriate control channel; δ_i - displacement of the corresponding controls; F_i - operator of control along the appropriate channel.

The supplementary complexities, not reflected in the written equations, consist in the fact that the forces and torque/moments are interconnected with motion characteristics along dependences (2.65), (2.68), (2.71), (2.75), (2.76) and by others.

The solution of the complete system of equations, which describes the three-dimensional/space flight of the guided missile, is very complex and laborious; most frequently during the solution of practical problems, system is simplified.

1.2. Simplified systems of equations, which describe spatial motion of the guided missile in the dense layers of the atmosphere.

During motion along missile trajectory with the well designed and correctly working control system, the angles α, β, γ and γ_c as a rule, are small, small also angular velocities, ω_{x_1}, ω_{y_1} and ω_{z_1}.

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This makes it possible to disregard as small second-order quantities the terms, which contain the products of the sizes of the named angles and product of angular velocities; with this system of equations it will take the form

$$\left.\begin{aligned}
&1.\ \dot{v}=\frac{1}{m}\Big[\Big(P_{x_1}-\sum X_{p1}\Big)\cos\alpha\cos\beta-\Big(P_{y_1}+\\
&+\sum Y_{p1}\Big)\sin\alpha\cos\beta+\Big(P_{z_1}+\sum Z_{p1}\Big)\sin\beta-X-Q\cdot\sin\theta\Big].\\
&2.\ \dot{\theta}=\frac{1}{mv}\Big[\Big(P_{x_1}-\sum X_{p1}\Big)\sin\alpha\cos\gamma_c+\Big(P_{y_1}+\\
&+\sum Y_{p1}\Big)\cos\gamma_c\cos\alpha-\Big(P_{z_1}+\sum Z_{p1}\Big)\sin\gamma_c\cos\beta+Y^*-\\
&-Q\cdot\cos\theta\Big].\\
&3.\ \dot{\Psi}=-\frac{1}{mv\cos\theta}\Big[-\Big(P_{x_1}-\sum X_{p_1}\Big)\cos\gamma_c\cos\alpha\times\\
&\times\sin\beta+\Big(P_{y_1}+\sum Y_{p1}\Big)\sin\gamma_c\cos\alpha+\Big(P_{z_1}+
\end{aligned}\right\}\quad(3.28)$$

$$+\sum Z_{p1}(\cos\gamma_c\cos\beta + Z').$$

$$
\begin{aligned}
&4.\ J_{x_1}\dot\omega_{x_1} = \sum M_{x_1} + \sum M_{p\,x_1}.\\
&5.\ J_{y_1}\dot\omega_{y_1} = \sum M_{y_1} + \sum M_{p\,y_1}.\\
&6.\ J_{z_1}\dot\omega_{z_1} = \sum M_{z_1} + \sum M_{p\,z_1}.\\
&7.\ \dot\vartheta = \omega_{y_1}\sin\gamma + \omega_{z_1}\cos\gamma.\\
&8.\ \dot\psi = \frac{1}{\cos\vartheta}(\omega_{y_1}\cos\gamma - \omega_{z_1}\sin\gamma).\\
&9.\ \dot\gamma = \omega_{x_1} - \mathrm{tg}\,\vartheta(\omega_{y_1}\cos\gamma - \omega_{z_1}\sin\gamma).\\
&10.\ \sin\vartheta = \sin\theta\cos\alpha\cos\beta + \cos\theta\sin\alpha\cos\gamma_c.\\
&11.\ \sin\psi\cos\gamma = \sin\Psi\cos\beta\cos\gamma_c + \cos\Psi(\sin\beta\cos\theta\\
&\quad + \sin\gamma_c\sin\theta\cos\beta) - \cos\psi\sin\vartheta\sin\gamma.\\
&12.\ \cos\vartheta\sin\gamma = \sin\gamma_c\cos\beta\cos\theta - \sin\beta\sin\theta.\\
&13.\ \dot x_3 = v\cos\theta\cos\Psi.\\
&14.\ \dot y_3 = v\sin\theta.\\
&15.\ \dot z_3 = -v\cos\theta\sin\Psi.\\
&16.\ r = \sqrt{x_3^2 + y_3^2 + z_3^2}.\\
&17.\ m = m_0 - \int_0^t |\dot m|\,dt.
\end{aligned}
\qquad (3.28)
$$

Although some equations were simplified, as before their number and the number of unknowns remained equal to seventeen. Further simplification can be carried out, if one assumes that the automatic

machine of roll capture provides the location of the body axis of the rocket Oy_1 in the vertical plane, passing through the axis of rocket Ox_1, i.e., to assume laterally level flight, with $\gamma=0$, $\dot{\gamma}=0$, $\omega_{x_1}=0$.

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System of equations in this case will take the form

$$\left.\begin{aligned}
\dot{v}&=\frac{1}{m}\Big[\Big(P_{x_1}-\sum X_{\text{pl}}\Big)\cos\alpha\cos\beta-\Big(P_{y_1}+\sum Y_{\text{pl}}\Big)\times\\
&\quad\times\sin\alpha\cos\beta+\Big(P_{z_1}+\sum Z_{\text{pl}}\Big)\sin\beta-X-Q\cdot\sin\theta\Big];\\
\dot{\theta}&=\frac{1}{mv}\Big[\Big(P_{x_1}-\sum X_{\text{pl}}\Big)\sin\alpha\cos\gamma_c+\Big(P_{y_1}+\sum Y_{\text{pl}}\Big)\times\\
&\quad\times\cos\alpha-\Big(P_{z_1}+\sum Z_{\text{pl}}\Big)\sin\gamma_c\cos\beta+Y^*-Q\cdot\cos\theta\Big];\\
\dot{\Psi}&=-\frac{1}{mv\cos\theta}\Big[-\Big(P_{x_1}-\sum X_{\text{pl}}\Big)\cos\gamma_c\cos\alpha\sin\beta+
\end{aligned}\right\}\quad(3.29)$$

$$\left.\begin{aligned}
&+\Big(P_{y_1}+\sum Y_{\text{pl}}\Big)\sin\gamma_c\cos\alpha+\Big(P_{z_1}+\sum Z_{\text{pl}}\Big)\times\\
&\quad\times\cos\gamma_c\cos\beta+Z^*\Big];\\
&J_{y_1}\dot{\omega}_{y_1}=\sum M_{y_1}+\sum M_{\text{p}\,y_1};\\
&J_{z_1}\dot{\omega}_{z_1}=\sum M_{z_1}+\sum M_{\text{p}\,z_1};\\
&\dot{\vartheta}=\omega_{z_1};\quad \dot{\psi}=\frac{1}{\cos\vartheta}\,\omega_{y_1};\\
&\sin\vartheta=\sin\theta\cos\alpha\cos\beta+\cos\theta\sin\alpha\cos\gamma_c;\\
&\sin\psi=\sin\Psi\cos\beta\cos\gamma_c+\cos\Psi(\sin\beta\cos\theta+\\
&\quad+\sin\gamma_c\sin\theta\cos\beta);\\
&\sin\gamma_c=\operatorname{tg}\beta\operatorname{tg}\theta.
\end{aligned}\right\}\quad(3.29)$$

The others of five equations will remain without change.

Another way of simplification in system (3.28) is connected with the concept of the motion of rocket in balancing conditions/mode.

If we in the small trajectory phase do not consider transverse vibrations of rocket of its relatively center of mass and rotation of relatively longitudinal axis, i.e., to consider that the flow of certain time its motion is completed, with

$$\omega_{x_1} \approx \omega_{y_1} \approx \omega_{z_1} \approx 0$$

and, correspondingly,

$$\dot{\omega}_{x_1} \approx \dot{\omega}_{y_1} \approx \dot{\omega}_{z_1} \approx 0,$$

then from equations (3.14)-(3.16) we will obtain

$$\dot{\vartheta} \approx \dot{\psi} \approx \dot{\gamma} \approx 0,$$

while from (3.10)-(3.12)

$$\sum M_{x_1} + \sum M_{p\,x_1} \approx 0; \quad \sum M_{y_1} + \sum M_{p\,y_1} \approx 0;$$
$$\sum M_{z_1} + \sum M_{p\,z_1} \approx 0. \qquad (3.30)$$

Last/latter equations are called the balancing dependences which reflect equality the moments of control forces to the moments of all remaining forces, which act on rocket. The angles of attack, slip and

task, determined by balancing dependences, are called trim angles - α_6, β_6 and γ_6, a the motion of rocket with such angles along trajectory - by motion in balancing conditions/mode.

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Actually, this motion of the inertia-free rocket which at each moment of controlled flight occupies in space such angular position which necessary for motion along programmed trajectory. Angles α_6, β_6 and γ_6 slowly change along trajectory, their instantaneous values depend on the surface position, velocity of the motion of rocket and height/altitude of its flight. In the well designed and correctly working system of control, as a rule, angles α_6, β_6, γ_6 and γ_6 are small, which makes it possible to drop/omit in equations the terms, which contain the products of the sines of the named angles. With this, system of equations will have the form

$$
\left.
\begin{aligned}
&\dot{v}=\frac{1}{m}\Big[\Big(P_{x_1}-\sum X_{p1}\Big)\cos\alpha_6\cos\beta_6-\Big(P_{y_1}+\sum Y_{p1}\Big)\times\\
&\quad\times\sin\alpha_6\cos\beta_6+\Big(P_{z_1}+\sum Z_{p1}\Big)\sin\beta_6-X-Q\cdot\sin\theta\Big];\\
&\dot{\theta}=\frac{1}{mv}\Big[\Big(P_{x_1}-\sum X_{p1}\Big)\sin\alpha_6\cos\gamma_c+\Big(P_{y_1}+\sum Y_{p1}\Big)\times\\
&\quad\times\cos\gamma_6\cos\alpha_6+\Big(P_{z_1}+\sum Z_{p1}\Big)\sin\gamma_c\cos\beta_6+Y^*-Q\cdot\cos\theta\Big];\\
&\dot{\Psi}=-\frac{1}{mv\cos\theta}\Big[-\Big(P_{x_1}-\sum X_{p1}\Big)\cos\gamma_c\cos\alpha_6\sin\beta_6+\\
&\quad+\Big(P_{y_1}+\sum Y_{p1}\Big)\sin\gamma_c\cos\alpha_6-\Big(P_{z_1}+\sum Z_{p1}\Big)\times\\
&\quad\times\cos\gamma_c\cos\beta_6+Z^*\Big];\\
&\qquad\sum M_{x_1}+\sum M_{p\,x_1}\approx 0;\\
&\qquad\sum M_{y_1}+\sum M_{p\,y_1}\approx 0;\\
&\qquad\sum M_{z_1}+\sum M_{p\,z_1}\approx 0;\\
&\sin\vartheta=\sin\theta\cos\alpha_6\cos\beta_6+\cos\theta\sin\alpha_6\cos\gamma_c;\\
&\sin\psi\cos\gamma_6=\sin\Psi\cos\beta_6\cos\gamma_c+\cos\Psi(\sin\beta_6\cos\beta_6+\\
&\quad+\sin\gamma_c\sin\theta\cos\beta_6-\cos\psi\sin\vartheta\sin\gamma_6;\\
&\cos\vartheta\sin\gamma_6=\sin\gamma_6\cos\beta_6\cos\theta-\sin\beta_6\sin\theta;\\
&\qquad\dot{x}_3=v\cos\theta\cos\Psi;\\
&\qquad\dot{y}_3=v\sin\theta;\\
&\qquad\dot{z}_3=-v\cos\theta\sin\Psi;\\
&\qquad r=\sqrt{x_3^2+y_3^2+z_3^2};\\
&\qquad m=m_0-\int_0^t|\dot{m}|\,dt.
\end{aligned}
\right\}\quad(3.31)
$$

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The obtained system consists of 14 equations, which contain 14

unknowns. If one assumes that in the process of flight the angles $\alpha_6, \beta_6, \gamma_6$, and also γ_c do not exceed the values with which it is possible to accept

$$\sin\alpha_6 \approx \alpha_6; \quad \sin\gamma_6 \approx \gamma_6; \quad \cos\alpha_6 \approx 1; \quad \cos\gamma_6 \approx 1;$$
$$\sin\beta_6 \approx \beta_6; \quad \sin\gamma_c \approx \gamma_c; \quad \cos\beta_6 \approx 1; \quad \cos\gamma_c \approx 1,$$

then in system (3.13) the form of equations substantially it will be simplified and it it will be written in the form

next page

$$\left.\begin{aligned}
&\dot{v}=\frac{1}{m}\Big[P_{x_1}-\sum X_{p1}-\Big(P_{y_1}+\sum Y_{p1}\Big)\alpha_6+\\
&\quad+\Big(P_{z_1}+\sum Z_{p1}\Big)\beta_6-X-Q\cdot\sin\theta\Big];\\
&\dot{\theta}=\frac{1}{mv}\Big[\Big(P_{x_1}-\sum X_{p1}\Big)\alpha_6+\Big(P_{y_1}+\sum Y_{p1}\Big)-\\
&\quad-\Big(P_{z_1}+\sum Z_{p1}\Big)\gamma_c+Y^*-Q\cdot\cos\theta\Big];\\
&\dot{\Psi}=-\frac{1}{mv\cos\theta}\Big[-\Big(P_{x_1}-\sum X_{p1}\Big)\beta_6+\Big(P_{y_1}+\sum Y_{p1}\Big)\gamma_c+\\
&\quad+\Big(P_{z_1}+\sum Z_{p1}\Big)+Z^*\Big];\\
&\sum M_{x_1}+\sum M_{p\,x_1}=0;\\
&\sum M_{y_1}+\sum M_{p\,y_1}=0;\\
&\sum M_{z_1}+\sum M_{p\,z_1}=0;\\
&\sin\vartheta=\sin\theta+\alpha_6\cos\theta;\\
&\sin\psi=\sin\Psi+\cos\Psi(\beta_6\cos\theta+\gamma_c\sin\theta)-\gamma_6\cos\psi\sin\vartheta;\\
&\gamma_6\cos\vartheta=\gamma_c\cos\theta-\beta_6\sin\theta;\\
&\dot{x}_3=v\cos\theta\cos\Psi;\\
&\dot{y}_3=v\sin\theta;\\
&\dot{z}_3=-v\cos\theta\sin\Psi;\\
&r=\sqrt{x_3^2+y_3^2+z_3^2};\\
&m=m_0-\int_0^t|\dot{m}|\,dt.
\end{aligned}\right\} \quad (3.32)$$

If is examined the action of the rotating or slow rolling rocket, then, obviously, $\omega_{x_1}\neq 0$, while the legitimacy of assumption $\omega_{y_1}\approx\omega_{z_1}\approx 0$ should be checked by supplementary investigations.

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Essential simplification in the system of equations, which describes spatial motion, can be obtained for axisymmetric rockets and projectiles whose thrust does not create moment with respect to the center of mass and it is directed along axis of rocket, i.e., when

$$P = P_{x_1}; \quad P_{y_1} = P_{z_1} = 0.$$

Forces $\Sigma X_{p1}, \Sigma Y_{p1}, \Sigma Z_{p1}$ let us consider in implicit form, after include/connecting them respectively in drag, hoisting and lateral forces. Then from (3.4)-(3.9) we obtain

$$\begin{aligned}
\dot{v} &= \frac{1}{m}(P\cos\alpha\cos\beta - X - Q\cdot\sin\theta);\\
\dot{\theta} &= \frac{1}{mv}[P(\sin\alpha\cos\gamma_c + \cos\alpha\sin\beta\sin\gamma_c) + \\
&\quad + Y\cos\gamma_c - Z\sin\gamma_c - Q\cdot\cos\theta];\\
\dot{\Psi} &= -\frac{1}{mv\cos\theta}[P(\sin\alpha\sin\gamma_c - \cos\alpha\sin\beta\cos\gamma_c) + \\
&\quad + Y\sin\gamma_c + Z\cos\gamma_c].
\end{aligned} \tag{3.33}$$

From (3.10)-(3.12)

$$J_{x_1}\dot{\omega}_{x_1}=\sum M_{x_1}-(J_{z_1}-J_{y_1})\,\omega_{y_1}\omega_{z_1}+\sum M_{p\,x_1};$$
$$J_{y_1}\dot{\omega}_{y_1}=\sum M_{y_1}-(J_{x_1}-J_{z_1})\,\omega_{z_1}\omega_{x_1}+\sum M_{p\,y_1}; \qquad (3.34)$$
$$J_{z_1}\dot{\omega}_{z_1}=\sum M_{z_1}-(J_{y_1}-J_{x_1})\,\omega_{x_1}\omega_{y_1}+\sum M_{p\,z_1}.$$

If the angular velccities ω_{y_1}, ω_{z_1} cf the rocket (projectile) in rotary motion are low and $\omega_{x_1}=0$, then equaticns (3.34) can be replaced with balancing dependences (3.30)., frcm which let us find trim angles α_6, β_6 and will introduce them in (3.33). Additionally, as before let us accept $\sin\alpha_6\approx\alpha_6$, $\sin\beta_6\approx\beta_6$, $\sin\gamma_c\approx\gamma_c$, a the cosines cf these angles let us equate tc unit. For hoisting and lateral fcrces let us accept the linear dependences

$$Y=Y^{\alpha}\alpha \quad и^{(1)} \; Z=-Z^{\beta}\beta, \qquad (3.35)$$

where

$$Y^{\alpha}=qSc_y^{\alpha} \quad и^{(1)} \; Z^{\beta}=qS\,|c_z^{\beta}|. \qquad (3.36)$$

Key: (1) and.

Introducing the adopted simplifications, we will obtain from (3.33)

$$\dot{v}\approx\frac{P-X}{m}-g\sin\theta; \qquad (3.37)$$
$$\dot{\theta}\approx\frac{(P+Y^{\alpha})\alpha_6+(P+Z^{\beta})\beta_6\gamma_c}{mv}-\frac{g\cos\theta}{v}; \qquad (3.38)$$
$$\dot{\Psi}\approx-\frac{(P+Y^{\alpha})\alpha_6\gamma_c-(P+Z^{\beta})\beta_6}{mv\cos\Psi}. \qquad (3.39)$$

From equations (3.17), (3.18) and (3.19) we will obtain

$$\sin\vartheta \approx \sin\theta + \cos\theta(\alpha_6 + \beta_6\gamma_c);$$
$$\sin\psi \approx \sin\Psi + \cos\Psi(\beta_6\cos\theta + \gamma_c\sin\theta); \qquad (3.40)$$
$$\gamma_c \approx \beta_6 \operatorname{tg}\theta.$$

Equations (3.20)-(3.24) will remain without change.

Thus, as a result of the simplifications conducted we will obtain system from 11 equations, the unknowns in it will remain 13:

$$v(t);\ \theta(t);\ \Psi(t);\ x_3(t);\ y_3(t);\ z_3(t);\ r(t);$$
$$\vartheta(t);\ \psi(t);\ \alpha_6(t);\ \beta_6(t);\ \gamma_c(t),\ m(t).$$

System can be solved, if we to the written equations add the kinematic equations, which determine the method of control, or to accept for two of these unknown values the specific law of a change in them in time (program). For example, for the rockets of class "Earth- Earth" can be additionally assigned $\vartheta(t)$ and $\psi(t)$.

In equations (3.38) and (3.39) the terms, which contain the products of small angles $\beta_6\gamma_c$ and $\alpha_6\gamma_c$, are considerably less than remaining terms, and when conducting of approximate computations them it is possible to drop/omit. Then instead of equations (3.38) and (3.39) we are have

$$\dot\theta \approx \frac{(P+Y^{\alpha})\alpha_6}{mv} - \frac{g\cos\theta}{v} \qquad (3.41)$$

$$\dot\Psi \approx \frac{(P+Z^{\beta})\beta_6}{mv\cos\theta}. \qquad (3.42)$$

The three-dimensional/space flight of the guided missile with engine off can be described by the system of equations, obtained from (3.28), if we place in it equal to zero thrust component torque/moments from thrust and to accept m = const. In this case, the first three equations take the form

$$\begin{aligned}
\dot{v}=&\frac{1}{m}\Big[-\cos\alpha\cos\beta\sum X_{p1}-\sin\alpha\cos\beta\sum Y_{p1}+\\
&+\sin\beta\sum Z_{p1}-X-Q\sin\theta\Big];\\
\dot{\theta}=&\frac{1}{mv}\Big[-\sin\alpha\cos\gamma_c\sum X_{p_1}+\cos\gamma\cos\alpha\sum Y_{p1}-\\
&-\sin\gamma_c\cos\beta\sum Z_{p1}+Y^*-Q\cos\theta\Big];\\
\dot{\psi}=&-\frac{1}{mv\cos\theta}\Big[\cos\gamma_c\cos\alpha\sin\beta\sum X_{p1}+\sin\gamma_c\times\\
&\times\cos\alpha\sum Y_{p1}-\cos\gamma_c\cos\beta\sum Z_{p1}+Z^*\Big],
\end{aligned}\qquad(3.43)$$

and the others of thirteen equations of system (3.28) can be undertaken without change.

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1.3. Systems of equations, which describe the spatial motion of the unguided rocket in the dense layers of the atmosphere.

The systems of equations, given in two preceding/previous sections, can be rebuilt in connection with the motion of the unguided rocket, if we drop/omit in them the equations of control (3.27) and from remaining equations to exclude members, who contain control forces and torque/moments. For example, on the base of system (3.28) we will obtain for the unguided rocket

$$
\left.\begin{aligned}
\dot{v} &= \frac{1}{m}[P_{x_1}\cos\alpha\cos\beta - P_{y_1}\sin\alpha\cos\beta + P_{z_1}\sin\beta - \\
&\qquad - X - Q\sin\theta];\\
\dot{\theta} &= \frac{1}{mv}[P_{x_1}\sin\alpha\cos\gamma_c + P_{y_1}\cos\gamma\cos\alpha - P_{z_1}\sin\gamma_c \times \\
&\qquad \times\cos\beta + Y^* - Q\cos\theta];\\
\dot{\Psi} &= -\frac{1}{mv\cos\theta}[-P_{x_1}\cos\gamma_c\cos\alpha\sin\beta + P_{y_1}\sin\gamma_c\cos\alpha + \\
&\qquad + P_{z_1}\cos\gamma_c\cos\beta + Z^*];\\
J_{x_1}\dot{\omega}_{x_1} &= \sum M_{x_1};\\
J_{y_1}\dot{\omega}_{y_1} &= \sum M_{y_1};\\
J_{z_1}\dot{\omega}_{z_1} &= \sum M_{z_1}.
\end{aligned}\right\} \quad (3.44)
$$

The others of 11 equations of system (3.28) will remain without change.

§2. The principles of the separation of spatial motion into forward/progressive and rotational, longitudinal are lateral.

The separation of spatial motion into the forward motion of the center of mass and rotary relative to the center of mass, into

longitudinal and lateral significantly simplifies the equations of external ballistics and their solution.

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With a strict theoretical approach to the solution of problem, the separation of the equations, which describe complex spatial motion, into the equations, which describe only forward motion of the center of mass or only rotary motion relative to the center of mass, and into the equations, which describe the separately longitudinal and transverse motions of the center of mass, obviously, is impossible. The rotary motion of the center of mass of solid body of variable mass (rocket) and of the body of constant mass (projectile) and the forward motion of the center of mass of these bodies mutually connected in principle. During the motion of bodies in air, basic communication/connection between these two forms of motions is realized through aerodynamic forces and the torque/moments (see §4 and §5, chapter II). Furthermore, the interdependence of forward/progressive and rotary motion for the guided and unguided missiles and projectiles is exhibited through the so-called cross couplings. Depending on the formulation of the problem during the compilation of the three-dimensional equations of motion of the rockets and projectiles, are considered inertia cross couplings and aerodynamic cross couplings, but during the compilation of the

three-dimensional equations of motion of the controlled objects in the expanded setting, must be considered also the cross couplings of the control system along the channels of pitch, yaw and bank.

In equations of motion, inertia cross couplings are exhibited through the terms, which contain products of inertia, and the terms, which contain the products of the angular velocities of form [see, for example, system of equations (1.20)]. To inertia cross couplings can be attributed also communication/connections, determined by forces and Coriolis's torque/moments, which appear during the displacement of masses of the relatively oscillating missile body [see equations (1.12) and (1.14)]. Aerodynamic cross couplings are determined by the dissymmetry of the flow around rocket or projectile in flight and connected with this field distribution of velocities according to their external enclosures. The cross couplings of the control system most noticeably are exhibited through a difference in readings of measuring and actuating elements.

The account of cross couplings in the systems of equations, which describe the motion of rockets and projectiles, becomes complicated by the fact that the acting forces, their torque/moments and the moments of inertia are determined relative to different coordinate systems and their communication/connection they are expressed by complex functions, changing in the process of moving of

rockets and projectiles.

The systematic generalization of investigations in the account of the effect of cross couplings on spatial motion of flight vehicles in air is stated in works [7-52].

For rockets and the projectiles of class the "surface - surface", as show theoretical and experimental studies, the separation of motions during the solution of ballistic problems into the forward motion of the center of mass and rotary motion relative to the center of mass and the separation of forward motion into longitudinal and lateral, makes it possible to obtain the virtually acceptable accuracy of the calculations whose results will agree well with the data of trial firings.

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2.1. Separation of motion into the forward motion of the center of mass and rotary motion relative to the center of mass.

Division of motion into forward/progressive and rotary is conducted most frequently during independent approximate solution of one of the basic ballistic tasks of determining of the motion characteristics of the center of mass of rocket (projectile) and of

the tasks, connected with stability of motion, by oscillation/vibrations and control.

In the written above systems of equations, it is possible to isolate the equations, which determine the motion of the center of mass, and the equations, which determine motion relative to the center of mass. For example, in system (3.26) the left sides of equations 1, 2, 3, 13, 14, 15 - the derivatives of the characteristics, which determine the motion of the center of mass of rocket; equation 4-9 contain the values, which determine motion relative to the center of mass. Trigonometric equations 10-12 determine the additional constraints between eight angles θ, Ψ, γ_c, ϑ, ψ, α and β. The first two angles can be determined during the joint solution of equations 1, 2, 3, 14 and 17; the others - during the solution of equations 4 -9. It is logical that the separate solution is possible with essential simplifications. In equations 1, 2, 3, which describe the motion of the center of mass (material point), it is necessary to accept $\alpha=\beta=\gamma=\gamma_c=0$, then

$$
\left.
\begin{aligned}
\dot{v}&=\frac{1}{m}\left[\left(P_{x_1}+\sum X_{p1}\right)-X-Q\sin\theta\right];\\
\dot{\theta}&=\frac{1}{mv}\left[\left(P_{y_1}+\sum Y_{p1}\right)+Y-Q\cos\theta\right];\\
\dot{\Psi}&=-\frac{1}{mv\cos\theta}\left[P_{z_1}+\sum Z_{p1}+Z\right].
\end{aligned}
\right| \qquad (3.45)
$$

After the addition of equations 14 and 17, system will be locked and it can be solved independently. For definition of six angles ϑ,

$\psi, \gamma, \alpha, \beta$ and γ_c and three angular velocities $\omega_{x_1}, \omega_{y_1}, \omega_{z_1}$ can be used nine equations 4-12. These equations cannot be solved independently, since for determining the torque/moments in the right sides of equations 4-6 it is necessary to know velocity and flight altitude. Taking into account the technical difficulties of the joint solution of nine equations, in practical work utilize an artificial method of the separation of system into the equations, which describe longitudinal, lateral and rotary motion into the longitudinal and lateral stated in following section. For describing the rotary motion of rocket or projectile (artillery and turbojet) is utilized equation (1.33).

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2.2. Separation of motion into longitudinal and lateral.

During the separation of the complex motion of rockets into the longitudinal and lateral, accept, that in the first approximation, the axial motion does not depend on the lateral. In accordance with this in equations for determining of longitudinal-behavior characteristics, are considered only those forces and the torque/moments, which act in range plane. With determination of the lateral-behavior characteristics of rocket, to consider it independent variable of the longitudinal is impossible; therefore in

equations are included all forces and the torque/moments, which to a certain degree can cause the deviations of rocket from range plane. For an example we will obtain from (3.28) the system of equations, which describes the longitudinal (flat/plane) motion of the guided missile.

$$\left.\begin{aligned}
&\dot{v}=\frac{1}{m}\Big[\Big(P_{x_1}-\sum X_{p1}\Big)\cos\alpha-\Big(P_{y_1}+\sum Y_{p1}\Big)\times\\
&\qquad\times\sin\alpha-X-Q\sin\theta\Big];\\
&\dot{\theta}=\frac{1}{mv}\Big[\Big(P_{x_1}-\sum X_{p_1}\Big)\sin\alpha+\Big(P_{y_1}-\sum Y_{p1}\Big)\times\\
&\qquad\times\cos\alpha+Y^{*}-Q\cos\theta\Big];\\
&J_{z_1}\dot{\omega}_{z_1}=\sum M_{z_1}+\sum M_{p\,z_1};\\
&\dot{\vartheta}=\omega_{z_1};\\
&\sin\vartheta=\sin\theta\cos\alpha+\cos\theta\sin\alpha=\sin(\theta+\alpha);\\
&\dot{x}_3=v\cos\theta;\\
&\dot{y}_3=v\sin\theta;\\
&r=\sqrt{x_3^2+y_3^2};\\
&m=m_0-\int_0^t|\dot{m}|\,dt.
\end{aligned}\right\}\quad(3.46)$$

Trigonometric equation in system (3.46), that connects angles ϑ, θ and α, in a series of the cases to conveniently replace with the equality

$$\vartheta=\theta+\alpha. \qquad (3.47)$$

If in the first two equations control forces are unknown, then to system one should add the equation of control, for example - (3.25). The obtained system of equations is locked and can be solved.

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Let us write now system of equations for the determination of the lateral-behavior characteristics of rocket, assuming laterally level flight ($\gamma=0$)

$$\left.\begin{aligned}
\dot{\Psi} &= -\frac{1}{mv\cos\theta}\Big[-\big(P_{x_1}-\sum X_{p1}\big)\cos\gamma_c\sin\beta+\\
&\quad+\big(P_{y_1}+\sum Y_{p1}\big)\sin\gamma_c-\big(P_{z_1}+\sum Z_{p1}\big)\cos\beta+Z^*\Big];\\
J_{y_1}\dot{\omega}_{y_1} &= \sum M_{y_1}-\sum M_{py_1};\\
\dot{\psi} &= \frac{1}{\cos\theta}\,\omega_{y_1};\\
\sin\psi &= \sin\Psi\cos\beta\cos\gamma_c+\cos\Psi(\sin\beta\cos\theta-\\
&\quad-\sin\gamma_c\sin\theta\cos\beta);\\
\sin\gamma_c &= \operatorname{tg}\beta\operatorname{tg}\theta;\\
\dot{z}_3 &= -v\cos\theta\sin\Psi.
\end{aligned}\right| \qquad (3.48)$$

Equations (3.48), which describe only yawing motion, cannot be solved independently, without the account of the basic values, determined by axial motion. For example, in system (3.48) the first equation is basic during parameter determination of yawing motion, but it is solved, only if are known $v(t)$, $\Psi(t)$ and $\theta(t)$. The aerodynamic forces and the torque/moments, which act in side direction, also cannot be determined, if are not known velocity and flight altitude.

System (3.48), which describes yawing motion, it was obtained

complex; furthermore, with large θ it is necessary to check the limits of the applicability of the formula, determining γ_c. Essential simplification in the system can be obtained, if we use artificial method and to accept $\vartheta=\theta=0$ in the equations, determining the angular parameters of the motion of rocket. This assumption is equivalent to the assumption that the rocket flies in horizontal position, its longitudinal axis comprises with the velocity vector of the center of mass angle β, and angle $\gamma_c=0$.

For obtaining the results, close to real, in the equations, which determine the conditional horizontal trajectory of the motion of the center of mass of rocket, its velocity must be undertaken equal $v = \cos\theta$ whose factors are determined during the solution of system of equations, which describe the axial motion of the center of mass of rocket (projectile). With the adopted assumptions from (3.48) it follows:

$$\left.\begin{aligned}
&\dot{\Psi}=-\frac{1}{mv\cos\theta}\Big[-\Big(P_{x_1}-\sum X_{p1}\Big)\sin\beta+\\
&\qquad+\Big(P_{z_1}+\sum Z_{p1}\Big)\cos\beta+Z^*\Big];\\
&J_{y_1}\dot{\omega}_{y_1}=\sum M_{y_1}+\sum M_{p\,y_1};\\
&\dot{\psi}=\omega_{y_1};\\
&\sin\psi=\sin\Psi\cos\beta+\cos\Psi\sin\beta=\sin(\Psi+\beta);\\
&\dot{z}_3=-v\cos\theta\sin\Psi.
\end{aligned}\right|\qquad(3.49)$$

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Penultimate equality it is possible to replace by the sum of the angles

$$\vartheta = \Psi + \beta. \quad (3.50)$$

If in the first equation are unknown the controlling forces, then to system (3.49) one should add the equation of control.

The flight control of rockets, as a rule, is realized along three separate channels - on pitch, to yaw and bank. Despite the fact that each control channel solves independent problems, their work proves to be interdependent that it is exhibited through the dynamic characteristics of rocket, dynamic and kinematic motion characteristics and cross aerodynamically communication/connections [see systems of equations (3.1)-(3.24) and formula (2.79)].

Therefore during the solution of problems regarding the action characteristics of the guided missiles, it can be divided into longitudinal and lateral only after setting of the legitimacy of this division on the basis of the preliminary analysis of the specific characteristics of rocket, its control system and the conditions of action.

The synthesis of complex spatial motion taking into account the special feature/peculiarities of control on each of the channels should be carried out for specific conditions.

§3. The simplified systems of equations longitudinal and yawing motions of the guided missile in the dense layers of the atmosphere.

In many instances it proves to be advisable to obtain the simplified systems of equations longitudinal and yawing motions not from the common/general/total system of spatial motion, but it is direct according to simplified diagram of the acting forces and torque/moments, comprised for a specific case.

3.1. Longitudinal controlled motion.

Let us examine the longitudinal controlled motion in the starting coordinate system which for contraction in the indexing let us designate Oxyz. Let us assume that roll and yaw control provides rocket flight in the vertical plane, passing through the place of start and the target/purpose. In this case, it is possible to count that the forces, applied to rocket, act in one plane - in range plane, and the flight trajectory - plane curve. Acceleration during flat/plane curvilinear motion can be presented as sum of the tangential (a_τ) and normal (a_n) of accelerations.

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These accelerations are directed along the natural (natural) axes of coordinates 0τn. Tangential acceleration is equal to $\dot{v}$ and it is directed tangentially toward trajectory. Normal acceleration is directed along the normal to trajectory to the side of instantaneous center of curvature and is equal v^2/ρ. Path curvature $1/\rho$ can be represented as

$$\frac{1}{\rho}=\left|\frac{d\theta}{dS}\right|,$$

where $d\theta$ - elementary change in the angle of arrival; dS - the segment element curved.

Carrying out replacement and transformations, we will obtain formula for the module/modulus of the normal acceleration

$$a_n=\frac{v^2}{\rho}=v^2\left|\frac{d\theta}{dt}\,\frac{dt}{dS}\right|=v\left|\frac{d\theta}{dt}\right|.$$

The sign of the derivative $d\theta/dt$ depends on the form of the trajectory: if θ decreases with the increase of arc - then $d\theta/dt < 0$, if vice versa - then $d\theta/dt > 0$.

Since the missile trajectory of class "surface - surface" are

convex with respect to the Earth curves, also, for them always $d\theta/dt < 0$, subsequently formula for normal acceleration we will take in the form

$$a_n = v\left|\frac{d\theta}{dt}\right| = -v\frac{d\theta}{dt}$$

(otherwise, sign "-" is a consequence of the fact that in the examination of motion in right system of coordinates Oxy to the positive direction of axis x corresponds the negative direction of the reference line of angles θ).

As an example let us comprise the system of equations, which describes the flight of winged missile in the work of its booster engine.

The circuit of the acting on winged missile forces, led to the center of mass, is represented on Fig. 3.1. Of the projections of forces or tangent and standard let us introduce in Table 3.1. The designations of forces and angles are undertaken from Fig. 3.1. The sign before the projection shows the direction comprising of the datum of force. If the direction that comprise coincides with reference direction, then stands plus, if it does not coincide, minus. control forces are given to the center of mass, force couple to figure are not shown.

In Table 3.1 by X_{p1} and Y_{p1} are implied the sums of the forces, created by one-type control devices; under P - gross thrust of all engines; gravitational force is undertaken in form Q=mg.

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Let us write equations of motion in projections on tangent and the standard

$$\left.\begin{aligned} m\dot{v} &= P\cos(\alpha-\xi) - X - mg\sin\theta - X_{p1}\cos(\vartheta-\theta) - \\ &\quad - Y_{p1}\sin(\vartheta-\theta); \\ -mv\dot{\theta} &= -P\sin(\alpha-\xi) - Y + mg\cos\theta + X_{p1}\sin(\vartheta-\theta) - \\ &\quad - Y_{p1}\cos(\vartheta-\theta). \end{aligned}\right\} \quad (3.51)$$

In the equation of rotary motion, let us consider the following torque/moments:

- moment of aerodynamic forces M_z, which is approximately equal to product $Yl_{ц.м}$, where $l_{ц.м}$ - distance between centers of masses (ts.m) and the resultant pressure (ts.d) (see Fig. 3.1);

- the moment of control forces M_{pz}, which is approximately equal to $Y_{p1}l_p$, where l_p - a distance from center of pressure control

devices (TsDR) to the center of mass of winged missile.

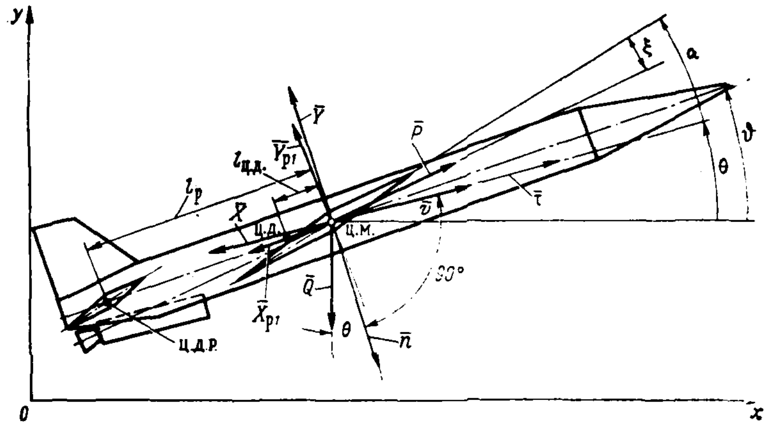

Fig. 3.1. The circuit of the action of forces on the winged missile, driving/moving in the work of the booster engine: τ - direction of tangent to trajectory; n - direction of standard to trajectory.

Tables 3.1. Force components which act on winged missile, along the axes τ and n.

(1) Сила	(2) Касательная составляющая (проекция на ось τ)	(3) Нормальная составляющая (проекция на ось n)
(4) Сила тяги	$P\cos(\alpha-\xi)$	$-P\sin(\alpha-\xi)$
(5) Лобовое сопротивление	$-X$	—
(6) Подъемная сила	—	$-Y$
(7) Сила тяжести	$-mg\sin\theta$	$mg\cos\theta$
(8) Потери тяги на рулях	$-X_{p1}\cos(\theta-\theta)$	$X_{p1}\sin(\theta-\theta)$
(9) Управляющая сила	$-Y_{p1}\sin(\theta-\theta)$	$-Y_{p1}\cos(\theta-\theta)$

Key: (1). force. (2). Tangential component (projection on axis τ). (3). Normal component (projection on axis n). (4). thrust. (5). Drag. (6). Lifting force. (7). Gravitational force. (8). Losses of thrust on controls. (9). Control force.

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The equation of rotary motion will have the form

$$J_z\ddot{\theta} = M_z + M_{pz}. \tag{3.52}$$

where J_z - axial moment inertia of winged missile relative to axis Oz_1, which in the case in question coincides with axis Oz.

To the written three equations it is necessary to add two obvious kinematic of the equation

$$\dot{x} = v\cos\theta; \quad \dot{y} = v\sin\theta \tag{3.53}$$

and of the relationship/ratio, which determine flight program.

The numerical values of the projections of all forces on the natural coordinate axes depend, except construction and the size/dimensions of flight vehicle, from the values of angles α, ε, θ and ϑ. Since booster engine to the torque/moment of its separation from winged missile is fixed, then angle ε to change in the process of flight is impossible. Angles α and θ change in the process of flight and we could be undertaken as progressed motion characteristics; however, their reliable measurement represents great difficulties.

Relatively simply and sufficiently reliably is measured in flight pitch angle ϑ. Therefore it are selected as the cell/element, which determines flight program in vertical plane.

Let us examine the axial motion of surface-to-surface missile, controlled along pitch angle. The schematic diagram of pitch control is shown on Fig. 3.2/

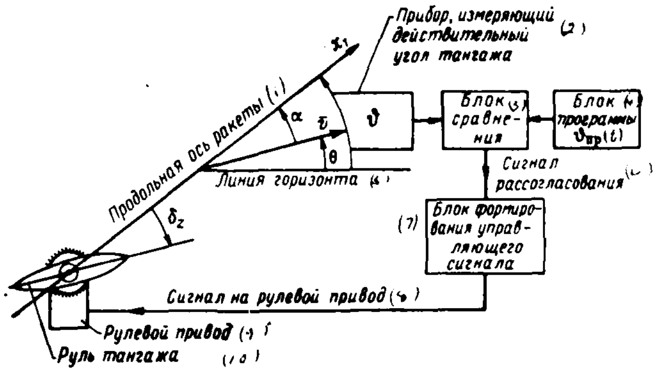

Fig. 3.2. Schematic diagram of rocket control of pitch angle.

Key: (1). Axis of rocket. (2). Instrument, which measures real angle of pitch. (3). Comparator. (4). Unit of program. (5). Line of horizon. (6). Error signal. (7). Shaping unit of control signal. (8). Signal to steering drive. (9). Steering drive. (10). Control of pitch.

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In the process of work, the unit of program develops the electrical signal, which corresponds to the programmed value of pitch angle at the given instant $\vartheta_{пр}(t)$. At the same time sensing element measures the real pitch angle ϑ and transfers the corresponding signal into the comparator which after the comparison of the signals,

proportional programmed and real pitch angles, issues the error signal. Intensified and converted in the shaping unit of control signal, it is transferred to the steering drive which correspondingly turns controls. In this case, the direction of the motion of the rocket will change so as to parry the emergent deviation from programmed trajectory. Displacement angle with respect to pitch can be approximately connected with the angle of rotation of control devices with the aid of the first component equation of control (3.25)

$$\delta_z = k_{0\vartheta}(\vartheta - \vartheta_{np}). \qquad (3.54)$$

where $k_{0\vartheta}$ - a static coefficient of the system of control.

From last/latter equality (3.30) we will obtain the balancing dependence between angles δ_z and α. For simplicity of writing, let us drop/omit the sign of sum, assuming that the rocket has only one control device, which determines its action in vertical plane.

Then

$$M_z + M_{pz} = 0. \qquad (3.55)$$

Utilizing first equation (2.86) and accepting approximately $M_{pz} = Y_{p1}^{\delta_z}\delta_z l_p$, where l_p it is taken on Fig. 3.1, we will obtain

$$\frac{\rho v^2}{2} S l m_z^{\alpha}\alpha_6 + Y_{p1}^{\delta_z}\delta_z l_p = 0.$$

Hence

$$\alpha_6 = -\frac{2Y_{p1}^{\delta} l_p \delta_z}{\rho v^2 S l m_z^{\alpha}} = -\varepsilon_\delta \delta_z. \qquad (3.56)$$

Minus sign shows that during the rotation of the governing controls, arrange/located behind the center of mass in the tail section of the rocket, to one side, for example, clockwise, the rocket will turn itself in the other direction (counterclockwise).

Utilizing (3.54), we will obtain

$$\alpha_6 = -\varepsilon_\delta k_{0\vartheta} (\vartheta - \vartheta_{np}) \qquad (3.57)$$

or, replacing in (3.57) $\vartheta = \theta + \alpha_6$, —

$$\alpha_6 = \frac{\varepsilon_\delta k_{0\vartheta} (\vartheta_{np} - \theta)}{1 + \varepsilon_\delta k_{0\vartheta}}. \qquad (3.58)$$

After taking as basis system of equations (3.51), let us suppose that force vector of thrust acts along axis of rocket.

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Furthermore, is include/connected X_{p1} in drag, and Y_{p1} - by lift. On the smallness of angle of attack, let us accept how it is earlier

$$\sin\alpha = \alpha_6;\quad \cos\alpha = 1;\quad Y = Y^{\alpha}\alpha_6.$$

Then

$$m\dot{v} = P - X - mg\sin\theta;$$
$$mv\dot{\theta} = (P + Y^{\alpha})\alpha_6 - mg\cos\theta.$$

After adding usual kinematic relationship/ratios and programmed equation for a pitch angle, we will obtain the system of equations, which approximately describes longitudinal controlled rocket flight

$$\left.\begin{aligned} &\dot{v} = \frac{P - X}{m} - g\sin\theta;\\ &\dot{\theta} = \frac{\varepsilon_{\vartheta}k_{0\vartheta}}{mv}\,\frac{P + Y^{\alpha}}{1 + \varepsilon_{\vartheta}k_{0\vartheta}}(\vartheta_{np} - \theta) - \frac{g\cos\theta}{v};\\ &\vartheta = \theta + \alpha_6 = \vartheta_{np}(t);\quad \dot{y} = v\sin\theta;\quad \dot{x} = v\cos\theta. \end{aligned}\right\} \quad (3.59)$$

System has five variable values $v(t)$; $\theta(t)$; $x(t)$; $y(t)$; $\alpha_6(t)$ can be solved, if are assigned: the program of the pitch angle $\vartheta_{np}(t)$, of dependence $c_y(M)$, ε_{ϑ}, $k_{0\vartheta}$, the dependence, which determines a change in the mass of rocket $m(t)$, and reference-flight conditions.

The simplest system of equations, which approximately describes the controlled flight of the center of mass of rocket, can be obtained from (3.59), if we consider that rocket it is control/guided ideally and its longitudinal axis it coincides with the velocity vector of flight ($\vartheta=\theta$).

Then

$$v = \frac{P-X}{m} - g\sin\theta;\quad \theta(t) \approx \theta_{np}(t);\quad \dot{y} = v\sin\theta;\ \dot{x} = v\cos\theta. \quad (3.60)$$

Here three unknowns - v(t), x(t), y(t), and if is known m(t), system it is solved simply.

If we in (3.60) assume X=0, then system of equations will describe the motion of the center of mass of the guided missile in the vacuum.

3.2. Action of the guided missile in side direction.

We utilize a system of equations (2.49). Since as the governing parameter is accepted the yaw angle ψ, the third equation in system can be omitted. Let the thrust coincide with axis of rocket, then

$$P_{x_1} - \sum X_p = P.$$

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In the absence of disturbance/perturbations in side direction, the missile trajectory coincides with range plane and, if there are no any special considerations, the programmed value of the yaw angle

should take as equal to zero. Displacement angle with respect to yaw let us connect with the angle of rotation of control devices with the aid of the static coefficient of the system of control - $k_{0\psi}$ and from (3.27) we will obtain

$$\delta_{p_1} = k_{0\psi}\psi. \qquad (3.61)$$

The second equation of system (3.49) let us replace with the balancing dependence utilizing which let us write

$$\frac{\rho v^2}{2} S l m_{y_1}^{\beta} \beta_0 + Z_{p_1}^{\delta_{p_1}} l_p \delta_{p_1} = 0.$$

Hence

$$\beta_0 = -\frac{2 Z_{p_1}^{\delta_{p_1}} l_p \delta_{p_1}}{\rho v^2 S l m_{y_1}^{\beta}} = -\varepsilon_{\psi} \delta_{p_1}. \qquad (3.62)$$

Introducing the coefficient of control and using (3.50), let us be, it is analogous (3.58), to have

$$\beta_0 = -\frac{\varepsilon_{\psi} k_{0\psi} \Psi}{1 + \varepsilon_{\psi} k_{0\psi}}. \qquad (3.63)$$

Replacing in the first equation of system (3.49) sin β on β_0 and selecting from (3.4) and (2.85) $Z^* = Z = -Z^{\beta}\beta_0$, we will obtain already known to us formula (3.42)

$$\Psi = \frac{(P + Z^{\beta})}{mv \cos \theta} \beta_0.$$

Replacing β_6 from (3.63), we will obtain the system of equations, which approximately describes the balanced flight of the guided missile in the side direction

$$\dot{\Psi} = \frac{\varepsilon_{\beta} k_{0\beta}(P + Z^{\beta})(\Psi_{np} - \Psi)}{mv \cos\theta (1 + \varepsilon_{\beta} k_{0\beta})}; \quad (3.64)$$

$$\dot{z}_3 = -v \cos\theta \sin\Psi. \quad (3.65)$$

In certain cases, for example, during the trajectory calculation of the missile targeting to the driving/moving target/purpose when are placed limitations on the orientation of the velocity vector of the center of mass of rocket in space, angles θ and Ψs can be determined apart from the written equations. Then, derivatives $\dot{\theta}$ and $\dot{\Psi}$ it is possible to find by directly numerical differentiation or analytically.

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The balance angles and slip approximately are determined from (3.41) and (3.42) from the formulas

$$\alpha_6 \approx \frac{m}{P + Y^{\alpha}} (v\dot{\theta} + g\cos\theta); \quad (3.66)$$

$$\beta_6 \approx \frac{mv\dot{\Psi}\cos\theta}{P + Z^{\beta}}. \quad (3.67)$$

Trim angles, as a rule, are small; however, with use of systems of the approximate equations, one should remember about those assumptions, with which they are obtained as far as possibility to produce the evaluation of errors. The equations of controlled flight with engine off easily can be obtained from preceding/previous, if we in them accept thrust P=0 and m = const.

§4. Systems of equations of the motion of the center of mass of rocket taking into account the rotation of the Earth.

Let us comprise system of equations in the connected geocentric system of coordinates Cxyz, convenient for the account of the effect of the form of the Earth and its rotation. The position of rocket let us determine by coordinates x, y, z, by geocentric latitude and by longitude λ (see Fig. 2.1). Let us assume that the control at each moment of time combines axis of rocket with velocity vector. During the well working control of the oscillation/vibration of the rocket relative to the center of mass it is possible not to consider and to consider the motion of rocket as material point.

The differential equation of motion of the center of mass in inertial coordinate system and the designations of formula (2.31) takes the form

$$m\left(a_{\text{от}} + a_{\text{пер}} + a_{\text{кор}}\right) = \sum F.$$

In relative motion $m\bar{a}_{отн}=\sum\bar{F}-m\bar{a}_{пер}-m\bar{a}_{кор}$. For the writing of scalar equations, let us determine the projections of the entering these dependences values to coordinate axes.

The projection of thrust on coordinate axes will be equal to the product of thrust on the cosine of the angle between the sense of the vector of force and the corresponding axis. The cosines of angles will be equal to respectively

$$\frac{v_{отн\,x}}{v_{отн}};\quad \frac{v_{отн\,y}}{v_{отн}};\quad \frac{v_{отн\,z}}{v_{отн}},$$

where $v_{отн\,x}$, $v_{отн\,y}$ and $v_{отн\,z}$ - projection of the velocity of relative motion of the center of mass of rocket on coordinate axes.

Omitting subsequently the indices of "otn", we will obtain

$$v=\sqrt{v_x^2+v_y^2+v_z^2}.$$

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The projections of thrust are determined by the formulas

$$P_x=P\frac{v_x}{v};\quad P_y=P\frac{v_y}{v};\quad P_z=P\frac{v_z}{v}.$$

For the air resistance, it is convenient to utilize in the case in question expression (2.43). Its projections on coordinate axes will be equal to force of R, multiplied on the cosines of the angles between the sense of the vector of thrust and the corresponding coordinate axis

$$R_x = \frac{\varrho v}{2} S c_R v_x; \quad R_y = \frac{\varrho v}{2} S c_R v_y; \quad R_z = \frac{\varrho v}{2} S c_R v_z. \qquad (3.68)$$

Force component of the gravity of the Earth let us determine, using formula for accelerating the force of gravity, obtained on the assumption that Earth - sphere

$$g_T = g_{T0} \frac{R_3^2}{r^2},$$

where g_{T0} - acceleration of the force of gravity on the surface of the Earth; R_3 - radius of the Earth; r - distance from the center of mass of the rocket to the conditional center of the Earth.

Force component of gravity will be equal to

$$F_{Tx} = mg_{T0} \frac{R_3^2}{r^2} \frac{x}{r}; \quad F_{Ty} = mg_{T0} \frac{R_3^2}{r^2} \frac{y}{r}; \quad F_{Tz} = mg_{T0} \frac{R_3^2}{r^2} \frac{z}{r}. \qquad (3.69)$$

where x/r; y/r; z/r - cosines of the angles between line of force F_T and the corresponding coordinate axis.

Utilizing these formulas, and also (2.33) and (2.35), we will obtain the system of the differential equations, which describe the motion of the center of mass of the rocket

$$\left.\begin{aligned}\dot{v}_x &= P\frac{v_x}{v}\frac{1}{m}-\frac{\varrho v}{2}Sc_R v_x\frac{1}{m}-g_{T0}\frac{R_3^2}{r^3}x+\Omega^2 x-2\Omega v_z;\\ \dot{v}_y &= P\frac{v_y}{v}\frac{1}{m}-\frac{\varrho v}{2}Sc_R v_y\frac{1}{m}-g_{T0}\frac{R_3^2}{r^3}y;\\ \dot{v}_z &= P\frac{v_z}{v}\frac{1}{m}-\frac{\varrho v}{2}Sc_R v_z\frac{1}{m}-g_{T0}\frac{R_3^2}{r^3}z+\Omega^2 z+2\Omega v_x.\end{aligned}\right\} \quad (3.70)$$

Let us establish communication/connection between the position of rocket in the selected rectangular coordinate system and its geocentric latitude and the longitude. From Fig. 2.1 and formulas (2.1) we will obtain

$$x=r\cos\varphi_{ru}\sin\lambda;\quad y=r\sin\varphi_{ru};\quad z=r\cos\varphi_{ru}\cos\lambda. \quad (3.71)$$

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Differentiating (3.71) twice on time, converting the obtained differential equations in accordance with dependences (3.70) and

remembering that

$$v_x=\dot{x};\ v_y=\dot{y};\ v_z=\dot{z} \text{ and } \dot{v}_x=\ddot{x};\ \dot{v}_y=\ddot{y};\ \dot{v}_z=\ddot{z},$$

we will obtain the system of the differential equations which describe the trajectory of relative motion of rocket in the connected geocentric spherical coordinates. Taking into account unwieldiness of system, let us write it in the form of the functional dependences

$$\left.\begin{aligned}\ddot{r}&=f_1(r,\ \varphi_{ru},\ \lambda,\ \dot{r},\ \dot{\varphi}_{ru},\ \dot{\lambda},\ t);\\ \ddot{\varphi}_{ru}&=f_2(r,\ \varphi_{ru},\ \lambda,\ \dot{r},\ \dot{\varphi}_{ru},\ \dot{\lambda},\ t);\\ \ddot{\lambda}&=f_3(r,\ \varphi_{ru},\ \lambda,\ \dot{r},\ \dot{\varphi}_{ru},\ \dot{\lambda},\ t).\end{aligned}\right| \quad (3.72)$$

The complexity of the solution of the written equations is obvious. However, difficulties are overcome with the aid of electronic computers.

From common/general/total system of equations (3.72) it is easy to obtain the systems, which describe special cases of motion. The motion characteristics of the center of mass of the rocket with the shut-down engine (on inactive leg) can be determined during the use of equations (3.70), in which it is necessary to equate P zero. During the calculation of motion at high altitudes in near-earth evacuated space when it is possible not to consider the air resistance, it is necessary in (3.70) to drop/omit the terms,

undertaken from equations (3.68).

§5. Equations of motion of the guided missile in flat/plane central gravitational field.

Let us comprise the system of equations, which describes the controlled flight of the center of mass of ballistic missile on powered flight trajectory. Let us examine two-dimensional problem and will consider the rotational effect of the earth partially, through the acceleration of gravity. Control forces Y_{pi} and X_{pi} separately consider we will not, but it is include/connected them in drag and lift. Thrust it is directed along axis of rocket. System of equations let us comprise into projection on the axis of the rectangular starting system of coordinates $0xy(0x_{cт}y_{cт})$ (Fig. 3.3). The projection of velocity vector on axis Ox let us desigrate through u, while projection on axis Oy through w. The projections of gravitational force on the same axis let us designate respectively through g_x and g_y; $g_x = g\sin\gamma$; $g_y = g\cos\gamma$.

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After writing equations of motion in projections on the selected coordinate axes and after adding usual kinematic and trigonometric ratio, obvious from Fig. 3.3, we will obtain system of equations

$$
\left.
\begin{aligned}
&\frac{du}{dt}=\frac{P\cos(\theta+\alpha)-X\cos\theta-Y\sin\theta}{m}-g\sin\gamma;\\
&\frac{dw}{dt}=\frac{P\sin(\theta+\alpha)-X\sin\theta+Y\cos\theta}{m}-g\cos\gamma;\\
&\frac{dx}{dt}=u;\ \frac{dy}{dt}=w;\\
&\operatorname{tg}\theta=\frac{w}{u};\quad \operatorname{tg}\gamma=\frac{x}{R_3+y};\\
&v=\sqrt{u^2+w^2};\\
&r=\sqrt{(R_3+y)^2+x^2};\\
&g=g_0\left(\frac{R_3}{r}\right)^2.
\end{aligned}
\right| \qquad (3.73)
$$

If we to the written equations add the equation, which determines tilt angle $(\alpha+\theta)=\vartheta=\vartheta_{np}(t)$ or programmed angle of attack $\alpha_{np}(t)$, dependence for determining the lift and the equation, which determines a change in the mass of rocket $m(t)$, then system can be solved numerically.

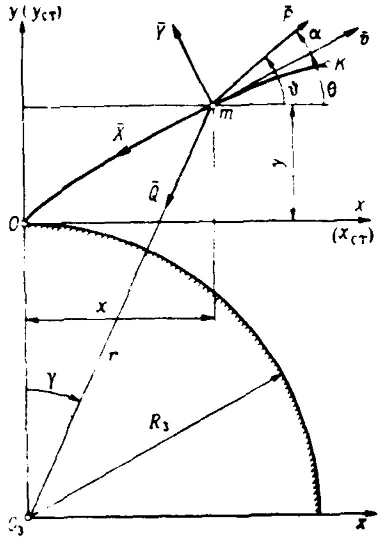

Fig. 3.3. Circuit of the forces, which act on the long range ballistic missile in the examination of its relative motion in central flat/plane gravitational field.

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The system of equations, which describes the controlled flight of the center of mass of ballistic missile (or its nose section) on inactive leg taking into account the effect of the air resistance, it is easy to obtain from system (3.73), if we accept thrust P=0 and m=const

$$
\left.
\begin{aligned}
&\frac{du}{dt} = -\frac{X\cos\theta + Y\sin\theta}{m} - g\sin\gamma;\\
&\frac{dw}{dt} = -\frac{X\sin\theta - Y\cos\theta}{m} - g\cos\gamma;\\
&\frac{dy}{dt} = w;\ \frac{dx}{dt} = u;\\
&v = \sqrt{u^2 + w^2};\\
&\mathrm{tg}\,\theta = \frac{w}{u};\ \mathrm{tg}\,\gamma = \frac{x}{R_3 + y};\\
&r = \sqrt{(R_3 + y)^2 + x^2};\\
&g = g_0\left(\frac{R_3}{r}\right)^2;\\
&\alpha + \theta = \vartheta = \vartheta_{np}(t).
\end{aligned}
\right| \quad (3.74)
$$

§6. Equations of motion of the unguided rocket in plane-parallel gravitational field.

In the absence of control or its disconnection on any reasons rocket flight becomes unguided. The unguided rockets are intended for a firing to comparatively small distances; therefore in many instances it is possible not to consider the surface curvature of the Earth, but its rotation to consider approximately through the acceleration of gravity, assuming by its constant in value and direction (i.e. considering that the gravitational field of the Earth - plane-parallel). With an immobile atmosphere the missile trajectory in this gravitational field will be plane curve, passing in vertical

plane of fire. During the solution of problems, frequently is applied the system of equations, written in the natural coordinate system. We will obtain it from system (3.51), after drop/omitting the terms, which consider the effect of control. Of the majority of the unguided rockets, force vector of thrust coincides with the axis of rocket, i.e., $\varepsilon=0$. If one considers that at low angles of attack $\sin\alpha\approx 0$, $\cos\alpha\approx 1$ and $Y=0$ and to add usual kinematic relationship/ratios, then we will obtain the unknown system of equations, which describes the motion of the center of mass of the unguided rocket

$$\left.\begin{aligned} &\frac{dv}{dt}=\frac{P-X}{m}-g\sin\theta; \quad \frac{d\theta}{dt}=-\frac{g\cos\theta}{v};\\ &\frac{dy}{dt}=v\sin\theta; \quad \frac{dx}{dt}=v\cos\theta. \end{aligned}\right\} \quad (3.75)$$

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With the adopted assumptions force vectors of thrust, of drag and velocity of the center of mass lie/rest on one straight line (Fig. 3.4). Figures 3.4 depicts simplified diagram of the forces, which act on the material point m, which coincides with the center of mass of the unguided rocket. The displacement of the center of mass relative to missile body it is not considered. On the basis of Fig. 3.4, let us write

$$\frac{du}{dt}=\frac{P-X}{m}\cos\theta.$$

After multiplying and after dividing the right side of the equation to m_0v, we will obtain

$$\frac{du}{dt}=\frac{1}{\mu}(D-E)u,$$

where

$$\mu=\frac{m}{m_0};\quad D=\frac{P}{m_0v};\quad E=\frac{X}{m_0v}.$$

Usually they designate $tg\theta=p$, and then

$$\frac{dp}{dt}=\frac{1}{\cos^2\theta}\frac{d\theta}{dt}.$$

Utilizing the second equation of system (3.75), we will obtain

$$\frac{dp}{dt}=-\frac{g}{u},$$

a from the third equation of system 3.75, we will obtain

$$\frac{dy}{dt}=v\sin\theta\frac{\cos\theta}{\cos\theta}=up.$$

The fourth equation of system (3.75) will remain without change. We will finally obtain system of equations in the form, convenient for the practical calculations of the powered flight trajectories of the unguided rockets

$$\left.\begin{aligned}&\frac{du}{dt}=\frac{1}{\mu}(D-E)u;\\&\frac{dp}{dt}=-\frac{g}{u};\ \frac{dy}{dt}=up;\ \ \frac{dx}{dt}=u.\end{aligned}\right\}\qquad(3.76)$$

From Fig. 3.4 it is possible also to obtain

$$\frac{du}{dt}=\frac{(P-X)\cos\theta}{m},$$
$$\frac{dw}{dt}=\frac{(P-X)\sin\theta}{m}-g.$$

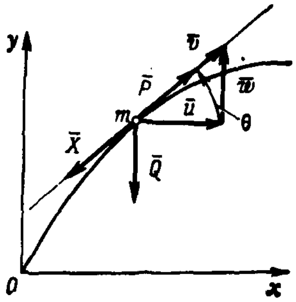

Fig. 3.4. Simplified diagram of the forces, which act on the material point, which coincides with the center of mass of the unguided rocket.

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Converting, we will obtain the system

$$\left.\begin{array}{l}\frac{du}{dt}=\frac{1}{\mu}(D-E)u;\quad \frac{dw}{dt}=\frac{1}{\mu}(D-E)w-g;\\ \frac{dy}{dt}=w;\quad \frac{dx}{dt}=u;\qquad v=\sqrt{u^2+w^2}.\end{array}\right\} \quad (3.77)$$

§7. Systems of equations with relative dimensionless arguments.

The tendency to simplify the solution of the problems of external ballistics and to make possible an application/use of various kinds of design schedules, which reduce the time of solution,

led to the development of systems of equations, in which as independent variables are accepted relative dimensionless quantities.

Let us give the system of equations for describing the motion of the center of mass of the unguided rockets, in which is utilized the independent variable

$$\lambda=\frac{\int_0^t Q_{\text{сек}}dt}{Q_0}, \qquad (3.78)$$

representing by itself the ratio of the weight of the burned down fuel/propellant to the initial weight of rocket. Let us take the first equation of system (3.75), also, on the basis (2.95) and (2.121) let us replace E and X. Let us replace also taking into account (3.78)

$$dt=\frac{Q_0}{Q_{\text{сек}}}d\lambda. \qquad (3.79)$$

After substitution let us have

$$\frac{Q_{\text{сек}}}{Q_0}\frac{dv}{d\lambda}=\frac{Q_{\text{сек}}w_e}{Q}-cH(y)F(v)-g\sin\theta. \qquad (3.80)$$

Current weight of the rocket

$$Q=Q_0-\int_0^t Q_{\text{сек}}dt.$$

Over-all payload ratio is equal to

$$\frac{Q}{Q_0}=1-\frac{\int_0^t Q_{\text{сек}}dt}{Q_0}=1-\lambda=\frac{m}{m_0}=\mu.$$

The ballistic coefficient of rocket it is possible to express by the over-all payload ratio

$$c=\frac{id^2}{Q}10^3=\frac{id^2 10^3}{Q_0(1-\lambda)}=\frac{c_0}{1-\lambda}=\frac{c_0}{\mu}, \qquad (3.81)$$

where $c_0=id^2/Q_0\ 10^3$.

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After dividing in (3.80) the right and left of part on $\frac{Q_{\text{сек}}}{Q_0}$ and after substituting into the last/latter three equations of system (3.75) dt from (3.79), we will obtain

$$\left.\begin{aligned}
&\frac{dv}{d\lambda}=\frac{w_e}{1-\lambda}-\left[\frac{c_0 H(y) F(v)}{1-\lambda}+g\sin\theta\right]\frac{Q_0}{Q_{\text{сек}}};\\
&\frac{d\theta}{d\lambda}=-\frac{Q_0}{Q_{\text{сек}}}\frac{g\cos\theta}{v};\\
&\frac{dx}{d\lambda}=\frac{Q_0}{Q_{\text{сек}}}v\cos\theta;\quad \frac{dy}{d\lambda}=\frac{Q_0}{Q_{\text{сек}}}v\sin\theta.
\end{aligned}\right\} \qquad (3.82)$$

System is applied during ballistic design. After conducting its integration, it is possible to ascertain that first term of right part one equation will correspond to formula of K. E. Tsiolkovskiy for determining the velocity of the motion of rocket without the account of the air resistance and gravitational force. The second term will consider the effect of air resistance, the third - a gravity effect.

For the guided missiles the system of equations of motion is

taken in a somewhat different form. With constant flow rate per second ($|\dot{m}|$ = const) from (3.24) we will obtain

$$m=m_0-|\dot{m}|t.$$

In this case, the over-all payload ratio of the rocket is equal to

$$\mu=\frac{m}{m_0}=1-\frac{|\dot{m}|}{m_0}t.$$

Dimensionless quantity μ frequently is selected as independent variable.

Let us designate $\tau_\phi=\frac{m_0}{|\dot{m}|}$ and then $t=\tau_\phi(1-\mu)$. Value τ_ϕ has dimensionality of time (s) and is called fictitious time. Determining from the last/latter equality

$$dt=-\tau_\phi d\mu$$

and carrying out replacement in system (3.66), we will obtain:

$$\left.\begin{aligned}&\frac{dv}{d\mu}=-\frac{P-X}{\mu m_0}\tau_\phi+g\tau_\phi\sin\theta;\\&\frac{dy}{d\mu}=-v\tau_\phi\sin\theta;\quad\frac{dx}{d\mu}=-v\tau_\phi\cos\theta.\end{aligned}\right\}\qquad(3.83)$$

It is obvious, in this case the tilt angle must be represented also on argument μ: $\theta=\theta_{np}(\mu)$.

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Chapter IV

MOTION OF ROCKETS WHEN GUIDED TO MOVING TARGETS

In the majority of the cases of the trajectory of induction, represent by themselves complex space curves. For convenience in their examination it is expedient to break into three basic sections.

The first let us consider the launching phase, the second - the section of conclusion/derivation and by the third - the section of motion along the calculated trajectory of induction. Figures 4.1 shows the circuits of trajectories of induction for the anti-tank guided missile (PTURS), an aircraft rocket and the AA guided missile (ZUR), controlled on ray/beam.

On the launching phase the flight most frequently either unguided or is realized according to the predetermined program. Of some types of rockets on the first phase of flight, works the special booster engine, sometimes detached after the end/lead of the work (for antiaircraft guided missiles).

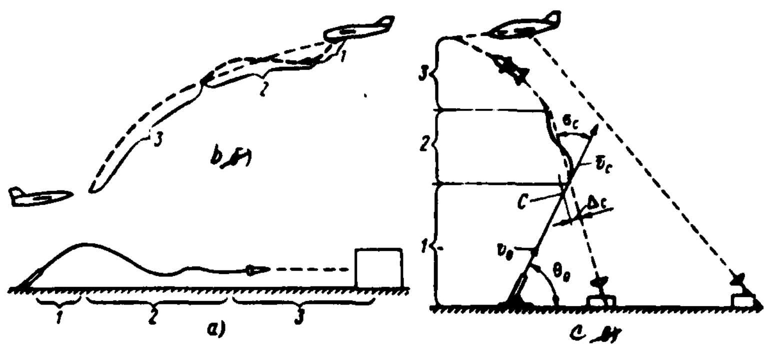

Fig. 4.1. The schematic of the trajectories of induction for PTURS (a), an aircraft rocket (b) and an antiaircraft guided missile (c): 1 - the launching phase; 2 - section of conclusion/derivation; 3 - section of induction.

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On the second section enters into action the control system and rocket it is derive/concluded to the trajectory, which corresponds to the selected guidance method to target/purpose.

By the force of different reasons toward the end of the launching phase, at the moment of the connection/inclusion of control, the position of the center of mass of rocket in space, the senses of the vector of speed and axis of rocket do not correspond to

the required trajectory of induction to target/purpose. For example, as shown in Fig. 4.1c, the control system is included at point C, when the center of mass of rocket is located at a distance Δ_C from the axis of lead beam, and the velocity vector of the center of mass with approach to ray/beam is directed toward it at angle σ_C; by the same, approximately, will be the angle of the slope of axis of rocket to lead beam.

By the force of the inertia properties of rocket and system of rocket control, emerges to the calculated trajectory of induction after certain transient process whose duration determines the length of the section of conclusion/derivation (section 2 in Fig. 4.1), that is the component part of the common trajectory.

The concrete/specific/actual form of the third phase of trajectory, section of the induction at the end of which must be provided the encounter of rocket with target, is determined by network of induction accepted to the target/purpose or by the guidance method which characterizes the kinematic relation between the parameters of the motion of rocket and target/purpose.

The differential equations, which describe the motion of rocket on the launching phase, depend on the construction of rocket. In the case of flight according to program, it is possible to use system of

equations (3.60), after introducing into it the appropriate programmed equations. In the case of the unguided flight calculation can be conducted, utilizing a system of equations (3.75). Initial conditions are determined according to the predicted circuit of the work of the complex of control and induction. For example, during calculations in connection with the circuit, presented in Fig. 4.1c, it is necessary to know the possible range of angles θ_0 and the velocity of the descent of rocket from the guides v_0. As a result of the calculation of the launching phase, must be obtained the characteristics of trajectory up to the torque/moment of the beginning of work the system of control, necessary for the calculation of transient process. So, in the examine/considered by us example it is necessary to know possible ranges of sizes ε_c, σ_c and Δ_c.

The second trajectory phase, the section of passage from the unguided flight to controlled, must be calculated with the enlistment of the methods of the theory of the flight control.

Let us return to an example in question. According to the circuit, depicted in Fig. 4.1c, the control must ensure the beam capture, i.e., expand/develop rocket so that the vector $\bar{v}$ (or the axis of rocket) would coincide with the direction of lead beam. Transient process, i.e., and the section of conclusion/derivation,

will be finished when Δ and σ they will become close to zero.

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The possible character of dependence $\Delta(t)$ on the section of conclusion/derivation for the case when $\bar{v}$ intersects ray/beam, it is shown on Fig. 4.2; qualitatively the same will be dependence $\sigma(t)$.

Under actual conditions dependence $\Delta(t)$ and $\sigma(t)$ they are more complex, since vector $\bar{v}$ and lead beam in the general case are located in different planes.

In this case, the dependence $\Delta(t)$ will have the same character, as shown on on Fig. 4.2, but the trajectory of the center of mass of rocket will represent by itself no longer plane, but space curve. Performance calculation of the motion of rocket along this trajectory taking into account the variability of the mass of rocket and rate of its motion is very complex. At the same time during well working control, the section of conclusion/derivation barely affects the subsequent trajectory of induction. Therefore, without examining here the tasks of the flight control, it is possible in the first stages of design the second trajectory phase to represent smooth curve, for example in the form of circular arcs with the radii, determined by the permissible normal accelerations of rocket. With minimum target

range; the calculation of the second section should be carried out more accurately with the fact in order to check the possibility of guiding rocket to target.

The third and basic trajectory phase - the section of induction, it is depicted in Fig. 4.1 in the form of the smoothly changing in the plane of drawing curves; in actuality under the effect on the rocket of all forces and torque/moments, its motion on the section of induction bears vibrational three-dimensional nature. Axis of rocket oscillates around the center of mass, and auto/self-n moves over the complex trajectory, which passes closely that depicted curve, as if winding it. Of rockets with the well working system of control of oscillation/vibration, they are completed smoothly, with small deviations from the calculated trajectory of motion. Therefore, examining the trajectories of induction, we in this section will not concern the effect of oscillation/vibrations on calculated trajectory.

Are well known four classes of methods of guidances, which are distinguished by the parameters on which are placed the limitations, which determine the motion of the rocket during its approach for target/purpose.

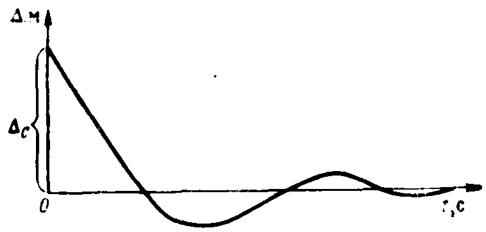

Fig. 8.2. Graph/diagram of dependence Δ(t) the process of transition from the unguided phase of flight to controlled.

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With the command methods of position control, of target/purpose and rocket in space is determined relative to guidance station. In the general case line "guidance station - target/purpose" and line "guidance station - rocket" they do not coincide. The position of each of the lines in space is determined by two angles. The first class of guidance methods, which sets limitations on these angles, is called the class of angular methods. In the particular case when angles coincide, the rocket during approach for target/purpose will be located on straight line "guidance station - target/purpose". This variety of angular method calls the method of coincidence.

The second class covers methods of guidances which set

limitations on the position of longitudinal axis of flight vehicle relative to the line of sighting (line, passing through the centers of mass of target/purpose and of rocket). The third class of guidance methods limits the sense of the vector of the velocity of the center of mass of rocket relative to the line of sighting. In the methods of the fourth class, they are placed limitations on the position of the very line of sighting in space.

§1. Guidance methods to the driving/moving target/purpose.

1.1. Angular guidance method.

The method of firing into set forward point is the basic method of the firing of cannon-type artillery at the rapidly moved target/purposes and detailed. As is known, the position of set forward point is selected so that for the time of the motion to it of projectile target/purpose also would move into set forward point (or into collision point $\dot{Y}$).

If we designate through ε_n and ε_p the angles of sighting and rocket, but through A_n and A_p - their azimuths, then communication/connection between angles will be determined, from Fig. 4.3, by the simple equalities

$$A_p = A_n + \Delta A_p; \quad (4.1)$$

$$\varepsilon_p = \varepsilon_n + \Delta\varepsilon_p. \quad (4.2)$$

In the process of guidance, the lead angles $\Delta\varepsilon_p$ and ΔA_p are alternating/variable. Is known many formulas, on which are assigned the lead angles. In general form functional dependences can be expressed thus:

$$\left.\begin{aligned}\Delta\varepsilon_p &= f_1(r_u, r_p, \dot{r}_u, \dot{r}_p, \varepsilon_u, A_u, a_1, b_1, \dots);\\ \Delta A_p &= f_2(r_u, r_p, \dot{r}_u, \dot{r}_p, \varepsilon_u, A_u, a_2, b_2, \dots),\end{aligned}\right\} \quad (4.3)$$

where r_u and r_p - a distance (radii) of target/purpose and rocket from the beginning of the coordinate system.

$\dot{r}_u$, $\dot{r}_p$ - rate of change of the values of radii into the process of guidance.

a_1, b_1, a_2, b_2 and so forth - constant values, characteristic for this control system.

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The concrete/specific/actual form of functional dependence is established during the design of entire missile complex. Here it is possible to indicate two common/general/total requirement. The direction of the motion of rocket must be changed smoothly, with the possible approach/approximation of trajectory to rectilinear. It is

lcgical also that the value of current angle-offs $\Delta\varepsilon_p$ and ΔA_p, as far as possibility, they must decrease in the process of guidance, and when $r_u = r_p$ must be made condition $\Delta\varepsilon_p = \Delta A_p = 0$. Otherwise the rocket will fly wide of the mark.

1.2. Guidance using the matching method.

In guidance using the matching method, the control system holds the missile on a straight line connecting the guidance station with the target. As a result of this, it is somethimes said that the missile is guided by the target-bracketing method and the guidance trajectory is called a three-point curve. The graphical method for constructing a plane three-point curve is explained by Fig. 4.4. The target's position is noted on its trajectory 0, 1, 2, 3, ..., which correspoind to the successive time values t_0, t_1, t_2,....

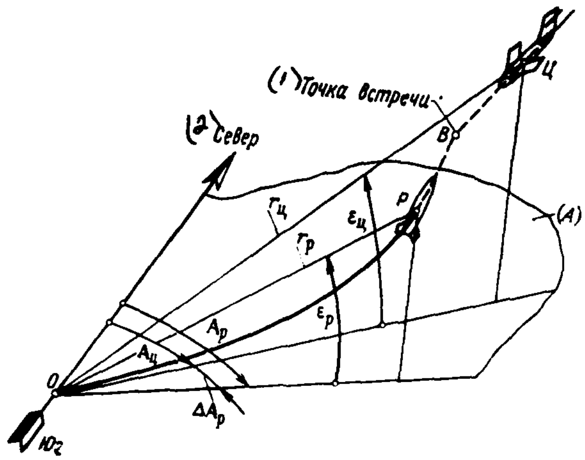

Fig. 4.3. The position of surface-to-air missile and target/purpose before the rendezvous: (A) - horizontal plane; $\varepsilon_ц$ - angle of sighting; ε_p - angle of elevation of rocket; $A_ц$ - azimuth of target/purpose; A_p - missile bearing.

Key: (1). Collision point. (2). North.

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If very guidance station is moved (for example, with missile targeting from ship or aircraft), then in its trajectory will be also deposited marks by 0, 1, 2, 3, ... for the same selected moments of time. Marks in the trajectories of the motion of target/purpose and guidance station, which correspond to the identical moments of time,

are connected by straight lines. In the particular case, with fixed guidance station, let us have the convergent at the point of its arrangement/position pencil of straight lines (see Fig. 44). On one of the straight lines by point P_0 they note the position of rocket, which corresponds to the beginning of the trajectory of the guidance (for the determination of this point one should focus special attention, if the place of missile takeoff is arrange/located at considerable removal/distance from guidance station and them it is not possible to accept combined). From the beginning of the trajectory of guidance P_0, is carried out the arc with a radius of $\bar{v}_{p0}\Delta t_0$ before intersection with following line at point P_1, which shows the place where will be located the rocket with $t = t_1$.

Then similar construction is made from point P_1, determining point P_2, and continue it before the intersection of the trajectories of target/purpose and rocket.

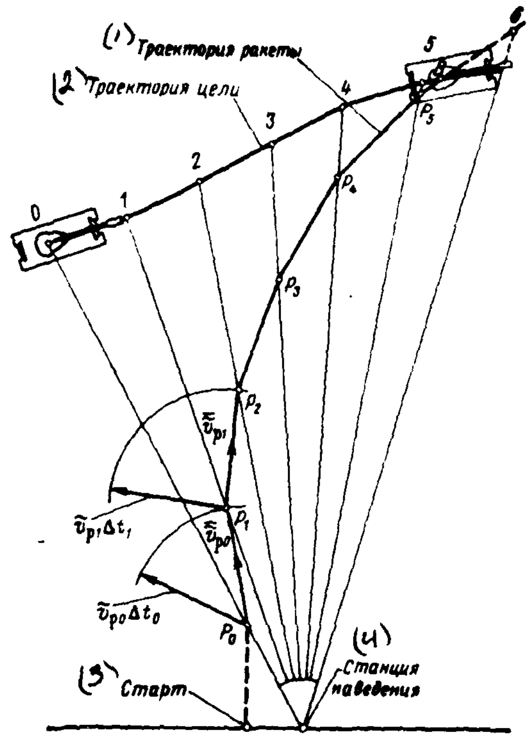

Fig. 4.4. Approximate construction of missile trajectory, aimed using the method of coincidence.

Key: (1). Missile trajectory. (2). Trajectory of target/purpose. (3). Start. (4). Guidance station.

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During the definition of displacements $\overline{P_iP_{i+1}}$ the average speed of

rocket on section can be undertaken as

$$\tilde{v}_{p\,i}=\frac{v_{p\,i}+v_{p\,(i+1)}}{2}, \qquad (4.4)$$

where v_{pi} and $v_{p(i+1)}$ - velocity of rocket in the beginning and end/lead of the trajectory phase in question.

Corresponding to the section of construction time interval $\Delta t_i = t_{i+1} - t_i$.

Within limit, accepting Δt that vanishing and increasing the number of sections to infinity, it is possible broken line to reduce to smooth curve of guidance. Approximated, with a sufficient for practice degree of accuracy, guidance curve can be constructed as curve, passing through the angles of broken line, constructed for an interval $\Delta t \leqslant 1$ s. During a smooth change in the motion characteristics of target/purpose and during large removal/distance of rocket from target/purpose, it is possible to take $\Delta t = 1$ s.

During the approach/approximation of the rocket to target/purpose and during the construction of the trajectory of guidance to the sharply maneuvering target Δt one should decrease, since in this case grows the slope/transconductance of missile trajectory, leading to 0.1 s. The applicability of the graphic method of the construction of trajectory is limited to the need for having previously designed dependence of the velocity of rocket on time $v_p(t)$.

At each moment of time, as it follows from Fig. 4.4, the velocity vector of rocket is directed to the side of the expected collision point of rocket for target/purpose at certain angle to the radius-vector, passing through the guidance station, the center of mass of rocket and the target/purpose. Is obtained guidance with certain lead angle, the line of sighting comprising the part of the radius of the vector, which rotates during the motion of target/purpose and rocket around the point in which is arrange/located the guidance station.

Let us determine the value of lead angle α_p. Let us assume, for infinitesimal time interval dt target/purpose will move from position of a to position of b (Fig. 4.5). In this case, the radius-vector will turn itself to angle $d\eta$, and rocket will move from position a' to position b'. On the basis of Fig. 4.5, with high degree of accuracy it is possible to write that

$$\sin\alpha_u = \frac{\overline{bc}}{v_u\,dt} \quad \text{and} \quad \sin\alpha_p = \frac{\overline{b'c'}}{v_p\,dt},$$

while ratio

$$\frac{\overline{b'c'}}{\overline{bc}} = \frac{r_p + dr_p}{r_u + dr_u} \approx \frac{r_p}{r_u}.$$

Hence

$$\sin\alpha_p = \frac{v_u}{v_p}\,\frac{r_p}{r_u}\,\sin\alpha_u. \tag{4.5}$$

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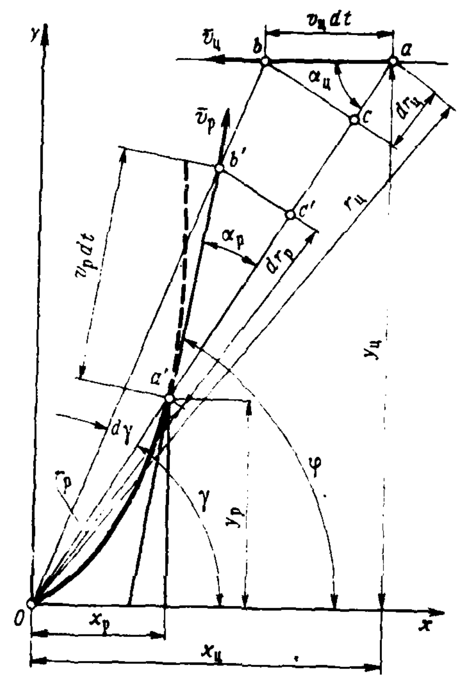

Fig. 8.5. Schematic of construction of angles of curve of coincidence.

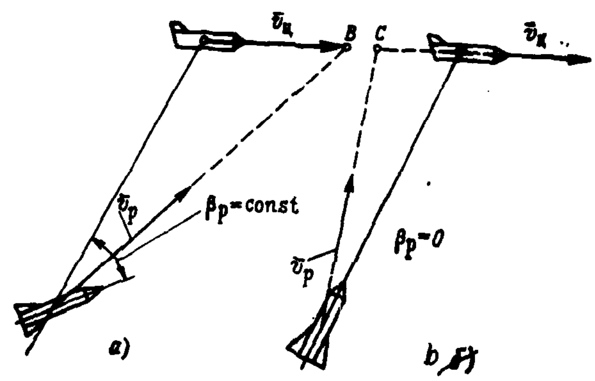

Fig. 4.6. Circuits of guidance with orientation of axis of rocket relative to line of sighting: a) at angle of bearing of target β_p=const; b) at angle of bearing of target β_p=0 (direct/straight guidance).

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1.3. Guidance with the orientation of axis of rocket.

For homing missiles is known the use of a guidance method with the constant angle of the bearing of target and method of direct/straight guidance (Fig. 4.6). In the first case in the process of the approach of rocket for target/purpose, must be retained constant angle β_p (angle of the target bearing) between the axis of rocket and the line of sighting. It is obvious that the velocity vector of target/purpose in the general case will not coincide with the direction of the line of sighting, but missile body will be always bored relatively β_p in the direction of the motion of target/purpose. The angle of bearing should select similar so that the velocity vector of the center of mass of rocket would be directed to set forward point B before the target/purpose (see Fig. 4.6a). With the method of direct/straight guidance, which is a special case

of guidance method with the constant angle of the target bearing, the angle of bearing $\beta_p=0$, and the axis of the rocket is directed along the line of sighting (see Fig. 4.6b). In this case, the velocity vector of the center of mass of the rocket is directed to point C, arrange/located behind the driving/moving target/purpose. This leads to the increased slope/transconductance of the trajectory of guidance and the increased normal accelerations, which act on rocket. Therefore method is applied for guidance to comparatively slow and fixed targets.

1.4. Guidance with the orientation of the velocity vector of the center of mass of rocket.

This class includes the majority of methods of guidances, by which is assigned the law of a change in the lead angle α_p, between the velocity vector and the line of sighting. Lead angle can be the function of many values - see formulas (4.3).

Let us examine the kinematics of the action of target/purpose and rocket on the last/latter section of the path before their rendezvous with guidance. Let us consider that the target/purpose and rocket move in one plane - guidance plane.

The relative attitude of rocket and target/purpose let us determine by distance of r and by two angles α_n and α_p between the

instantaneous velocity vectors of rocket and target/purpose and the line of sighting, which coincides with r (Fig. 4.7). Angle α_p determines advance/prevention. For orienting the line of the sighting of relatively fixed coordinate system, let us introduce, as earlier, angle γ let us designate the angle of the slope of the velocity vector of rocket and horizontal line Px, which lies at guidance plane, through $\varphi=\gamma+\alpha_p$.

So that the rocket and the target/purpose would be met, it is necessary to ensure the identical time of the action of rocket t_p and of target/purpose t_u from starting points to collision point B

$$t_p=t_u.$$

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Let target speeds v_u and of rocket v_p to collision point do not change; then during rectilinear motion of target/purpose and rocket (see Fig. 4.7) we obtain

$$t_p=\frac{\overline{PB}}{v_p} \text{ and } t_u=\frac{\overline{ЦB}}{v_u};$$

$$\overline{PB}\sin\alpha_p=\overline{ЦB}\sin\alpha_u;$$

$$v_p\sin\alpha_p=v_u\sin\alpha_u. \qquad (4.6)$$

Equality (4.6) is called the condition of the ideal

advance/prevention during accomplishing of which is provided the rendezvous of rocket and target/purpose, if they will move to collision point rectilinear with constant velocities. Lead angle will be determined from the equality

$$\sin \alpha_p = \frac{v_u}{v_p} \sin \alpha_u. \quad (4.7)$$

During the curvilinear motion of target/purpose or change in the velocities of the motion of target/purpose and rocket, the position of collision point will always change. In this case condition (4.6) must answer the various moments of time of guidance. For the determination of the changing in process guidance of lead angle α_p it is must, besides v_u and v_p, to know angle α_u. The examined method was called the method of consecutive advance/preventions.

A change in the distance between the rocket and the target/purpose for infinitesimal time interval is equal to a difference in the projections of speeds v_u and v_p on direction r

$$\frac{dr}{dt} = v_u \cos \alpha_u - v_p \cos \alpha_p. \quad (4.8)$$

For the approach of rocket for target/purpose, it is necessary to maintain the inequality

$$\frac{dr}{dt} < 0.$$

Rate of change in the angle γ will be equal to

$$\frac{d\gamma}{dt} = \frac{v_u \sin \alpha_u - v_p \sin \alpha_p}{r}. \quad (4.9)$$

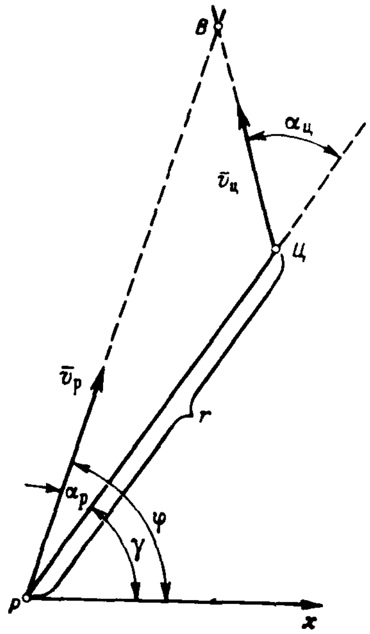

Fig 4.7.

Fig. 4.7. Schematic of the angles, which determine the relative position of rocket and target/purpose.

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The normal accelerations of target/purpose and rocket are determined by the products of the corresponding tangential and angular velocities

$$a_{n\,u} = v_u\left(\frac{d\alpha_u}{dt} + \frac{d\gamma}{dt}\right); \tag{4.10}$$

$$a_{n\,p} = v_p\left(\frac{d\alpha_p}{dt} + \frac{d\gamma}{dt}\right). \tag{4.11}$$

Is known the particular case of the method of consecutive advance/preventions, the so-called method of the half approach by which the lead angle is taken as equal to one-half angle, which corresponds to the condition of the ideal advance/prevention:

$$\alpha_p = \frac{1}{2}\arcsin\left(\frac{v_u}{v_p}\sin\alpha_u\right). \tag{4.12}$$

To this same the class of the methods, united by the sign/criterion of the orientation of the velocity vector of the center of mass of rocket relative to the line of sighting, one should relate a method of guidance with fixed-lead angle $\alpha_p = \text{const}$ and its special case, called the Stearn-Chase method when $\alpha_p = 0$.

According to the principle of the plotting of curves pursuit at each moment of time, the velocity vector of the center of mass of rocket must be directed toward target/purpose, i.e., is realized guidance with the zero angle of advance/prevention.

With the known characteristics of the trajectory of the motion of target/purpose, graphic construction of missile trajectory is not caused work (Fig. 4.8). To the trajectory of target/purpose, will be deposited its positions (0, 1, 2, 3, ...), which correspond to the consecutive values of time t_0, t_1, t_2, ... On the initial line of sighting, they plot/deposit in direction in target/purpose cut $\overline{P_0P_1}$, equal to $v_{p0}\Delta t_0$, where $\Delta t = t_1 - t_0$, and v_{p0} - is determined from formula (4.4).

The obtained point P_1 determines the place, occupied by rocket at the moment of time t_1. Then similar construction is repeated before obtaining of the intersection of the missile trajectories and target/purpose.

With high plotting scale and low time intervals Δt_i, with the large number of points, the curve can be constructed with the accuracy, sufficient for the solution of some practical problems. For

the method of pursuit guidance, the angle φ is determined from the kinematic dependence

$$\mathrm{tg}\,\varphi = \frac{y_{u} - y_{p}}{x_{u} - x_{p}}.$$

In certain cases to more expedient utilize the trajectories of guidance, constructed in relative motion. Let us examine an example of the construction of linear curved in the coordinate system, connected with target/purpose.

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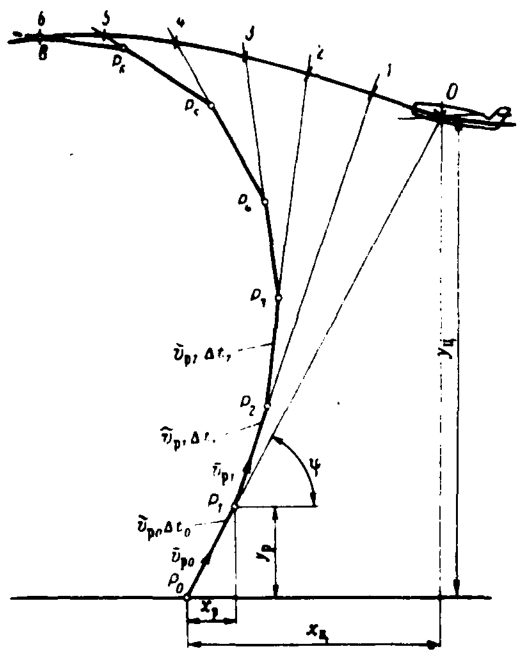

Fig. 4.8. Diagram of interception of target/purpose on pursuit curve.

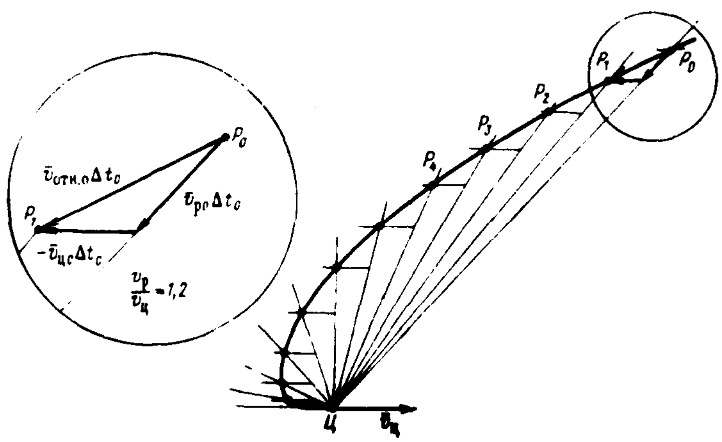

Fig. 4.9. Schematic of trajectory of relative motion of rocket, aimed

along pursuit curve.

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Path in relative movement for time interval dt is located from vector equality $\bar{v}_{отн}dt=\bar{v}_{р}dt-\bar{v}_{ц}dt,$ written on the assumption that for time dt $\bar{v}_{р}$ and $\bar{v}_{ц}$ they do not change.

Transfer/converting to final low cuts, from the initial position of rocket P_0, let us construct vector $\bar{v}_{р0}\Delta t_0$ in direction in target/purpose (Fig. 4.9). From the terminus of this vector, we plot/deposit vector $\bar{v}_{ц0}\Delta t_0$ in the direction, opposite to direction $\bar{v}_{ц}$ in absolute motion. End line, obviously, will be $\bar{v}_{отн}\Delta t_0$. Continuing the construction to the encounter of rocket with target and connecting points P_0, P_1, P_2, ... smooth curved, we will obtain the form of the which interests us trajectory. In Fig. 4.9 is well visible sharp passage from smooth pursuit curve to rectilinear motion into the tail of target/purpose.

4.5. Guidance with the orientation of the line of sighting.

If we use the condition of ideal advance/prevention to formula

(4-9); then we will obtain $d\gamma/dt=0$, and this means that in the process of guidance the line of sighting will be moved, remaining parallel to itself. This method of guidance with advance/prevention was called the method of the constant-bearing approach. During rectilinear motion of target/purpose, the rocket will fly along straight line. In more general case of the curvilinear motion of target/purpose and the parallel displacement of the line of sighting, the lead angle α_p cannot remain constant, but it must change depending on a change in angle α_u. Taking into account the variability of angle, this method of guidance sometimes also calls the method of consecutive advance/preventions. With the known characteristics of the trajectory of the motion of target/purpose, approximate graphic construction of missile trajectory difficulties is not caused. The method of construction is shown on Fig. 4.10. To the trajectory of the motion of target/purpose, will be deposited its positions: 0, 1, 2, 3, ..., that correspond to the values of flight time t_0, t_1, t_2: from each point are conducted the direct/straight, parallel to initial direction lines of sighting. On the initial line of sighting, will be deposited the position of the center of mass of rocket (point P_0), and from it on the following line of sighting by compass is made cut, giving in the point of intersection point P_1; then similar constructions are repeated from points P_1, P_2 and so forth. Opening of compasses for each space of construction is established on the basis of the conditions

$$\overline{P_0P_1}=\tilde{v}_{p0}\Delta t_0;\quad \overline{P_1P_2}=\tilde{v}_{p1}\Delta t_1\ldots\overline{P_iP_{i+1}}=\tilde{v}_{pi}\Delta t_i.$$

Construction continues before the intersection of the trajectories of target/purpose and rocket in collision point V. average speed $\bar{v}_{pi}$ of rocket on each of the sections can be defined as half-sum of the velocities in the beginning and end/lead of the section in question.

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Let us examine one additional method of guidance to target/purpose at the alternate angle of advance/prevention α_p, by the determined any predetermined dependence. Is widely known the use as this limitation of a linear dependence between the angular rate of rotation of the velocity vector of the center of mass of rocket and the angular rate of rotation of the line of the sighting of the form

$$\frac{d\varphi}{dt} = a\frac{d\gamma}{dt}. \qquad (4.13)$$

This method of guidance to target/purpose was called the method of proportional approach. Utilizing equation (4.9), we will obtain

$$\frac{d\varphi}{dt} = a\frac{v_u \sin\alpha_u - v_p \sin\alpha_p}{r}. \qquad (4.14)$$

From Fig. 4.7 $\varphi = \alpha_p + \gamma$, then

$$\frac{d\alpha_p}{dt} = (a-1)\frac{v_u \sin\alpha_u - v_p \sin\alpha_p}{r} \qquad (4.15)$$

and

$$\alpha_p = \alpha_{p0} + (a-1)\int_{t_0}^{t} \frac{v_u \sin \alpha_u - v_p \sin \alpha_p}{r}\, dt.$$

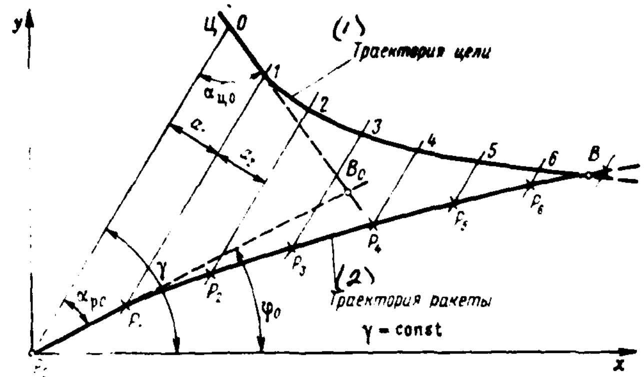

Fig. 4.10. Approximate construction of missile trajectory, aimed according to parallel approach method.

Key: (1). Trajectory of target/purpose. (2). Missile trajectory.

§2. Three-dimensional/space guidance to the driving/moving target/purpose.

At variable speed of the flight of target/purpose under conditions of its maneuvering, the missile trajectory will be space curve. The system of equations, which describes the spatial motion of rocket, can be comprised in starting rectangular coordinate system Cxyz and in the starting polar system r, ε, β (see Fig. 2.2). The place of start can be fixed or that moving (ship, aircraft, etc.).

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We will obtain first the system of equations, which describes the three-dimensional/space missile targeting to the driving/moving target/purpose in the rectangular terrestrial starting coordinate system. The system of equations of the spatial motion of rocket in the general case will consist of seventeen fundamental equations (3.7)-(3.12), (3.14)-(3.24). Supplementary equations for determining of control forces $\sum X_{p1}, \sum Y_{p1}, \sum Z_{p1}$ and the moments of control forces $\sum M_{px_1}, \sum M_{py_1}, \sum M_{pz_1}$ entering equations (3.7)-(3.12), must be comprised in accordance with guidance method and consider the dependence of forces and torque/moments on the motion characteristics of target/purpose. In this complete form the system of equations proves to be very complex and in engineering practice is simplified.

Let us take the system, which consists of equations (3.14)-(3.24) and simplified equations (3.33) and (3.34)

$$\begin{aligned}
&\dot{v}=\frac{1}{m}(P\cos\alpha\cos\beta-X-Q\sin\theta);\\
&\dot{\theta}=\frac{1}{mv}[P(\sin\alpha\cos\gamma_c+\cos\alpha\sin\beta\sin\gamma_c)+Y\cos\gamma_c-\\
&\quad-Z\sin\gamma_c-Q\cos\theta];\\
&\dot{\Psi}=-\frac{1}{mv\cos\theta}[P(\sin\alpha\sin\gamma_c-\cos\alpha\sin\beta\cos\gamma_c)+\\
&\quad+Y\sin\gamma_c+Z\cos\gamma_c];\\
&J_{x_1}\dot{\omega}_{x_1}=\sum M_{x_1}-(J_{z_1}-J_{y_1})\omega_{y_1}\omega_{z_1};\\
&J_{y_1}\dot{\omega}_{y_1}=\sum M_{y_1}-(J_{x_1}-J_{z_1})\omega_{z_1}\omega_{x_1};\\
&J_{z_1}\dot{\omega}_{z_1}=\sum M_{z_1}-(J_{y_1}-J_{x_1})\omega_{x_1}\omega_{y_1};\\
&\qquad\dot{\vartheta}=\omega_{y_1}\sin\gamma+\omega_{z_1}\cos\gamma;\\
&\qquad\dot{\psi}=\frac{1}{\cos\vartheta}(\omega_{y_1}\cos\gamma-\omega_{z_1}\sin\gamma);\\
&\qquad\dot{\gamma}=\omega_{x_1}-\operatorname{tg}\vartheta(\omega_{y_1}\cos\gamma-\omega_{z_1}\sin\gamma);\\
&\sin\vartheta=\sin\theta\cos\alpha\cos\beta+\cos\theta(\sin\alpha\cos\gamma_c+\\
&\qquad+\cos\alpha\sin\beta\sin\gamma_c);\\
&\sin\psi\cos\gamma=\sin\Psi\cos\beta\cos\gamma_c+\cos\Psi(\sin\beta\cos\theta+\\
&\qquad+\sin\gamma_c\sin\theta\cos\beta)-\cos\psi\sin\vartheta\sin\gamma;\\
&\cos\vartheta\sin\gamma=\sin\gamma_c\cos\beta\cos\theta-\sin\beta\sin\theta;\\
&\dot{x}=v\cos\theta\cos\Psi;\\
&\dot{y}=v\sin\theta;\\
&\dot{z}=-v\cos\theta\sin\Psi;\\
&r=\sqrt{x^2+y^2+z^2};\\
&m=m_0-\int_0^t|\dot{m}|\,dt.
\end{aligned}\qquad(4.16)$$

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In the written form the system is locked and it is suitable for the calculation of the trajectory phases during the unguided flight (in the systems of equations of motion (4.16-4.23) of all motion characteristics of rocket index "r", which was being applied in §1, it is lowered).

In connection with the trajectories of the guidance in a series of the cases, is utilized spherical coordinates, in which directly is determined slant range to rocket r (see Fig. 2.2). For the writing of kinematic equations in spherical system, we utilize conclusion of A. A. Lebedev [36]. Let us examine two systems of rectilinear coordinates - terrestrial Oxyz and mobile O'x'y'z' whose axes remain parallel to the axes of the first system in the process of motion (Fig. 4.11). The point with which coincides the center of mass of rocket, let us designate P; its position in moving coordinate system will be determined by a radius by the vector

$$\bar{r}_{0'} = \overline{0P} - \overline{00'}.$$

Differentiating vector equality, we will obtain

$$\dot{\bar{r}}_{0'} = \bar{v} - \bar{v}_{0'}, \qquad (4.17)$$

where $\bar{v}$ - a velocity vector of the center of mass of rocket;

$\bar{v}_0$ - velocity vector of the beginning of moving coordinate system 0'.

With $v_0=0$, $\bar{r}_0=\bar{r}$, $\dot{\bar{r}}=\bar{v}$. We plan last/latter equality to the spherical coordinate axes:

$$v_r = \dot{r};\quad v_\varepsilon = r\dot{\varepsilon};\quad v_\lambda = r\dot{\lambda}\cos\varepsilon.$$

The projections of velocity vector on the axis of earth-based

coordinate system will be determined by formulas (2.4). Using the table of the cosines of the angles between the axes of the terrestrial rectangular coordinate system and spherical coordinates (see Table 2.1), we will obtain

$$\begin{aligned}
\dot{r} &= v(\cos\theta\cos\Psi\cos\varepsilon\cos A + \sin\theta\sin\varepsilon - \cos\theta\sin\Psi\cos\varepsilon\sin A);\\
r\dot{\varepsilon} &= v(-\cos\theta\cos\Psi\sin\varepsilon\cos A + \sin\theta\cos\varepsilon + \cos\theta\sin\Psi\sin\varepsilon\sin A);\\
r\dot{A}\cos\varepsilon &= -v(\cos\theta\cos\Psi\sin A + \cos\theta\sin\Psi\cos A).
\end{aligned}$$

After transformations we will obtain the following kinematic equations

$$\left.\begin{aligned}
\dot{r} &= v\cos\theta\cos(\Psi + A)\cos\varepsilon - v\sin\theta\sin\varepsilon;\\
r\dot{\varepsilon} &= -v\cos\theta\cos(\Psi + A)\sin\varepsilon + v\sin\theta\cos\varepsilon;\\
r\dot{A}\cos\varepsilon &= -v\cos\theta\sin(\Psi + A).
\end{aligned}\right\} \qquad (4.18)$$

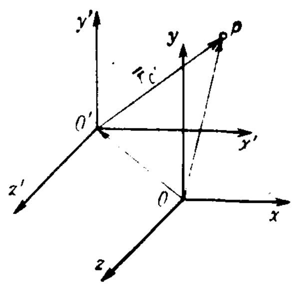

Fig. 4.11. Schematic of the relative position of fixed coordinate system Oxyz and of mobile system O'x'y'z'.

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We focus reader's attention to some difficulties, appearing during use in the calculations of equations (4.18) and connected with the reading of angles Ψ and A. According to the rule of mechanics the positive value of angle Ψ it is accepted to count off in direction against the motion of hour hand. At the same time azimuth A is usually reckoned on the motion of hour hand from direction in north. If we accept for an azimuth the reading of angles in the direction, opposite to the motion of hour hand, i.e., in the direction of the axis [A] in Fig. 2.2, then in equations (4.18) it is necessary to replace

$$(\Psi + A) \overset{(1)}{\text{на}} [\Psi + (-A)] = (\Psi - A)$$

and

$$\dot{A}\ \overset{(1)}{\text{на}}\ \frac{d}{dt}(-A_*) = -\frac{dA_*}{dt} = -\dot{A}_*.$$

Key: (1). On.

Recall that the obtained system corresponds to the fixed origin of coordinates 0'. During the action of the origin of coordinates with a a rate of of $\bar{V}_0$ kinematic equations we will obtain, after designing vector equality (4.17) or the spherical coordiante axis [r], [ε] and [A*]

$$\left.\begin{aligned}\dot{r}_{0'} &= v\cos\theta\cos(\Psi - A_*)\cos\varepsilon + v\sin\theta\sin\varepsilon -\\ &\quad - v_{0'}\cos\theta_{0'}\cos(\Psi_{0'} - A_*)\cos\varepsilon - v_{0'}\sin\theta_{0'}\sin\varepsilon;\\ r_{0'}\dot{\varepsilon} &= -v\cos\theta\cos(\Psi - A_*)\sin\varepsilon + v\sin\theta\cos\varepsilon +\\ &\quad + v_{0'}\cos\theta_{0'}\cos(\Psi_{0'} - A_*)\sin\varepsilon - v_{0'}\sin\theta_{0'}\cos\varepsilon;\\ r_{0'}\dot{A}_*\cos\varepsilon &= v\cos\theta\sin(\Psi - A_*) - v_{0'}\cos\theta_{0'}\sin(\Psi_{0'} - A_*).\end{aligned}\right| \quad (4.19)$$

Equations (4.18) and (4.19) are conveniently applied in the implementation of the remote control of guidance of rocket in accordance with fixed and mobile of guidance stations [3].

For obtaining the kinematic equations, which describe relative motion of target/purpose and rocket, the beginning of moving coordinate system is placed to the center of mass of rocket. Distance between centers of the masses of rocket 0' (R) and target/purposes (Ts) on the line of sighting let us designate through r_0' (Fig. 4.12)

$$\bar{r}_{0'} = \overline{0\text{Ц}} - \overline{0\text{P}}.$$

Differentiating, we will obtain

$$\dot{\bar{r}}_{0'} = \bar{v}_{\text{ц}} - \bar{v}. \qquad (4.20)$$

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Let us design last/latter equality on the spherical coordiante axis [r], [ε] and [A] whose beginning let us place to the center of mass of the target/purpose

$$\left.\begin{aligned}\dot{r}_0 &= v_u \cos\theta_u \cos(\Psi_u - A_*)\cos\varepsilon + v_u \sin\theta_u \sin\varepsilon - \\ &- v\cos\theta\cos(\Psi - A_*)\cos\varepsilon - v\sin\theta\sin\varepsilon;\\ r_0\dot{\varepsilon} &= -v_u\cos\theta_u\cos(\Psi_u - A_*)\sin\varepsilon + v_u\sin\theta_u\cos\varepsilon + \\ &+ v\cos\theta\cos(\Psi - A_*)\sin\varepsilon - v\sin\theta\cos\varepsilon;\\ r_0\dot{A}_*\cos\varepsilon &= v_u\cos\theta_u\sin(\Psi_u - A_*) - v\cos\theta\sin(\Psi - A_*).\end{aligned}\right\} \quad (4.21)$$

The obtained equations are utilized during the trajectory calculation of rockets with homing/self-induction.

During the writing of the common/general/total system, which describes the motion of rocket, kinematic equations are selected depending on the principle, placed as the basis of the flight control. Let us write common/general/total system of equations, after using simplified system (4.16) and after replacing in it the kinematic equations

$$\dot{x} = v\cos\theta\cos\Psi;\ \dot{z} = -v\cos\theta\sin\Psi$$

and

$$\dot{y} = v\sin\theta$$

by three kinematic equations, obtained in the spherical system of coordinates [r], [ε] and [A_{*}] and by those corresponding the fixed guidance station, combined with the origin of coordinates (i.e. with $\bar{r}_{e}' = \bar{r}$). First nine and the last/latter three equations of system (4.16) will remain without the change

$$
\left.
\begin{aligned}
&\dot{v} = \frac{1}{m}(P\cos\alpha\cos\beta - X - Q\sin\theta);\\
&\dot{\theta} = \frac{1}{mv}[P(\sin\alpha\cos\gamma_c + \cos\alpha\sin\beta\sin\gamma_c) + \\
&\quad + Y\cos\gamma_c - Z\sin\gamma_c - Q\cos\theta];\\
&\dot{\Psi} = -\frac{1}{mv\cos\theta}[P(\sin\alpha\sin\gamma_c - \cos\alpha\sin\beta\cos\gamma_c) + \\
&\quad + Y\sin\gamma_c + Z\cos\gamma_c];\\
&J_{x_1}\dot{\omega}_{x_1} = \sum M_{x_1} - (J_{z_1} - J_{y_1})\omega_{y_1}\omega_{z_1};\\
&J_{y_1}\dot{\omega}_{y_1} = \sum M_{y_1} - (J_{x_1} - J_{z_1})\omega_{x_1}\omega_{z_1};
\end{aligned}
\right\}
\quad (4.22)
$$

$$
\left.
\begin{aligned}
&J_{z_1}\dot{\omega}_{z_1} = \sum M_{z_1} - (J_{y_1} - J_{x_1})\omega_{x_1}\omega_{y_1};\\
&\dot{\vartheta} = \omega_{y_1}\sin\gamma + \omega_{z_1}\cos\gamma;\\
&\dot{\psi} = \frac{1}{\cos\vartheta}(\omega_{y_1}\cos\gamma - \omega_{z_1}\sin\gamma);\\
&\dot{\gamma} = \omega_{x_1} - \operatorname{tg}\vartheta(\omega_{y_1}\cos\gamma - \omega_{z_1}\sin\gamma);\\
&\dot{r} = v\cos\theta\cos(\Psi - A_{*})\cos\varepsilon + v\sin\theta\sin\varepsilon;\\
&r\dot{\varepsilon} = -v\cos\theta\cos(\Psi - A_{*})\sin\varepsilon + v\sin\theta\cos\varepsilon;\\
&r\dot{A}_{*}\cos\varepsilon = v\cos\theta\sin(\Psi - A_{*});\\
&y = r\sin\varepsilon;\\
&\sin\vartheta = \sin\theta\cos\alpha\cos\beta + \cos\theta(\sin\alpha\cos\gamma_c + \\
&\quad + \cos\alpha\sin\beta\sin\gamma_c);\\
&\sin\psi\cos\gamma = \sin\Psi\cos\beta\cos\gamma_c + \cos\Psi(\sin\beta\cos\theta + \\
&\quad + \sin\gamma_c\sin\theta\cos\beta) - \cos\psi\sin\vartheta\sin\gamma;\\
&\cos\vartheta\sin\gamma = \sin\gamma_c\cos\beta\cos\theta - \sin\beta\sin\theta.
\end{aligned}
\right\}
\quad (4.22)
$$

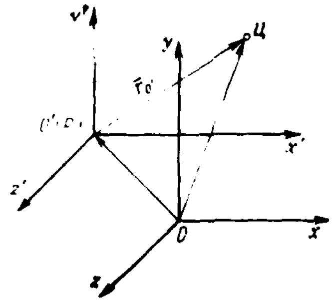

Fig. 4.12. The schematic of the relative position of fixed coordinate system Oxyz and of the mobile system O'x'y'z' whose beginning coincides with the center of mass of rocket B.

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Written system (4.22) just as system (4.16), does not contain in the equations of forces and torque/moments, determined by the work of control devices, and it is suitable for performance calculation of the motion of the unguided rockets. The introduction in (4.16) or (4.22) of equations for value determination of control forces and the torque/moments, which reflect their dependence on the motion characteristics of the center of mass of target/purpose and guidance method, leads to the difficulties of a calculated-technical order.

Let us examine the relatively simple method of calculation of

the three-dimensional/space trajectories of guidance, which makes it possible to obtain concrete/specific/actual numerical results. Method is not set any limitations on the action characteristics of target/purpose and rocket. Let us assume that as before that the balance angles and slip do not exceed the values with which it is possible to accept

$$\sin\alpha_6\approx\alpha_6;\quad \sin\beta_6\approx\beta_6;\quad \cos\alpha_6\approx\cos\beta_6\approx 1.$$

Let us assume also that the hoisting and lateral forces are linearly connected with the angles of attack and slip along (2.85), let us drop/omit the products of small angles $\alpha_6\gamma_c$ and $\beta_6\gamma_c$. With the adopted assumptions the spatial motion of the rocket can be described by six by the differential equations

$$\left.\begin{aligned} \dot v&=\frac{P-X}{m}-g\sin\theta;\\ \dot\theta&=\frac{(P+Y^{\alpha})\alpha_6}{mv}-\frac{g\cos\theta}{v};\end{aligned}\right\}\quad(4.23)$$

$$\left.\begin{aligned} \dot\Psi&=\frac{(P+Z^{\beta})\beta_6}{mv\cos\theta};\\ \dot y&=v\sin\theta;\\ x'_y&=\frac{\cos\Psi}{\operatorname{tg}\theta};\quad z'_y=-\frac{\sin\Psi}{\operatorname{tg}\theta}.\end{aligned}\right\}\quad(4.23)$$

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Recall that angles α_6 and β_6 can be found from the second and third equations of system (4.23) in the form (3.66) and (3.67).

For determining of angles θ and Ψ and their derivatives during spatial motion, it is logical to assume that the guidance system provides the conditions according to which at each moment of time the velocity vector of the center of mass of rocket and the line of sighting lie/rest at the plane, passing through the velocity vector of the center of mass of target/purpose. Let us call/name this plane the plane of interception (P). The position of the plane of interception in space in the process of guidance will be determined by the position of the velocity vector of the center of mass of target/purpose and by the point, which corresponds to the position of the center of mass of rocket. For determining the position of the plane of the interception of relatively horizontal plane (A) let us introduce two angles: the angle of the slope of the plane of interception to horizontal plane let us designate through χ. the angle between the selected direction of horizontal axis 0x and the intersection (in the general case) of the inclined plane of interception with horizontal plane let us designate through Ψ_0. The orientation of axis 0x relative to direction in north let us define that as usual, by azimuth A_{0x} [17]. One of the possible positions of the plane of interception it is shown on Fig. 4.13.

The position of the line of sighting r in the plane of

interception let us determine by angle γ between the line of sighting and the intersection of the plane of interception with horizontal plane. We additionally utilize two angles α_u and α_p, the lying horizontal at the plane of interception and determining instantaneous positions of the velocity vectors of the centers of mass of target/purpose and rocket relative to the line of sighting.

The position of the velocity vector of the center of mass of rocket $\bar{v}_p$ in space is defined that as usual, by angle θ (between vector $\bar{v}_p$ and the horizontal plane) and the angle of rotation of trajectory Ψ (between the projection of vector $\bar{v}_p$ on horizontal plane and the axis 0x).

The positions of the centers of mass of target/purpose and rocket are assigned in terrestrial system Oxyz by coordinates $x_u, y_u, z_u, x_p, y_p, z_p$, which subsequently let us call basic unlike coordinates in the planes of interception which let us supply with supplementary index «χ», for example $y_{p\chi}$ and so forth.

Angles θ and Ψ_0 can be determined through the named previously angle of the slope of the plane of interception χ and the angle ϕ, which determines the position of vector $\bar{v}_p$ in the plane of interception.

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From $\triangle aPP_{(A)}$, $\triangle PP_{(A)}b$ and $\triangle aPb$ (see Fig. 4.13), let us have

$$\sin\theta = \sin\chi \sin\varphi. \qquad (4.24)$$

Replacing φ, we will obtain

$$\sin\theta = \sin\chi \sin(\gamma + \alpha_p). \qquad (4.25)$$

Angle α_p will be determined by guidance method. For example, for the condition of ideal advance/prevention it is necessary to use (4.6).

From Fig. 4.13, utilizing additionally $\triangle abP_{(A)}$, we will obtain

$$\sin\xi = \frac{\cos\chi \sin\varphi}{\cos\theta} = \operatorname{tg}\theta \operatorname{ctg}\chi. \qquad (4.26)$$

Angle of rotation of the trajectory

$$\Psi = \Psi_0 + \xi. \qquad (4.27)$$

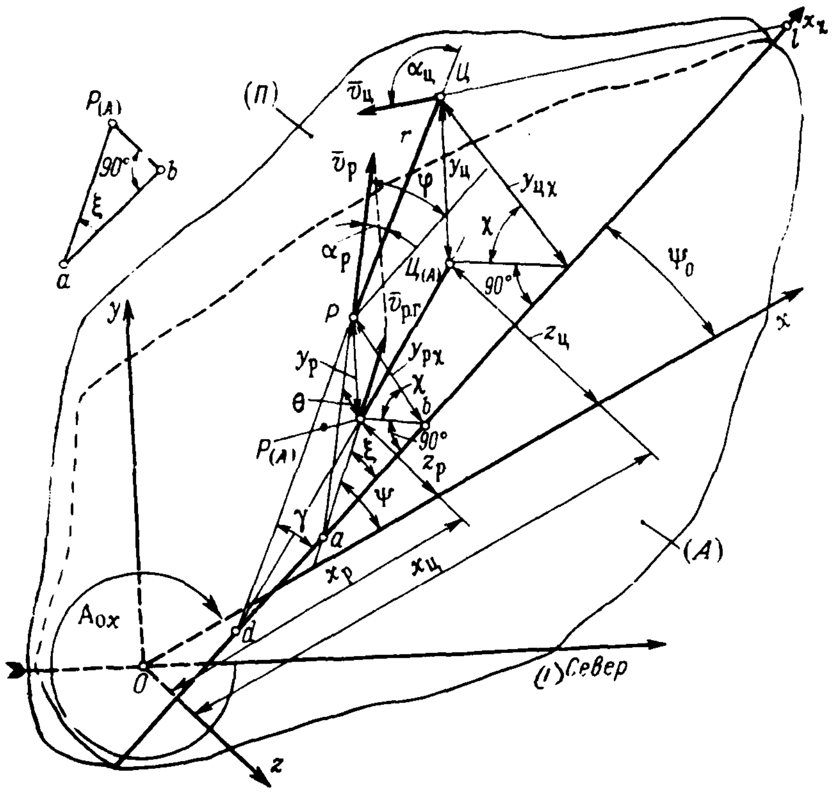

Fig. 4.13. Schematic of the angles, which determine the position of the line of sighting and velocity vector the center of mass of rocket in space and on the plane of interception (A).

Key: (1). North.

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For determination χ and γ, it is necessary to know the coordinates of target and rocket. During the designed trajectory calculations of the guidance of the motion characteristic of target/purpose, they must be assigned, and the motion characteristics of rocket are located in the process of the solution of problem. When conducting of the firings of the coordinate of target and rocket, are determined from the data of the servo after target/purpose equipment which measures: the angle of sighting and rocket $\varepsilon_{ц}, \varepsilon_{р}$; inclined target ranges and rocket $r_{ц}$ and $r_{р}$; Doppler velocities $\dot{r}_{ц}$ and $\dot{r}_{р}$; the azimuths of target/purpose and rocket or angles $A_{x ц}, A_{x р}$ from the initial line of fire, which coincides with axis Ox (Fig. 4.14).

Let us find the angle of the slope of the plane of interception to the horizon. From $\triangle ЦЦ_{(A)}c$, $\triangle dЦ_{(A)}c$ and $\triangle dЦЦ_{(A)}$ in Fig. 4.14:

$$\operatorname{tg}\chi=\frac{y_{ц}}{cЦ_{(A)}}=\frac{\operatorname{tg}\mu}{\sin\nu}; \qquad (4.28)$$

$$\operatorname{tg}\mu=\frac{y_{ц}-y_{р}}{r_{г}}. \qquad (4.29)$$

The projection of the line of sighting on horizontal plane $r_{г}$ let us determine from Fig. 4.15:

$$r_{г}=\overline{P_{(A)}Ц_{(A)}}=\sqrt{(x_{ц}-x_{р})^{2}+(z_{ц}-z_{р})^{2}} \qquad (4.30)$$

or, after transformation, we will obtain

$$r_{г}=(x_{ц}-x_{р})\sqrt{1+\operatorname{tg}^{2}(\Psi_{0}-\nu)}.$$

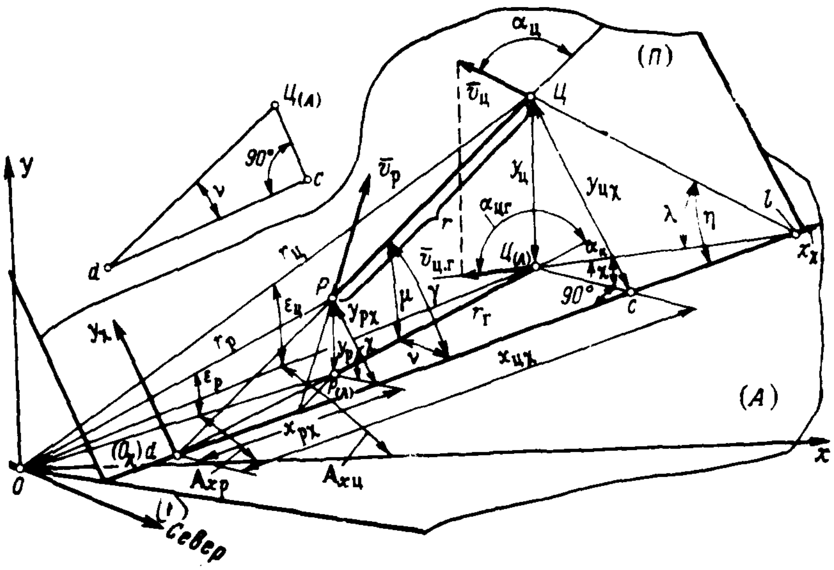

Fig. 4.14. Schematic of coordinates and angles, which determine the position of the centers of mass and rocket in space.

Key: (1). North.

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In accordance with Fig. 4.14 and 4.15, it is possible to write:

$$\left.\begin{array}{ll} y_ц = r_ц \sin \varepsilon_ц; & y_p = r_p \sin \varepsilon_p; \\ x_ц = r_ц \cos \varepsilon_ц \cos A_{x\,ц}; & x_p = r_p \cos \varepsilon_p \cos A_{xp}; \\ z_ц = -r_ц \cos \varepsilon_ц \sin A_{x\,ц}; & z_p = -r_p \cos \varepsilon_p \sin A_{x\,p}, \end{array}\right\} \quad (4.31)$$

where $A_{x\,p} = A_{0x} - A_p$ and $A_{x\,ц} = A_{0x} - A_ц$.

Let us find the value of angle ν between the projection of the line of sighting on horizontal plane r_r and of the intersections of the plane of interception (E) with horizontal plane (A) (by line d¶). If we designate angle off in the horizontal plane through α_K, a the angle of the pitching (or diving) of the target/purpose through λ, then of ∆dfl, $\triangle l\text{ЦЦ}_A$, $\triangle \text{Ц}_{(A)}lf$ and $\triangle d\text{ЦЦ}_{(A)}$ (see Fig. 4.14 and 4.15) we will obtain

$$\operatorname{tg}\nu=\frac{\sin\alpha_\kappa}{\operatorname{tg}\lambda\operatorname{ctg}\mu+\cos\alpha_\kappa}. \qquad (4.32)$$

The angle between the projection of the line of sighting on horizontal plane and the axis Ox will be determined (see Fig. 4.15) from the formula

$$\operatorname{tg}(\Psi_0+\nu)=\frac{z_\text{ц}-z_\text{p}}{x_\text{ц}-x_\text{p}}. \qquad (4.33)$$

Substituting in (4.28) expression for $\sin\nu$ from (4.32), we will obtain formula for determining the angle of the slope of the plane of interception to the horizon

$$\operatorname{tg}\gamma=\frac{\operatorname{tg}\lambda+\operatorname{tg}\mu\cos\alpha_\kappa}{\sin\alpha_\kappa\cos\nu}. \qquad (4.34)$$

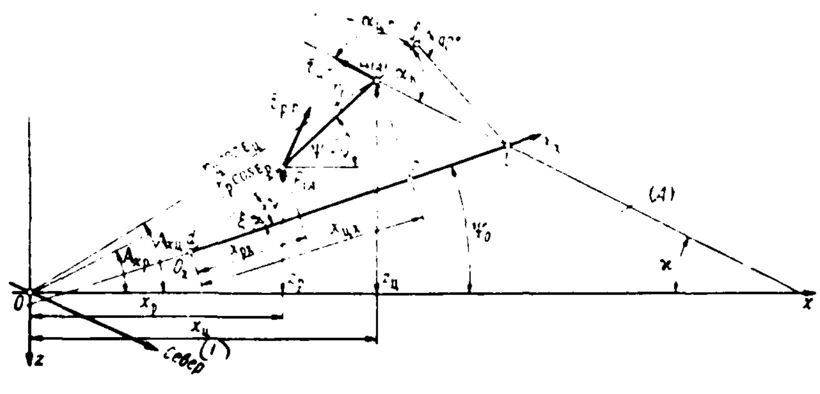

Fig. 4.15. Schematic of coordinates and angles, which determine the position of the projection of the line of sighting on horizontal plane.

Key: (1). North.

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Angle γ is conveniently determined through the coordinates of target and rocket in the plane of interception. Communication/connection between the real coordinates of target and rocket and their coordinates in the plane of interception is established through angles χ, ν and μ (see Fig. 4.14 and 4.15):

$$\left.\begin{aligned} x_{u\gamma} &= \frac{y_u \cos \nu}{\operatorname{tg} \mu}; & y_{u\gamma} &= \frac{y_u}{\sin \chi}; \\ x_{r\chi} &= \frac{y_r \cos \nu}{\operatorname{tg} \mu}; & y_{r\chi} &= \frac{y_r}{\sin \chi}. \end{aligned}\right\} \quad (4.35)$$

the origin of coordinates 0_γ in the plane of interception corresponds to the point of intersection of the continuation of the line of sighting with horizontal plane (point d in Fig. 4.13-4.15).

Necessary for many calculations angle α_u let us find from ΔdTsL (see Fig. 4.14):

$$\alpha_u = 180° - \gamma - \eta; \quad (4.36)$$

$$\operatorname{tg} \gamma = \frac{y_{u\gamma} - y_{r\gamma}}{x_{u\gamma} - x_{r\gamma}}. \quad (4.37)$$

while angle η will be determined according to the theorem of sines from the same triangle:

$$\sin \eta = \frac{\sin \lambda \sin \gamma}{\sin \mu}. \quad (4.38)$$

The projection of angle α_u on horizontal plane let us designate $\alpha_{u.r.}$, then the course angle

$$\alpha_k = 180° - \alpha_{u.r.}$$

In turn, from Fig. 4.15:

$$\alpha_{u.r} = 180° - (\nu + \Psi_0) - \varkappa,$$

where $\varkappa$ - angle between the axis Ox and the direction of the projection of velocity vector $\bar{V}_u$ on horizontal plane. Angles $\varkappa$ and λ determine the direction of the action of target/purpose and they must be assigned.

Those necessary subsequently for the solution of the problem of the three-dimensional/space guidance of an increase in basic

coordinates Δx_p and Δz_p depending on increments of coordinates Δy_p will be determined of two last/latter formulas of system (4.23)

$$\Delta x_p \approx \Delta y_p \frac{\cos \Psi}{\operatorname{tg} \theta}; \qquad (4.39)$$

$$\Delta z_p \approx -\Delta y_p \frac{\sin \Psi}{\operatorname{tg} \theta}. \qquad (4.40)$$

The order of numerical solution is presented in Chapter VI.

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§3. Special feature/peculiarities of special cases of moving the target/purpose.

To special cases let us relate such case with which the target/purpose and the aimed at it rocket fulfill maneuvering in constant/invariable plane. Maneuvers can be carried out in inclined, vertical and horizontal planes. In all three versions the position of the plane of interception will be determined by angle χ constant in the course of the solution of problem. For vertical plane $\chi=\frac{\pi}{2}$, for horizontal $\chi=0$ and for inclined $0<\chi<\frac{\pi}{2}$.

The case by which the target/purpose retains the plane of the fulfilled maneuver, and rocket is located out of this plane, obviously, it must be referred to the common/general/total task of

spatial motion. During the solution of two-dimensional problem of guidance with $\chi=\text{const}$ is expedient axis 0x to combine with the intersection of planes (A) and (P) (with line d1 in Fig. 4.14), and it began basic coordinate system to combine with point d. In this case,

$$\Psi_0=0;\ \Psi=\xi;\ x_{p\chi}=x_p\ \text{and}\ x_{u\chi}=x_u.$$

If the plane of interception is vertical, then, additionally to that noted, we will obtain

$$\Psi=\xi=\nu=0;\ \varphi=\theta;\ z_u=z_p=0.$$

With the guidance in horizontal plane supplementary conditions they will be:

$$\varepsilon_p=\varepsilon_u=\mu=\lambda=\gamma=\theta=0;\ \nu=\gamma;\ \varphi=\Psi;\ \alpha_{u\,\gamma}=\alpha_u;$$
$$y_p=y_u=y_{p\chi}=y_{u\chi}=\text{const}$$

(in the particular case of value y_i they can be equal to zero). One should distinguish two characteristic versions of the motion of the target/purpose: more complex, when during motion in inclined plane change the coordinates of target x_u, y_u, z_u (during motion in vertical plane, respectively - y_u and x_u, but during motion in horizontal - x_u and z_u), and the simpler case, with which the target/purpose moves in parallel to the coordinate axis Ox, i.e., rectilinearly and at constant height/altitude; in this case $y_u=\text{const}$ and $z_u=\text{const}$, but

angle $\alpha_n=180°-\gamma$.

In the case of the inclined plane of interception when $\chi=\text{const}$ the fundamental equations, which describe the spatial motion of rocket (4.23), will remain without change.

While maneuvering of rocket in the vertical plane of interception, which coincides with the coordinate plane xOy, we have $\beta_6=\Psi=\dot{\psi}=z=0$. Consequently, in system (4.23) of equation for Ψ and z'_y they must be lowered, equations for $\dot{v}$, $\ddot{\theta}$ and $\dot{y}$ will remain without change, but equation for x'_y will obtain form $x'_y=\text{ctg}\,\theta$.

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In the case of the maneuver of rocket in horizontal plane $\theta=0$, $\dot{\theta}=0$; $\dot{y}=0$; $y_n=y_p=\text{const}$; $\Delta y_p=0$. In this case, fundamental equations are converted and accept the following form:

$$\dot{v}_p=\frac{1}{m}(P-X); \qquad (4.41)$$

$$\dot{\Psi}=\frac{P+Z^{\beta}}{mv_p}\beta_6; \qquad (4.42)$$

$$\dot{x}_p=v_p\cos\Psi; \qquad (4.43)$$

$$z'_{px_p}=-\text{tg}\,\Psi. \qquad (4.44)$$

Let us examine other special feature/peculiarities of special

cases. Let $0<\chi=\text{const}<\frac{\pi}{2}$, then r_r and $tg\ \mu$ be determined from formulas (4.30) and (4.29), but

$$\operatorname{tg} \nu = \frac{z_u - z_p}{x_u - x_p}. \qquad (4.45)$$

The coordinates of target and rocket in the plane of interception on the basis (4.35) will be equal to

$$x_{u\chi} = x_u = \frac{y_u \cos \nu}{\operatorname{tg}\mu}; \qquad y_{u\chi} = \frac{y_u}{\sin \chi};$$

$$x_{p\chi} = x_p = \frac{y_p \cos \nu}{\operatorname{tg}\mu}; \qquad y_{p\chi} = \frac{y_p}{\sin \chi},$$

while from (4.37) follows

$$\operatorname{tg} \gamma = \frac{y_{u\chi} - y_{p\chi}}{x_u - x_p}.$$

The definition of angles α_p and φ depends, just as during the solution of the common/general/total problem of guidance, from the selection of guidance method. For a method with fixed-lead angle

$$\alpha_p = \alpha_{p0} = \text{const} \quad \textbf{and} \quad \varphi = \gamma + \alpha_{p0}.$$

With guidance on linear curved

$$\varphi = \gamma; \qquad \nu = \xi;$$

$$\theta = \mu; \qquad \operatorname{tg} \varphi = \frac{y_{u\chi} - y_{p\chi}}{x_u - x_p}. \qquad (4.46)$$

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Differentiating last/latter equation, it is possible to obtain

analytical formula for the angular velocity $d\varphi/dt$

$$\frac{d\varphi}{dt}=\frac{\cos^2\varphi}{(x_u-x_p)^2}\left[(x_u-x_p)(w_{u\chi}-w_{p\chi})-(y_{u\chi}-y_{p\chi})(u_u-u_p)\right], \quad (4.47)$$

where u_u and u_p - horizontal component target speeds and rocket;

$w_{u\chi}$ and $w_{p\chi}$ - comprising target speeds and rocket in plane (P), determined by angle χ, in the direction, perpendicular to axis 0x.

With constant bearing guidance additionally it is possible to write

$$\gamma=\gamma_0=\text{const};\quad \nu=\nu_0=\text{const};\quad \mu=\mu_0=\text{const}.$$

Since μ=const=μ_0, then from formula (4.29) it is possible to find

$$r_r=\frac{y_u-y_p}{\operatorname{tg}\mu_0},$$

after which from converted formula (4.30) we find

$$x_p=x_u-\frac{r_r}{\sqrt{1+\operatorname{tg}^2\nu_0}} \quad (4.48)$$

also, through (4.45)

$$z_p=z_u-(x_u-x_p)\operatorname{tg}\nu_0.$$

With matching guidance the lead angle in the plane of interception is determined from formula (4.5).

If version $0<\chi=\text{const}<\frac{\pi}{2}$ is supplemented by condition $y_u=\text{const}$, we consider that the target/purpose moves rectilinearly and in parallel to axis $0x$ $(\bar{v}_u\|0x)$, then $\lambda=\eta=\varkappa=0$; $\alpha_\kappa=\nu$ and

$$\operatorname{tg}\chi=\operatorname{tg}\gamma_0=\frac{\operatorname{tg}\mu}{\sin\alpha_\kappa}. \quad (4.48)$$

In this case, with constant bearing guidance, let us have

$$v=v_0=\alpha_к=\text{const};\quad \alpha_ц=180°-\gamma_0=\text{const}.$$

Entering similarly, it is possible to obtain solutions, also, for other special cases of moving the target/purpose.

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Chapter V.

THE FREE FLIGHT OF ARTILLERY SHELLS AND UNGLIDED ROCKETS ON FREE-FLIGHT PHASE OF THE TRAJECTORY

On inactive leg when there is no consumption of mass, the motion of the center of mass of rocket or unguided nose section and artillery shell is described by one-type differential equations.

§1. Spatial motion in the dense layers of the atmosphere.

During free flight in the dense layers of the atmosphere on the projectile of constant mass, act two groups of forces - aerodynamic

forces and forces, determined by the effect of the Earth. The system of equations, which describes the three-dimensional/space flight of the unguided projectile, can be obtained from common/general/total system of equations (3.7)-(3.12), (3.14)-(3.23), if we in them drop/omit control forces and torque/moments and to place m = const and P = 0. Of the well stabilized projectiles the change in the angles α and β, caused by the oscillation/vibrations of projectile relative to the center of mass, leads to the insignificant deviations of the center of mass from the calculated trajectory, obtained without the account of the oscillation/vibrations of projectile.

Noticeable effect on the form of the trajectory of the center of mass of projectile during free flight can exert the perturbation

factors, not taken into consideration in equations (3.7)-(3.12) and (3.14)-(3.23): variability ξ and Coriolis acceleration, the given rise to by rotation Earth, eccentricity of the masses of projectile, aerodynamic eccentricity and disregarded aerodynamic force component, for example, Magnus' force. The rotational effect of the Earth on rocket flight and projectiles have examined we in chapter II can be taken into account during the writing of system of equations in relative motion. Eccentricity of the masses of projectile can be taken into account by changing the moments of inertias; aerodynamic eccentricity - by change in the value aerodynamic force component and torque/moments.

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Perturbing forces and torque/moments (for example, Magnus' force) let us consider by the introduction of separate terms into equations (3.7)-(3.12)

$$\left.\begin{aligned}\dot{v}&=\frac{1}{m}\left(-X+Q_x-\sum X_{\text{в}}\right);\\ \dot{\theta}&=\frac{1}{mv}\left(Y^*+Q_{y^*}+\sum Y^*_{\text{в}}\right);\\ \dot{\Psi}&=-\frac{1}{mv\cos\theta}\left(Z^*+\sum Z^*_{\text{в}}\right);\end{aligned}\right\} \quad (5.1)$$

$$\left.\begin{aligned}J_{x_1}\dot{\omega}_{x_1}+(J_{z_1}-J_{y_1})\,\omega_{y_1}\omega_{z_1}&=\sum M_{x_1}+\sum M_{\text{в}\,x_1};\\ J_{y_1}\dot{\omega}_{y_1}+(J_{x_1}-J_{z_1})\,\omega_{x_1}\omega_{z_1}&=\sum M_{y_1}+\sum M_{\text{в}\,y_1};\\ J_{z_1}\dot{\omega}_{z_1}+(J_{y_1}-J_{x_1})\,\omega_{x_1}\omega_{y_1}&=\sum M_{z_1}+\sum M_{\text{в}\,z_1}\end{aligned}\right\} \quad (5.2)$$

In the written equations, besides values, is earlier than known, $\sum X_{\text{в}}$, $\sum Y^*_{\text{в}}$, $\sum Z^*_{\text{в}}$ - the projection of perturbing forces on the half-speed coordinate axes; $\sum M_{\text{в}\,x_1}$, $\sum M_{\text{в}\,y_1}$, $\sum M_{\text{в}\,z_1}$ - sum of the projections of the moments of perturbing forces on the body axes of coordinates.

Equations (3.14)-(3.23) will remain without change and system will consist of 16 equations. If are known the aerodynamic and perturbing forces and torque/moments, then system is locked and can be solved, since in it 16 unknowns

$$\begin{gathered}v(t),\ \theta(t),\ \Psi(t),\ x_3(t),\ y_3(t),\ z_3(t),\\ r(t),\ \vartheta(t),\ \psi(t),\ \gamma(t),\ \alpha(t),\ \beta(t),\\ \gamma_c(t),\ \omega_{x_1}(t),\ \omega_{y_1}(t),\ \omega_{z_1}(t).\end{gathered}$$

For finned unrotative projectiles and min in equations (5.2) it is possible to drop/omit the terms, which contain the product of

angular velocities, and to write them thus:

$$J_{x_1}\dot{\omega}_{x_1}=\sum M_{x_1}+\sum M_{\text{в}\,x_1};$$
$$J_{y_1}\dot{\omega}_{y_1}=\sum M_{y_1}+\sum M_{\text{в}\,y_1};$$
$$J_{z_1}\dot{\omega}_{z_1}=\sum M_{z_1}+\sum M_{\text{в}\,z_1}.$$

If torque/moments the moments of the aerodynamic and perturbing forces will be known, then each of these equations can be solved independently.

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§2. Equations, which describe the motion of the center of mass of projectile in the dense layers of the atmosphere.

The disregard of perturbing forces and torque/moments, and also the assumption of plane-parallel gravitational field leads to the planar trajectory of motion. If we accept for low angles of attack cos $\alpha \approx 1$; sin $\alpha \approx 0$ and $\gamma = 0$ and to add to (5.1) usual kinematic relationship/ratios, then we will obtain system of equations (in the starting system of coordinates Oxy), that describes the motion of the center of mass of the unguided projectile:

$$\left.\begin{aligned} &\frac{dv}{dt}=-\frac{X}{m}-g\sin\theta; \quad \frac{d\theta}{dt}=-\frac{g\cos\theta}{v};\\ &\frac{dy}{dt}=v\sin\theta; \quad \frac{dx}{dt}=v\sin\theta. \end{aligned}\right\} \tag{5.3}$$

The first two equations are written in the high-speed/velocity coordinate system, i.e., in projections on tangent and standard to trajectory. In many instances they prove to be convenient for the solutions of system of equations, written in the rectangular starting coordinate system. In accordance with Fig. 5.1 $m\frac{du}{dt}=-X\cos\theta$. Multiplying numerator and the denominator of right side to v, we will obtain

$$\frac{du}{dt}=-\frac{Xu}{mv}.$$

The value of full speed in the known value u can be found as $v=u\sqrt{1+p^2}$, where p = tg θ. Therefore as the second equation of the system in question let us take the differential equation

$$\frac{dp}{dt}=\frac{d(\mathrm{tg}\,\theta)}{dt}=\frac{1}{\cos^2\theta}\frac{d\theta}{dt}=\frac{1}{\cos^2\theta}\left(-\frac{g\cos\theta}{v}\right)=-\frac{g}{u}.$$

Designating for simplicity of writing E=X/mv and adding kinematic relationship/ratios, we will obtain known system of equations

$$\frac{du}{dt}=-Eu;\quad \frac{dp}{dt}=-\frac{g}{u};\quad \frac{dy}{dt}=up;\quad \frac{dx}{dt}=u. \tag{5.4}$$

If we use for X equality (2.95) and to introduce function G(v), then

the first equation of system will take the form

$$\frac{du}{dt} = -cH(y)G(v)u = -Eu, \qquad (5.5)$$

where

$$E = cH(y)G(v).$$

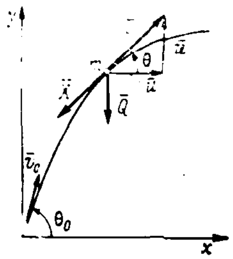

Fig. 5.1. Simplified diagram of the forces, which act on the projectile of constant mass, driving/moving in air with $\bar{g}$ = const.

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With the wish to consider a change in the speed of sound with height/altitude it is necessary to use (2.108) and then

$$\frac{du}{dt}=-cH_{\tau}(y)G(v_{\tau})u.$$

Respectively:

$$E=cH_{\tau}(y)G(v_{\tau}). \qquad (5.6)$$

The second equation of system (5.4) can be replaced by differential equation for determining vertical component velocity

$$\frac{dw}{dt}=-\frac{X\sin\theta}{m}-g.$$

Introducing into the first member of right side that composing

velocities $w=v\sin\theta$, let us have a system of equations:

$$\frac{du}{dt}=-Eu;\quad \frac{dw}{dt}=-Ew-g;\quad \frac{dy}{dt}=w;\quad \frac{dx}{dt}=u. \qquad (5.7)$$

Systems of equations with independent alternating/variable t it is expedient to utilize for performance calculation of the motion of the anti-aircraft shells of constant mass. For performance calculation of the motion of the projectiles of class "Earth- Earth" (the inactive legs of rockets and trajectories of artillery shells) for independent variable usually is taken coordinate x. We will obtain system of equations with argument x, after conducting the obvious transformations of the first and second equations of system (5.4):

$$\frac{du}{dx}=\frac{du}{dt}\,\frac{dt}{dx}=(-Eu)\left(\frac{1}{u}\right)=-E;$$
$$\frac{dp}{dx}=\frac{dp}{dt}\,\frac{dt}{dx}=-\frac{g}{u}\,\frac{1}{u}=-\frac{g}{u^2}.$$

System of equations with independent alternating/variable x will take the form

$$\frac{du}{dx}=-E;\quad \frac{dp}{dx}=-\frac{g}{u^2};\quad \frac{dy}{dx}=p;\quad \frac{dt}{dx}=\frac{1}{u}. \qquad (5.8)$$

For obtaining the approximate analytical solutions of the basic problem of external ballistics of the projectiles of constant mass, is utilized the system of equations for independent variable θ. Let us present

$$\frac{du}{d\theta}=\frac{du}{dt}\,\frac{dt}{d\theta}=-Eu\left(-\frac{v}{g\cos\theta}\right)=\frac{Ev^2}{g}.$$

After replacing E, using 5.5, we will obtain

$$\frac{du}{d\theta}=\frac{c}{g}\,H(y)\,vF(v). \tag{5.9}$$

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After introducing intermediate derivatives, we will obtain respectively

$$\frac{dx}{d\theta}=\frac{dx}{dt}\,\frac{dt}{d\theta}=v\cos\theta\left(-\frac{v}{g\cos\theta}\right)=-\frac{v^2}{g};$$
$$\frac{dy}{d\theta}=\frac{dy}{dx}\,\frac{dx}{d\theta}=\operatorname{tg}\theta\left(-\frac{v^2}{g}\right).$$

System of equations with argument θ will take the form

$$\frac{du}{d\theta}=\frac{c}{g}\,H(y)\,vF(v),\quad \frac{dy}{d\theta}=-\frac{v^2\operatorname{tg}\theta}{g};\quad \frac{dx}{d\theta}=-\frac{v^2}{g};$$
$$\frac{dt}{d\theta}=-\frac{v}{g\cos\theta}. \tag{5.10}$$

For the account of a change in the speed of sound with height/altitude, it is necessary to use (2.108), and then the first equation of system (5.10) should replace with following:

$$\frac{du}{d\theta}=\frac{c}{g}\,H(y)\,vF(v_\tau). \tag{5.11}$$

In systems (5.3), (5.4), (5.7) and (5.8) joint (i.e. being subject joint solution) are the first three equations. In system (5.10) joint are only two first equation. During the supplementary simplifications, for example when $H(y) \approx H(y_{cp})$, in the first equation it is possible to divide variables. this property of system (5.10) makes it possible to utilize it for the approximate analytical solutions.

The solution of equation (5.9) is the functional dependence v = f(6), which determines the hodograph of the velocity of the center of mass of projectile. Therefore it is accepted (5.9) to call the hodograph equation of velocity.

During the compilation of all preceding/previous equations of present chapter, was assumed $\bar{g}$=const. However, this assumption is correct during the trajectory calculation of the motion of the projectiles, intended for a firing to relatively short distances. The system of equations, which describes the free flight of the center of mass of the projectile of constant mass, intended for a firing to long range, can be obtained from (3.73), if we in it accept thrust P = 0

$$\left.\begin{aligned}
&\frac{du}{dt} = -\frac{X\cos\theta + Y\sin\theta}{m} - g\sin\gamma;\\
&\frac{dw}{dt} = -\frac{X\sin\theta - Y\cos\theta}{m} - g\cos\gamma;\\
&\frac{dx}{dt} = u;\quad \operatorname{tg}\theta = \frac{w}{u};\quad g = g_0\left(\frac{R_3}{r}\right)^2;\\
&\frac{dy}{dt} = w;\quad \operatorname{tg}\gamma = \frac{x}{R_3+y};\quad J_{z_1}\ddot{\vartheta} = \sum M_{z_1};\\
&\alpha = \vartheta - \theta;\quad v = \sqrt{u^2+w^2};\quad r = \sqrt{(R_3+y)^2+x^2}.
\end{aligned}\right| \quad (5.12)$$

If we assume $\alpha=0$, $Y=0$, then:

$$\frac{du}{dt} = -\frac{X\cos\theta}{m} - g\sin\gamma;\quad \frac{dw}{dt} = -\frac{X\sin\theta}{m} - g\cos\gamma. \quad (5.13)$$

The remaining equations of system (5.12) will remain without change.

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§3. Equations describing the motion of the center of mass of projectile in the vacuum.

3.1. Motion in conditional plane-parallel gravitational field.

For the approximate computation of the small free trajectories, which pass in the vacuum or in the medium, which exerts negligible

resistance, when it is possible not to consider variability $\bar{g}$, Coriolis acceleration and the curvature of the Earth, they are utilized the equations, which do not consider the air resistance. The system of equations, comprised under the assumption $\bar{g}$=const and X=Y=0, can be utilized for the trajectory calculation of the projectiles of constant mass, driving/moving in the rarefied layers of the atmosphere, with the expected firing distance to 500 km. Error in this case will comprise not more than 10o/o [35]. At the low speeds of the motion of bodies in air (approximately to 50 m/s) it is possible also not to consider the air resistance and to count that the body flies as in the vacuum. In these cases the only acting force will be the gravitational force, and the differential equations of motion, comprised in projections on the axis of the rectangular starting coordinate system, take the form

$$\frac{d^2x}{dt^2}=0; \quad \frac{d^2y}{dt^2}=-g. \qquad (5.14)$$

Equations (5.14) are initial equations in the so-called parabolic theory of the motion of the projectiles of constant mass.

3.2. Motion in flat/plane central gravitational field.

Common/general/total equations (3.70) make it possible to obtain the system of the differential equations, which describe the flight of ballistic missile or its nose section without the account of the

rotation of the Earth in the passive, unguided phase of flight beyond the limits of the dense layers of the atmosphere. If we in system (3.70) $v_x, \dot{v}_x$ and G make equal to zero, to drop/omit the terms, which consider thrust and aerodynamic drag, then we will obtain the case in question, which corresponds to the planar trajectory of the motion of relatively inertial coordinate system Cxy

$$\dot{v}_x = -g_{T0}\frac{R_3^2}{r^3}x; \quad \dot{v}_y = -g_{T0}\frac{R_3^2}{r^3}y.$$

On Fig. 5.2

$$x = r\sin\gamma; \quad y = r\cos\gamma \tag{5.15}$$

and then

$$\dot{v}_x = -g_{T0}\left(\frac{R_3}{r}\right)^2\sin\gamma, \qquad \dot{v}_y = -g_{T0}\left(\frac{R_3}{r}\right)^2\cos\gamma.$$

Remembering that $\ddot{x}=\dot{v}_x$ and $\ddot{y}=\dot{v}_y$ and twice differentiating (5.15), let us pass to polar coordinates r and γ:

$$\ddot{r}\sin\gamma + 2\dot{r}\dot{\gamma}\cos\gamma - r\dot{\gamma}^2\sin\gamma + r\ddot{\gamma}\cos\gamma = -g_{T0}\left(\frac{R_3}{r}\right)^2\sin\gamma; \tag{5.16}$$

$$\ddot{r}\cos\gamma - 2\dot{r}\dot{\gamma}\sin\gamma - r\dot{\gamma}^2\cos\gamma - r\ddot{\gamma}\sin\gamma = -g_{T0}\left(\frac{R_3}{r}\right)^2\cos\gamma. \tag{5.17}$$

Multiplying (5.16) by $\sin\gamma$, and (5.17) by $\cos\gamma$ and adding, we will have

$$\ddot{r} - r\dot{\gamma}^2 = -g_{T0}\left(\frac{R_3}{r}\right)^2. \tag{5.18}$$

Multiplying (5.16) on cos γ, a (5.17) on sin γ and to deduct of the second equation the first, then we will obtain

$$2\dot{r}\dot{\gamma}+r\ddot{\gamma}=0. \tag{5.19}$$

Multiplying last/latter equation on r and converting, let us write system of equations

$$\ddot{r}-r\dot{\gamma}^2=-g_{T0}\left(\frac{R_3}{r}\right)^2; \quad \frac{d}{dt}(r^2\dot{\gamma})=0. \tag{5.20}$$

These are the known differential equations of motion of the projectile of constant mass in the central gravitational field of the Earth without the account of air resistance and rotation of the Earth, written in polar coordinates r and γ. System (5.20) is the basis of the elliptical theory, which makes it possible to approximately determine the motion characteristics of ballistic missiles and Earth satellites.

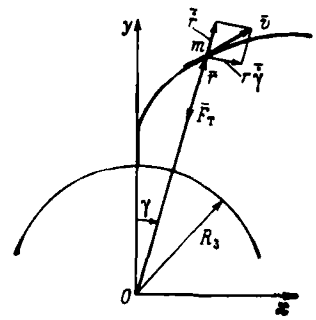

Fig. 5.2. Simplified diagram of the action of gravity force in central gravitational field.

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In the theory of the flight of the projectiles of constant mass, system (5.20) frequently is utilized; therefore is of interest her immediate conclusion, besides obtaining from common/general/total equations (3.70). Are convenient to obtain these system in the form of the equations of Lagrange of second kind (1.33), after taking as generalized coordinates r and γ and after considering that on projectile acts only one external gravity force F_T.

In accordance with Fig. 5.2.

$$v=\sqrt{\dot{r}^2+r^2\dot{\gamma}^2} \quad (5.21)$$

and kinetic energy of the projectile

$$T=\frac{m}{2}\left(\dot{r}^{2}+r^{2}\dot{\gamma}^{2}\right). \qquad (5.22)$$

Differential equation for coordinate r let us write, after obtaining preliminarily the values of terms in (1.33)

$$\frac{\partial T}{\partial r}=mr\dot{\gamma}^{2};\quad \frac{\partial T}{\partial \dot{r}}=m\dot{r};\quad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{r}}\right)=m\ddot{r};$$

$$F_{\tau r}=-g_{\tau}m=-mg_{\tau 0}\left(\frac{R_{3}}{r}\right)^{2}.$$

Summarizing and reducing to m, we will obtain the first equation of system (5.20)

$$\ddot{r}-r\dot{\gamma}^{2}=-g_{\tau 0}\left(\frac{R_{3}}{r}\right)^{2}.$$

Differential equation for a coordinate γ we will obtain, after substituting in (1.33) the values

$$\frac{\partial T}{\partial \gamma}=0;\quad \frac{\partial T}{\partial \dot{\gamma}}=mr^{2}\dot{\gamma} \text{ and } F_{\tau\gamma}=0.$$

This will be the second equation of system (5.20)

$$\frac{d}{dt}\left(r^{2}\dot{\gamma}\right)=0.$$

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Chapter VI

NUMERICAL INTEGRATION OF THE EQUATIONS OF EXTERNAL BALLISTICS AND THE USE OF ELECTRONIC COMPUTERS.

The motion characteristics of rockets and projectiles: velocity, acceleration, the coordinate of the center of mass, etc. are calculated during the integration of the corresponding differential equations of motion. Of the special feature/peculiarities of the integration of the equations of external ballistics, comprised it is strict (without essential assumptions), they consist in the complexity of equations themselves, and also in the fact that the functions, which determine the air resistance, the thrust and some other values do not have simple analytical form, but they are usually assigned by the complex curve/graphs, constructed according to experimental data, or by tables. In connection with this the equations of flight are solved usually by the methods of numerical integration either in electronic digital computers (ETsVM [digital computer]), or by means of manual count.

Manual calculation with the aid of the key-actuated digital

computers, logarithmic tables and small ETsVM is applied for comparatively small according to volume calculations when the use of large universal ETsVM, taking into account the complexity of the adjustment of program, is economically not justified. Manual count is applied also with the adjustment of program in electronic computer and for the control/check of separate calculations.

The complexity of the phenomena of controlled and unguided rocket flight and projectiles, the requirement of speed and high accuracy of performance calculations of motion, and also the large volume of calculations lead to the need for conducting serious research in the field of external ballistics with use of ETsVM and the electronic analog computers of continuous action.

Is known many methods of the numerical integration of differential equations. Are known the methods of Euler, Runge, Kutta, Adams, Stormer, Chaplygin, Krylov, etc. In contemporary works in calculating mathematics, the part of the methods, close in practical accomplishing, is united. More frequent than others in ballistic calculations are utilized the methods of Euler, Runge - Kutta, Adams - Stormer.

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In Russia numerical integration of the equations of external ballistics carried out for the first time A. N. Krylov. In 1917 he read a report, while in 1918 was published work "About the approximate numerical integration of ordinary differential equations", in which given thorough solution of one of the tasks of external ballistics. As the basis of solution, was placed adamsa - Stormer's method. During the subsequent years A. N. Krylov repeatedly returned to the numerical integration of the equations of external ballistics and he considerably improved him. For the determination of the initial values of functions, A. N. Krylov proposed the method of successive approximations.

The method of the numerical integration of A. N. Krylov received further development in N. A. Upornikov's works (was known the version of the solution of Krylov-Upornikov). Original solution was suggested by S. A. Zazakov. In the field of the numerical integration of the equations of external ballistics, worked V. M. Trofimov, V. V. Mechnikov, D. A. Venttsel', Ya. M. Shapiro, A. E. Komarov (version, obtained in the academy im. A. N. Krylov), V. S. Ustinov et al. enumerated authors's works in essence were directed toward the solution of new systems of equations, the development of the methods of calculating the initial values of functions and for the decrease of the volume of calculations with the preservation/retention/maintaining of the necessary accuracy of the

obtained results.

On the basis of all preceding/previous works in the contemporary calculating practice of the manual calculation of trajectory elements, was manufactured the general, sufficiently simple method of the numerical integration of the equations of external ballistics. This method, just as enumerated is earlier, are utilized the theory of interpolation and the theory of finite differences. Therefore, subsequently, let us call it the method of numerical integration with the aid of the tables of finite differences or, it is simpler, by difference method of numerical integration. For performance calculations of motion by ETsVM, is is been commonly used the method of the numerical integration of Runge - Kutta.

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§1. Numerical integration of the differential equations of external ballistics.

Preparation for ~~solution and~~ very solution of differential equations flight can be represented by those carrying out in following sequence.

1. Development of assignment, selection of system of equations and its analysis.

2. Selection of method of numerical integration and analysis of its applicability with manual and machine count.

3. Determination of space of integration.

4. Determination of initial conditions.

5. Determination of initial values of functions.

6. Strictly numerical integration (growth of lines).

7. Determination of cell/elements of characteristic points in the trajectory: cell/elements of end/lead of powered flight trajectory, trajectory elements during separation of used-ups stage, etc.

The first of six stages are tightly interconnected. For example, complexity and the labor expense of the solutions, which are completely reveal/detected only in the process of calculations (in the fifth and sixth stages), they can lead to the need for the review of system of equations for the direction of its simplification or to passage to another method of integration. The development of

assignment consists in the setting of task, the determination of initial conditions and functions, necessary for solution. During the development of assignment, is checked the possibility and the advisability of using the method of numerical integration.

1.1. Numerical integration by difference method.

The methods of numerical integration make it possible to calculate the value of integral of the function, assigned tabular. For this purpose is utilized the so-called interpolating function, which under integral sign replaces the real function whose analytical form is unknown. When this replacement occurs on low section curved, accuracy in the integration can be sufficient high.

Let the interpolating function will be function y = f(x). Then an increase in the definite integral within limits from x_n to x_{n+1} is equal

$$\Delta J_n = \int_{x_n}^{x_{n+1}} y\,dx. \qquad (6.1)$$

The very idle time is the linear interpolating function

$$\frac{y-y_n}{y_{n+1}-y_n} = \frac{x-x_n}{x_{n+1}-x_n}, \qquad (6.2)$$

where $x_{n+1}-x_n=h_x$ - the interval of the variation in the argument, or the space of argument;

$y_{n+1}-y_n=\Delta y_n$ - difference between the values of function, which correspond to datum (n) and to that following (n + 1) the values of argument.

From formula (6-2) the unknown value of function, which corresponds to the value of argument x, is equal

$$y=y_n+\frac{x-x_n}{h_x}\Delta y_n. \tag{6.3}$$

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During linear interpolation the area under curve, that depicts function f(x), will be broken into a series of trapezoids. Integration for trapezoidal rule gives significant error and, as a rule, it is not applied during ballistic calculations.

The simplest method of the numerical integration of differential equations is L. Euler's method. Let be required to find the solution of differential first-order equation

$$y'_x=f(x, y)$$

under the initial conditions: $x = x_0$, $y = y_0$. During integration the

space on argument h_x is selected so that within the limits of this space it would be possible to assume that function f(x, y) retains constant value. Replacing derivative by the relation of low finite increments, it is possible to write

$$y'_x = \frac{dy}{dx} \approx \frac{\Delta y}{\Delta x}.$$

For the first section of integration $\Delta x_0 = x_1 - x_0 = h_x$; $\Delta y_0 = y_1 - y_0$ and then

$$f(x_0, y_0) = \frac{y_1 - y_0}{h_x}.$$

After this let us have

$$y_1 = y_0 + h_x f(x_0, y_0).$$

Repeating process/operation for the subsequent sections of integration, are obtained the consecutive values of the functions:

$$y_2 = y_1 + h_x f(x_1, y_1);$$
$$y_3 = y_2 + h_x f(x_2, y_2).$$
$$\cdots\cdots\cdots\cdots$$

In general form the formula of the numerical integration of the differential equation of I orders for L. Euler's method will be written so

$$y_{n+1} = y_n + h_x f(x_n, y_n).$$

During the solution of the problems of external ballistics, L. Euler's method can lead to considerable errors. The accuracy of

method is raised with the decrease of the space of integration; however, in this case, increases the total volume of calculations and is retained the storage of the errors in the process of integration. Therefore in the most widely used methods of the numerical integration of the equations of external ballistics, are utilized the special interpolating functions, which make it possible to increase the space of integration, in comparison with integration for trapezoidal rule and L. Euler's method, for the preservation/retention/maintaining of the necessary accuracy.

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The interpolating function is comprised in the form of the whole polynomial whose degree per unit is smaller than the number of assigned values of function on the section of interpolation in question. Curve, that corresponds to polynomial, must pass through all the utilized points, which are called of interpolation points. Satisfaction of these conditions gives rise to the uniqueness of the interpolating function. Interpolation formulas provide the possibility of the approximation calculus of the values of function for the values of argument, different from the assemblies of interpolation. In this case, is distinguished the interpolation in the narrow sense when argument x lie/rests within assigned time interval $x_n - x_{n+1}$, and extrapolation - at the value of argument, which

is located beyond the limits of interval.

Is most universal the interpolating function of Lagrange. During its use on the selection of interpolation points are not placed any special limitations. Lagrange's interpolation formula takes the form

$$y=f(x)\approx\frac{(x-x_1)(x-x_2)\ldots(x-x_m)}{(x_0-x_1)(x_0-x_2)\ldots(x_0-x_m)}y_0+$$
$$+\frac{(x-x_0)(x-x_2)\ldots(x-x_m)}{(x_1-x_0)(x_1-x_2)\ldots(x_1-x_m)}y_1+\ldots+$$
$$+\frac{(x-x_0)(x-x_1)\ldots(x-x_{m-1})}{(x_m-x_0)(x_m-x_1)\ldots(x_m-x_{m-1})}y_m. \quad (6.5)$$

Are known other interpolation formulas: the first and second formulas of Newton, formula of Gauss, Bessel, Stirling. Newton's interpolation formulas, comprised with the use of tables of finite differences, are obtained under the condition of the constancy of the space of argument $(h_x=\text{const})$.

If certain function y =f (x) is assigned tabular with a change in the argument through the constant space h_x, then a finite difference in the function of the first order, or the first difference, is called value $\Delta y=\Delta f(x)=f(x+h_x)-f(x)$. Second difference in function $\Delta^2 y=\Delta^2 f(x)=\Delta[\Delta f(x)]$ and in so forth. For obtaining the table of finite differences, it is necessary from each value of function to deduct it that preceding and the obtained result to write in chair to the right in one line with subtrahend, after giving to it the number of the

latter (table 6.1). Comprised similarly table calls the horizontal table of finite differences.

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Introducing the new variable

$$\xi=\frac{x-x_0}{h_x},$$

where x_0 - initial value of argument, let us write formula for Newton's first interpolating polynomial to degree not higher than n-th

$$P_n(x)=y_0+\xi\Delta y_0+\frac{\xi(\xi-1)}{2!}\Delta^2 y_0+\ldots+\frac{\xi(\xi-1)\ldots(\xi-n+1)}{n!}\Delta^n y_0. \qquad (6.6)$$

Newton's second interpolation formula is applied for interpolation at the end of the known values of function (i.e. for an extrapolation):

$$P_n(x)=y_n+\xi\Delta y_{n-1}+\frac{\xi(\xi+1)}{2!}\Delta^2 y_{n-2}+\frac{\xi(\xi+1)(\xi+2)}{3!}\Delta^3 y_{n-3}+\ldots+\frac{\xi(\xi+1)\ldots(\xi+n-1)}{n!}\Delta^n y_0. \qquad (6.7)$$

where y_n - a last/latter known value of function.

When the table of finite differences is present, the principle of the compilation of the interpolating function consists in the following:

1. Is extracted the line of consecutive cell/elements, i.e., numbers from the table of the differences, which directly depend on the preceding/previous numbers (line is called that set of successive cell/elements, into which of each column it is taken only on one cell/element).

2. Before each row element, except the first, is placed coefficient in the form of fraction. Its denominator is the factorial whose order is equal to the order of cell/element, and in numerator it is contained so many the factors of form $\xi-i$, which order of difference.

Table 6.1. Table horizontal of finite differences in function y=f(x).

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
x_{n-3}	y_{n-3}	Δy_{n-3}	$\Delta^2 y_{n-3}$	$\Delta^3 y_{n-3}$
x_{n-2}	y_{n-2}	Δy_{n-2}	$\Delta^2 y_{n-2}$	$\Delta^3 y_{n-2}$
x_{n-1}	y_{n-1}	Δy_{n-1}	$\Delta^2 \overset{*}{y}_{n-1}$	$\Delta^3 \overset{*}{y}_{n-1}$
x_n	$\overset{*}{y}_n$	$\Delta \overset{*}{y}_n$	$\Delta^2 y_n$	$\Delta^3 y_n$
x_{n+1}	y_{n+1}	Δy_{n+1}	$\Delta^2 y_{n+1}$	
x_{n+2}	y_{n+2}	Δy_{n+2}		
x_{n+3}	y_{n+3}			

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3. First factor of numerator must take form $\xi-i_0$, where i_0 — index of preceding/previous cell/element of finite difference (for example, if preceding/previous cell/element has number n-3, by index of which they call number "n-3", then i_0=-3).

All the subsequent factors decrease per unit.

Utilizing this rule, let us write as an example interpolation formula for the broken line, which corresponds to value argument x_n. The finite differences, used for the writing of interpolation formula, noted are noted in table 6.1 by asterisk.

$$y = y_n + \frac{\xi}{1!}\Delta y_n + \frac{\xi(\xi-1)}{2!}\Delta^2 y_{n-1} + \frac{(\xi+1)\xi(\xi-1)}{3!}\Delta^3 y_{n-1}. \quad (6.8)$$

The written formula answers the broken line of consecutive cell/elements, designated by symbol With independent variable ξ from (6.1) we will obtain an increase in the definite integral for one space of integration in the form

$$\Delta J_n = h_x \int_0^1 y d\xi, \qquad (6.9)$$

since $d\xi = \frac{dx}{h_x}$.

For determination of ΔJ_n we can use one of the interpolation formulas, comprised on the basis of the lines of consecutive cell/elements. thus, for instance, after substituting (6.8) dependence (6.9) it is possible to obtain the following expression for calculating the increase in the definite integral

$$\Delta J_n = h_x \left(y_n + \frac{1}{2} \Delta y_n - \frac{1}{12} \Delta^2 y_{n-1} - \frac{1}{24} \Delta^3 y_{n-1} \right). \qquad (6.10)$$

It is analogous for the horizontal line of the consecutive cell/elements

$$\Delta J_n = h_x \left(y_n + \frac{1}{2} \Delta y_n - \frac{1}{12} \Delta^2 y_n + \frac{1}{24} \Delta^3 y_n \right). \qquad (6.11)$$

During the use of an inclined extrapolation line, the formula takes the form

$$\Delta J_n = h_x \left(y_n + \frac{1}{2} \Delta y_{n-1} + \frac{5}{12} \Delta^2 y_{n-2} + \frac{3}{8} \Delta^3 y_{n-3} \right). \qquad (6.12)$$

last/latter formula makes it possible to interpolate forward (to extrapolate), being based on the character of the preceding/previous change in the function.

Comprising the table of finite differences in the derivatives $y'=y'_x$, it is possible to obtain the formulas, utilized during the numerical integration of the equations of external ballistics (table 6.2).

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Adding the difference in the function Δy_n, determined in the appropriate formula, to known value y_n, we obtain the subsequent unknown value of the function

$$y_{n+1}=y_n+\Delta y_n. \quad (6.13)$$

The formulas, placed in table 6.2 and the containing differences in the third order, can be used for the growth of lines, if it is known not less than four initial values of derivative. The first value is determined by the initial conditions $x=x_0$ and $y=y_0$ for the integrated differential equation. The missing initial values of derivatives usually find by the method of successive approximations

whose essence is presented below.

Let be assigned differential equation 1 of order $y'_x=f(x, y)$, which for decreasing the indexing further let us record/write in the form $y'=f(x, y)$. The initial values x_0 and y_0 are known, therefore, known value y'_0. The determination of the missing cell/elements of the table of differences begins from the first approximation

$$\Delta y_0=hy'_0; \quad y_1=y_0+\Delta y_0.$$

Through the known value y_1, we find

$y'_1=f(x_1, y_1)$ we compute $\Delta y'_0=y'_1-y'_0$.

Table 6.2. Formulas for difference method of numerical integration.

Символ строки последовательных элементов (1)	Формулы численного интегрирования (2)
	$\Delta y_n = h\left(y'_n + \frac{1}{2}\Delta y'_n - \frac{1}{12}\Delta^2 y'_n + \frac{1}{24}\Delta^3 y'_n\right)$
	$\Delta y_n = h\left(y'_n + \frac{1}{2}\Delta y'_n - \frac{1}{12}\Delta^2 y'_{n-1} - \frac{1}{24}\Delta^3 y'_{n-1}\right)$
	$\Delta y_n = h\left(y'_n + \frac{1}{2}\Delta y'_{n-1} + \frac{1}{12}\Delta^2 y'_{n-1} - \frac{1}{24}\Delta^3 y'_{n-2}\right)$
	$\Delta y_n = h\left(y'_n + \frac{1}{2}\Delta y'_n - \frac{1}{12}\Delta^2 y'_{n-1} - \frac{1}{24}\Delta^3 y'_{n-2}\right)$
	$\Delta y_n = h\left(y'_n + \frac{1}{2}\Delta y'_{n-1} + \frac{5}{12}\Delta^2 y'_{n-2} + \frac{3}{8}\Delta^3 y'_{n-3}\right)$

Key: (1). Symbol of the line of consecutive cell/elements. (2). Formulas of numerical integration.

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During accomplishing of the second approach/approximation on known from the first approximation y'_0, y'_1 and $\Delta y'_0$ we determine

$$\Delta y_0 = h_x\left(y'_0 + \frac{1}{2}\Delta y'_0\right); \quad y_1 = y_0 + \Delta y_0$$

and

$$\Delta y_1 = h_x\left(y'_1 + \frac{1}{2}\Delta y'_0\right); \quad y_2 = y_1 + \Delta y_1.$$

Substituting in initial equation known x_1, y_1 and x_2, y_2, we find

~~and~~ and

y_1' ~~,~~ y_2' ~~,~~ $\Delta y_0' = y_1' - y_0'$; $\Delta y_1' = y_2' - y_1'$; $\Delta^2 y_0' = \Delta y_1' - \Delta y_0'$.

The third approach/approximation is control. In terms of the values

$$\Delta y_0 = h_x \left(y_0' + \frac{1}{2} \Delta y_0' - \frac{1}{12} \Delta^2 y_0' \right); \quad \Delta y_1 = h_x \left(y_1' + \frac{1}{2} \Delta y_1' - \frac{1}{12} \Delta^2 y_0' \right)$$

we determine y_1 and y_2 we compute y_1', y_2', $\Delta y_0'$, $\Delta y_1'$, $\Delta^2 y_0'$.

For clarity that presented is given in Table by 6.3.

After accomplishing of approach/approximations, is realized the consecutive solution of problem whose essence consists of the calculation of cell/elements n + 1 of line on the cell/elements of n-th line and preceding/previous table rows. The differences, found from inclined line for the leading equation, must be refined in the broken line (see Table 6.3).

Table 6.3. Order of calculation during the determination of the initial values of derivatives $y'_{xi}=y_i$ by successive approximations.

Приближение (1)	№	x	y	Δy	y'	Δy'	Δ²y'
I	0	x_0	y_0	Δy_0	y'_0	$\Delta y'_0$	
	1	x_1	y_1		y'_1		
II	0	x_0	y_0	Δy_0	y'_0	$\Delta y'_0$	$\Delta^2 y'_0$
	1	x_1	y_1	Δy_1	y'_1	$\Delta y'_1$	
	2	x_2	y_2		y'_2		
III	0	x_0	y_0	Δy_0	y'_0	$\Delta y'_0$	$\Delta^2 y'_0$
	1	x_1	y_1	Δy_1	y'_1	$\Delta y'_1$	$\Delta^2 y'_1$
	2	x_2	y_2	Δy_2	y'_2	$\Delta y'_2$	
	3	x_3	y_3		y'_3		

Key: (1). Approach/approximation.

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Calculation usually concludes with the determination of trajectory elements at end point. If the latter falls on the value of argument, multiple for whole space, then of no supplementary calculations it is required. But if end point does not correspond to this value of argument, then its cell/elements are determined with the aid of reverse/inverse interpolation. Under reverse/inverse interpolation is understood the determination of argument in terms of the known value of function. In this case into any of the interpolation formulas

unknown becomes the value of the interpolating factor ξ. Calculation of the interpolating factor ξ is done by the method of iterations as follows.

From the second member of the utilized interpolation formula (for example, 6.8) we find ξ, taking into account only finite differences in the second order:

$$\xi = \frac{y - y_n}{\Delta y_n} - \frac{\xi(\xi - 1)}{2!} \cdot \frac{\Delta^2 y_{n-1}}{\Delta y_n}. \qquad (6.14)$$

In the first approximation, let us have

$$\xi_1 = \frac{y - y_n}{\Delta y_n},$$

which corresponds to linear interpolation. Value ξ_2 in the second approach/approximation is determined from the formula

$$\xi_2 = \xi_1 - \frac{\xi_1(\xi_1 - 1)}{2!} \cdot \frac{\Delta^2 y_{n-1}}{\Delta y_n}. \qquad (6.15)$$

Upon consideration of the third differences, the problem of finding the interpolating factor ξ is solved in connection with (6.8) according to the similar schematic:

$$\xi_3 = \xi_2 - \frac{\xi_2(\xi_2 - 1)}{2!} \frac{\Delta^2 y_{n-1}}{\Delta y_n} - \frac{(\xi_2 + 1)\xi_2(\xi_2 - 1)}{3!} \frac{\Delta^3 y_{n-1}}{\Delta y_n}.$$

The number of approach/approximations, required for determination ξ with the established/installed accuracy, corresp to the number of the differences considered. The unknown value

argument is calculated with known ξ completely simply:

$$x = x_n + \xi h_x. \quad (6.16)$$

Basic advantages of difference methods of numerical integration - comparative simplicity of count, reliable means control/check of the course of calculations (course of changing the value of differences) and the high accuracy of the obtained results. To deficiency/lacks one should relate unwieldiness of solution, and also the fact that the beginning of calculation is conducted according to another network, than basic solution. This to a considerable degree impedes the realization of difference methods during calculations by EIsVM.

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1.2. Selection and analysis of system of equations.

The system of differential equations must with the largest possible accuracy to describe physical process (in our case - rocket flight) and to consider the larger possible number of its determining parameters (during the study of the action of the rockets - the acting on them forces and torque/moments). However, as is known, complex systems of equations lead to the bulky and laborious solutions, during which is raised the possibility of miscalculations

and errors. Therefore when selecting of system of equations, it is expedient to attempt to previously rate/estimate the effect of the acting forces and torque/moments with the fact in order to consider essential in datum investigation factors and to drop/omit secondary. There are other general requirements, presented on the integrated system of equations.

Most simply are solved the differential equations of first order; therefore the selection of system it is necessary to check that can it be converted into the system of equations of the first order. The equalizations of external ballistics are usually of the order not higher than the second and can be easily given to the system of equations of the first order in the manner that this was done above (for example, see chapter III). The directly solved system must be, naturally, locked and consist of differential first-order equations.

When selecting of system of equations and independent variable, it is necessary to also rate/estimate a change in the entering the equation functions and derivatives in all range of integration. For example, the use of system (5.8) is inexpedient at large angles of departure, since at $\theta_0 > 60°$ value dp/dx and p sharply changes for the preservation/retention/maintaining of the accuracy of calculations necessary to take very fine pitch, which increases the

volume of calculations. Cannot be used systems of equations, in the process of solution of which derived or functions themselves become equal to infinity. For example, named system (5.8) is unsuitable for trajectory calculation at the values θ_0 of close to 90°, since in this case $tg\theta_0 \to \infty$. Is unsuitable for the calculation of the ground-based trajectories, for example, system with independent alternating/variable y, since in peak of the trajectory $\frac{dx}{dy} = \infty$.

Systems of equations which can be integrated under virtually any initial conditions, are called universal. Such systems include (3.75), (3.77), (5.4) and other systems, comprised with independent alternating/variable t (time).

For integration it is desirable to select system with the smaller number of simultaneous, together solved equations, with the most convenient for this task argument.

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For example, for the calculation of the powered flight trajectory of the guided and unguided missiles and zenith trajectories of the projectiles of barrel systems it is convenient to take systems with independent by the variable t (time). For the calculation of the inactive legs of the rockets of class "surface-surface" when $\theta_K \leq 60°$

and of the ground-based trajectories of the projectiles of barrel systems when $\theta_0 \leqslant 60°$ usually takes system (5.8), comprised with independent alternating/variable x.

The important condition of evaluating the selective system is the number of leading equations. Leading is called the equation from solution of which begins the process of the growth of lines during the use of difference method of numerical integration. In the right side of the leading equation, is contained in explicit (or implicit) form function itself whose increase is determined during solution. An increase in the function can be found from the leading equation in two approach/approximations; first - from the formula of numerical integration, which corresponds to extrapolation (inclined) line, and after the solution of remaining simultaneous equations and growth of the table of differences - it is more precisely formulated on the formula of broken line. During a comparative evaluation of different systems of equations, the preference other conditions being equal should return to the system, which has one the leading equation, since in this case considerably are simplified calculations. For example, in system (5.8) - one the leading equation du/dx=-E, and in system (5.7) their two:

$$\frac{dw}{dt} = -Ew - g \text{ and } \frac{du}{dt} = -Eu.$$

In the presence of several leading equations one of them it is

selected by basic and with it begin the calculations. The basic from the leading equations is considered similar, which determines the parameter of motion, necessary for integrating the greatest number of simultaneous equations. So, in the examined above example the fundamental leading equation, the first, since w defines coordinate $y(\dot{y}=w)$, required for the calculation of force of X, while coordinate x, determined on $u(\dot{x}=u)$, for the solution of the simultaneous equations of system (5.7) it is not necessary. If trajectory ground-based and $\theta_0<60°$, then preference should return to system (5.8).

All the calculations when conducting of numerical integration are conducted by hand on basic and auxiliary forms. Basic form contains vertical columns for the location of the value of argument, tables of differences in the derived computed functions and functions themselves. Basic form's form is determined by system of equations and does not depend on the method of calculation. The leading equation is placed, as a rule, in the extreme right side of the form.

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In left extreme column are placed the numbers of lines, and together - the value of argument, which is changed through the constant space. Basic form's filling is conducted from right to left, beginning with

the leading equation.

Auxiliary form must be comprised so as to accomodate all the calculations regarding differences in the unknown functions in the formulas of numerical integration and the calculation of derivatives for the differential equations of system. Basic and auxiliary forms must be comprised in such a way as to shorten to the minimum the recording of any calculations out of forms. Recording on separate leaflets leads to errors and complicates testing solution. Auxiliary form's form depends on the integrated system of equations, method of calculation and computer technology used. If calculation is conducted with the help of logarithmic tables, then in auxiliary form must be provided lines for the recording in them of logarithms, actions with them and line for involution with the aid of the tables of antilogarithms. Auxiliary form, intended for calculation with the aid of key-actuated automatic machines or simplest PTsVM, is simpler, since do not contain the lines, intended for actions with logarithms.

With any method of calculation in form, must be the lines, intended for actions with the tabulated values. Considerable simplification in the calculations is achieved at use of tables for $\pi(y), H_\tau(y), \sqrt{\tau_{0N}/\tau}$, standand functions $F(v_\tau), G(v_\tau)$ etc. and logarithmic tables of the named functions.

The number of horizontal lines of basic form and the number of auxiliary form's vertical columns must correspond to the number of calculation points in the trajectory and certain supplementary lines and columns for conducting the approaches. Auxiliary form's first column does not usually have number and is intended for the calculation of the intermediate quantities, determined on the basis of initial conditions.

Basic forms's specimen/samples for the solution of systems (5.76) and (5.8) are visible from an example of calculation by the numerical integration of the motion characteristics of the unguided rocket on active and inactive legs.

1.3. Accuracy of calculations and selection of space of integration.

An error in the results of the calculations of the motion characteristics of flight vehicles is determined by the following standard groups of errors:

1. By errors in the formulation of the problem, determined by the degree of approximation of mathematical model to the real process of moving the flight vehicle. This group of errors, called errors in the task, is given rise to by the established/installed assumptions and the selection of system of equations.

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2. Second group is determined by errors in method of solution, for example, by introduction of approximating functions for air resistance, by replacement of numerical solution by approximate analytical and so forth.

3. Large group of errors appears during introduction into solution of series or other mathematical sequences and use only of their initial terms. Such errors are called remanent/residual and are given rise to by sum value of the omitted terms of a series.

4. Accuracy of solution is determined to a considerable extent by accuracy of initial data, for example, accuracy of determining aerodynamic characteristics. Such errors are called initial.

5. To technical errors in calculations usually are related rounding off errors and large group of errors in actions on approximate numbers.

Different fields of ballistic research and different methods of the solution of ballistic problems have their specific errors. During

the application/use of a finite-difference method, an error in the calculations is determined in essence by the errors for approximation, by the errors for mathematical operations and by rounding errors. Errors in the mathematical operations depend on the common/general/total schematic of solution and are virtually nonremovable. With rounding is applied the rule of Gauss. The errors for mathematical operations and rounding error usually little affect the accuracy of final results during trajectory calculation.

The errors for approximation are caused by the replacement of real function or by its derived interpolating function of the determined order. The value of the error for approximation is determined by the order of the held difference in the interpolating function and by the space of argument. In the process of the growth of the lines of the error for approximation, error for mathematical operations and rounding error, they are accumulated. The selection of the space of integration is the important stage of entire calculation, since its value determines not only accuracy, but also the labor expense of calculations.

With the low pitch of integration, it is possible to forego the use of differences in the derived high orders (the second and above); however, in this case grow/rises the number of calculation points and in the process of calculations increases the storage of errors. With

steep pitch is decreased the volume of calculations, but decreases the accuracy of calculations.

The value of the space of integration depends on many values and is not determined unambiguously. Let us establish communication/connection of the space of integration with the accuracy of calculations and the number of calculation points during definition of one of the most important characteristics - firing distance.

The expected firing distance x_c and the maximum standard deviation of the impact point in the projectile from the calculated - ε_x. An absolute error in the calculations in the process of design - Δ_x must be considerably less.

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In the assigned magnitude Δ_x or on connected with it relative error $\delta_x=\frac{\Delta_x}{x_c}100\%$ can be determined the number of accurate signs, held during calculations. According to the general rules of calculating mathematics, intermediate calculations must be conducted with one (two) spare sign. $\Delta_x\approx 0{,}1\,\varepsilon_x$. Then with one spare sign a maximum practical error in the calculations must be

$$\Delta_x^{\bullet}\approx 0{,}1\Delta_x\approx 0{,}01\varepsilon_x. \qquad (6.17)$$

During probabilistic estimation of error, resultant error is determined from the formula

$$\Delta_x^* = \sqrt{n}\,\Delta_{x_i}, \qquad (6.18)$$

where n - a number of terms during numerical integration (number of calculation points);

Δ_{x_i} - maximum absolute error, taken identical for each of the terms.

For example, if for one space of integration $\Delta_{x_i}=0{,}1$ m and during integration render/showed 49 calculation points, then complete distance x_C will be determined with the maximum practical error

$$\Delta_x^* = \sqrt{49}\cdot 0{,}1 = 0{,}7 \text{ м.}$$

Usually the numerical integration of the equations of external ballistics is conducted with the use of the second differences, and the space of integration is selected similar so that the rejected third differences in the derivatives barely would affect increases in the function (usually so that they would not exceed several units of the last/latter held sign in an increase in the function). Let with independent alternating/variable t the term in an increase in

function x, determined by the third difference, does not exceed the permissible absolute error at each space of integration - Δ_{x_i}. In the formulas of numerical integration, used for the growth of lines, the third differences have coefficient of $\frac{1}{24}$ (see Table 6.2). Then

$$\frac{h_t}{24}\Delta^3 u < \Delta_{x_i}. \qquad (6.19)$$

The space of integration and the number of calculation points are mutually connected through the expected finite value of the complete time of integration (duration of the operation of engine - t_K for an active section or complete flight time for a ground-based trajectory - t_C). For example, the number of calculation points for a powered flight trajectory is equal

$$n=\frac{t_K}{h_t}.$$

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After transformation (6.19) taking into account (6.18) for testing of space we will obtain the condition

$$h_t < \frac{24\Delta_x^*}{\Delta^3 u \sqrt{n}}. \qquad (6.20)$$

Since $\Delta^3 u$ in the beginning of calculations it is unknown, then also the space of integration cannot be determined unambiguously prior to the beginning of calculations. It is necessary to assign value of space, being oriented toward the complex of the similar

preceding/previous calculations. In the process of integration with an increase in the number of line, the third differences usually rapidly decrease; therefore testing the selected space on formula (6.20) one should make in the beginning of calculations with increase of the lines of the consecutive cell/elements of the table of differences in the derivatives.

During further calculations with the decrease of the third differences in the derivatives, the space of integration can be increased. The supplementary sign/criterion of the correctness of the selected space is the convergence of process during the determination of the initial values of function. With the correctly selected space it proves to be sufficient of two or three approach/approximations. As the criterion of the correctness of the selected space in the process of integration during the growth of lines can serve also the value of the refinement of the increase in the function, determined according to the leading equation. With the correctly selected space the difference in the values of increases, determined in inclined and broken lines, must not exceed several units of the last/latter held sign (see an example of calculation in table 6.4). Besides satisfaction to the named requirements on spacing accuracy must be, as far as possible, by the convenient number not which it is easy to multiply when conducting of calculations. At increase and a decrease of the space in the process of calculations usually are carried out

two times (or into the multiple two number once); therefore the initial value of space is expedient to select similar so that the double space and they would be double smaller also convenient for multiplication. The need of decreasing the space in the process of calculations is usually determined by abrupt change in one of the values, for example, by a change of value $c_x(M)$ in the velocity band of the motion of projectile, close to the speed of sound. After the passage of the area of the transonic speeds, during further smooth change $c_x(M)$, the space of integration it is expedient to again increase.

In the practice of the numerical calculation of relatively small trajectories (the inactive legs of the unguided rockets and the trajectories of the projectiles of barrel systems) by manual calculations were established the following values of the space of integration. During the integration of system (5.8) for independent alternating/variable x, for trajectories with relatively larger angles of departure ($10° < \theta_0 < 60°$) the space is usually taken $h_x = 500$ by m; during the calculation of the short low trajectories of small arms during the integration of the same system of equations, the space on x is taken $h_x = 100-200$ m.

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During the integration of systems (3.76) and (3.77), that describe the motion of the center of mass of the unguided rocket, with relatively small distances space or time can be taken $h_t=0.1-0.5$ s.

For surface-to-air missiles and projectiles in the zone of rendezvous for the maneuvering target, sometimes proves to be necessary to take $h_t=0.01-0.001$ s.

During the integration of the system of equations, which describes the motion of the center of mass of projectile together with the study of short-period oscillatory motions relative to the center of mass, the space of integration for low projectiles $h_t=0.001-0.0005$ s. In view of large labor expense, the solution of a similar problem by manual calculations is conducted in the exceptional cases.

The accuracy of calculations depends on the quantity of accurate significant digits which is determined from the inequality

$$\Delta \leqslant 10^{k}, \qquad (6.21)$$

where k=m-n+1;

Δ - absolute error in the number;

m - old decimal digit of number;

n - number of accurate significant digits.

Example. Let the approximate number L=12480 km be must be calculated with relative accuracy $\delta_L = 1\%, 0{,}1\%, 0{,}01\%$ and by 0.001o/o.

Let us determine, how many in each case must be taken accurate signs during calculations. Absolute error in each case will be determined according to formula $\Delta_l = \frac{L\delta_L}{100}$ and it will be equal to:

$$\Delta_{L1}=125<10^3;\ \Delta_{L2}=12{,}5<10^2;\ \Delta_{L3}=1{,}25<10^1;\ \Delta_{L4}=0{,}125<10^0.$$

Calculated number in position recording takes the form

$$12\,480=1\cdot 10^4+2\cdot 10^3+4\cdot 10^2+8\cdot 10^1+0\cdot 10^0.$$

Consequently, the old decimal digit m = 4. The number of accurate signs on formula n=m-k+1 for each calculated case will be equal $n_1=2$, $n_2=3$; $n_3=4$ and $n_4=5$. Taking into account one spare sign, intermediate calculations must be conducted in accordance with three, four, five and six accurate signs. The correct recording of number L in each case will take the form

$$L=1{,}25\cdot 10^4\ \text{км};\ L=1{,}248\cdot 10^4\ \text{км};\ L=1{,}2480\cdot 10^4\ \text{км};\ L=1{,}24800\cdot 10^4\ \text{км}.$$

During calculation the tables of common logarithms, one should

use tables with so many signs, how many accurate signs is expedient to have in the computed most important function, for example distance L. During calculations of the relatively small trajectories of artillery shells and rockets and for initial estimate and training calculations they frequently use the widespread four-place tables of common logarithms.

During the evaluation of the common/general/total accuracy of solution, it is necessary to keep in mind that, besides the accuracy of calculations, it is determined by the correctness of the description of physical process by differential equations, the accuracy of the assignment of initial data, etc.

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The unjustified increase of the quantity of significant digits in numbers does not raise the common/general/total accuracy of solution, but only increases the volume of recording and possibility of errors.

1.4. Determining motion characteristics at assigned points in the trajectory.

During the trajectory calculation of different types of projectiles, always is encountered the need of determining the

cell/elements of motion for assigned characteristic points in the trajectory. It is necessary to know the cell/elements of the end/lead of powered flight trajectory, trajectory elements during the separation of the used up stageren of rocket, trajectory elements at the point of firing jet engine (of ABS and ARR) and at the end of its work, the cell/elements of peak of the trajectory, impact point and point of impact. As a rule, the named characteristic points do not coincide with the points, obtained in the process of integration, and the calculation of motion characteristics in them is conducted with the aid of the formulas of direct/straight and reverse/inverse interpolation.

Motion characteristics at the end of the work of the jet engine of the unguided rocket are determined from interpolation formulas. In the case of the solution of problem with independent alternating/variable t (time) the interpolating factor is determined from the formula

$$\xi_{\kappa}=\frac{t_{\kappa}-t_{n}}{h_{t}},$$

where t_{κ} - a time of the end/lead of the engine operation;

t_n - near to t_{κ} preceding/previous tabular value of time;

h_t - space of integration for time. The abscissa of the

end/lead of the active section will be determined according to common/general/total formula (6.8)

$$x_к = x_n + \xi_к \Delta x_n + \frac{\xi_к(\xi_к - 1)}{2} \Delta^2 x_{n-1}.$$

The ordinate of the end/lead of the active section and the velocity at the end of powered flight trajectory are determined also from (6.8)

$$y_к = y_n + \xi_к \Delta y_n + \frac{\xi_к(\xi_к - 1)}{2} \Delta^2 y_{n-1};$$
$$v_к = v_n + \xi_к \Delta v_n + \frac{\xi_к(\xi_к - 1)}{2} \Delta^2 v_{n-1}.$$

Similarly are determined other trajectory elements at the end of the active section.

But if the independent variable is undertaken mass ratio μ, then interpolation factor will be determined according to the formula

$$\xi_к = \frac{\mu_к - \mu_n}{h_\mu},$$

where $\mu_к$ - corresponds to the end/lead of the engine operation;

h_μ - space of integration for μ.

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The cell/elements of peak of the trajectory S and of impact point C, i.e., the point of intersection of trajectory with the

surface of the Earth (with the coordinate plane Oxz) are determined from the formulas of inverse interpolation. In peak of the trajectory $\theta_S=0$ and, therefore, $p_S=0$. Then the interpolating factor is determined according to (6.14)-(6.16). In the first approximation,

$$\xi_{S1}=-\frac{p_n}{\Delta p_n},$$

where p_n - last/latter positive value from those obtained in the course of integration.

In the second approach/approximation

$$\xi_S\approx\xi_{S11}=\xi_{S1}-\frac{\xi_{S1}(\xi_{S1}-1)}{2}\frac{\Delta^2 p_{n-1}}{\Delta p_n}.$$

After this the motion characteristics, which correspond to peak of the trajectory, will be determined according to usual interpolation formulas. During integration with independent alternating/variable x the abscissa

$$x_S=x_n+\xi_S h_x,$$

while the ordinate of peak of the trajectory

$$y_S=y_n+\xi_S\Delta y_n+\frac{\xi_S(\xi_S-1)}{2}\Delta^2 y_{n-1}.$$

Similarly can be determined the others of motion characteristics.

The interval of integration, which includes impact point, is

characterized by a change in the calculated ordinate from $y_n>0$ to $y_{n+1}<0$. At impact point the ordinate of trajectory $y_C=0$. Then the interpolating factor in the first approximation,

$$\xi_{CI}=-\frac{y_n}{\Delta y_n}, \text{ a}$$

In the second approach/approximation

$$\xi_C\approx\xi_{CII}=\xi_{CI}-\frac{\xi_{CI}(\xi_{CI}-1)}{2}\frac{\Delta^2 y_{n-1}}{\Delta y_n}.$$

The basic cell/elements of motion at impact point, in the case of integration with independent to the variable x, are equal to

$$x_C=x_n+\xi_C h_x;$$
$$p_C=p_n+\xi_C\Delta p_n+\frac{\xi_C(\xi_C-1)}{2}\Delta^2 p_{n-1};$$
$$t_C=t_n+\xi_C\Delta t_n+\frac{\xi_C(\xi_C-1)}{2}\Delta^2 t_{(n-1)}.$$

Remaining cell/elements will be determined according to similar formulas.

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If for the guided ballistic missile given speed at the cutoff point of engine - v_K, then the interpolating factor in the first approximation, it is equal to

$$\xi_{\kappa I} = \frac{v_\kappa - v_n}{\Delta v_n},$$

while in the second

$$\xi_\kappa \approx \xi_{\kappa II} = \xi_{\kappa I} - \frac{\xi_{\kappa I}(\xi_{\kappa I} - 1)}{2} \frac{\Delta^2 v_{n-1}}{\Delta v_n}.$$

Time, which corresponds to the end/lead of powered flight trajectory,

$$t_\kappa = t_n + \xi_\kappa h_t.$$

Coordinates of the end/lead of powered flight trajectory

$$x_\kappa = x_n + \xi_\kappa \Delta x_n + \frac{\xi_\kappa(\xi_\kappa - 1)}{2} \Delta^2 x_{n-1};$$

$$y_\kappa = y_n + \xi_\kappa \Delta y_n + \frac{\xi_\kappa(\xi_\kappa - 1)}{2} \Delta^2 y_{n-1}.$$

Flight path angle at the cutoff point to conveniently determine through $\operatorname{tg}\theta_\kappa = p_\kappa$:

$$p_\kappa = p_n + \xi_\kappa \Delta p_n + \frac{\xi_\kappa(\xi_\kappa - 1)}{2} \Delta^2 p_{n-1}.$$

1.5. Example of calculation of motion characteristics of center of mass of unguided rocket by difference method of numerical integration.

Initial data of hypothetical rocket let us give in unit system of SI:

$$m_0 = 1174 \text{ кг}^{(1)}; \quad |\dot{m}| = 86{,}6 \text{ кг/с}^{(2)}; \quad d = 196 \text{ мм}; \quad v_0 = 55 \text{ м/с}^{(3)}; \quad \theta_0 = 40°; \quad t_k = 4{,}6 \text{ с};$$

$$\frac{S_a p_{0N}}{m_0} = 16{,}15 \text{ м/с}^2; \quad \frac{P_0}{m_0} = 148 \text{ м/с}^2; \quad i_{43} = 1{,}45.$$

Key: (1) kg. (2) kg/s. (3) m/s.

As usual, let us break trajectory into two section - active and passive. For the calculation of the first section, it is convenient to utilize a system of equations (3.76), for the second - (5.8). The air resistance on the first section let us determine, after using the curve/graph $c_x(M)$, presented in Fig. 6.1; on the second - by the standard function $G(v_\tau)$, comprised in connection with the "law of air resistance 1943". Calculations let us conduct with the aid of four-place logarithmic tables and auxiliary tables for $H_\tau(y)$, $\pi(y)$ etc.

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It is checked the dimensionality of the values, entering system (3.76), taking into account that many auxiliary tables were comprised before the introduction of the system of SI. The system of differential equations takes the following form:

$$\dot{u} = \frac{1}{\mu}(D - E)u; \quad \dot{p} = -\frac{g}{u}; \quad \dot{y} = up; \quad \dot{x} = u.$$

Values μ and $\pi(y)$ - dimensionless and do not depend on unit used

system. The dimensionality of value E 1/s, i.e., is identical for MKGSS system and SI. During the use of a curve/graph $c_x(M)$ value

$$E = \frac{S\varrho_{0N}}{2m_0} v \frac{\varrho}{\varrho_{0N}} c_x(M).$$

Ratio $\frac{\varrho}{\varrho_{0N}}$ and $c_x(M)$ - value dimensionless. To the dimensionality of values

$$S\ \text{м}^2,\ v\ \text{m/s},\ u\ \text{m/s},\ \theta\ \text{rad}.$$

are identical for both systems. Testing shows that the relation $\frac{\varrho_{0N}}{m_0}$ also does not depend on unit system, but during calculation it is necessary to bear in mind, which in the majority of handbooks ϱ_{0N} is given in the system of units MKGSS and then

$$\frac{\varrho_{0N}}{m_0} \left| \frac{\text{кг}\cdot\text{с}^2}{\text{м}^4} \middle/ \frac{\text{кг}\cdot\text{с}^2}{\text{м}} \right| \text{ or } \text{m}^{-3}.$$

Besides initial data, it is necessary to have a series of values, determined on them and, as a rule, into assignment not connected. In our example it is necessary to know $p_0, u_0, \dot{y}_0, a_0, c_x(M_0), X_0, P_0$ - if initial thrust is not directly assigned. It is necessary to determine also D_0, F_0 and $\dot{u}_0$. The named values are conveniently calculated on auxiliary form, after leaving for them column of the precomputations (see form on page 202).

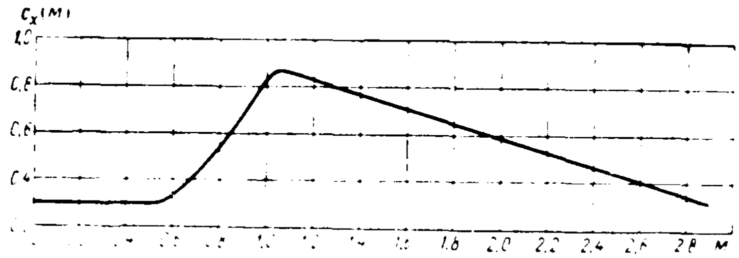

Fig. 6.1. Curve/graph $c_x(M)$ accepted for the assigned hypothetical rocket.

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The indexing of values during precomputations and during calculations by approaches does not coincide with the indexing of the calculated values, which are contained in auxiliary form. Indexing is usually develop/processed in connection with increase of lines by numerical integration, i.e., in connection with basic calculation.

Let us observe the schematic of the calculation of preliminary values. For calculation u_0, is brought in v_0 into auxiliary form's line (20) and $\sqrt{1+p_0^2}$ into line (19). After conducting division (20):(19), let us write in line (11) u_0 and will transfer its value into basic form. Value $\dot{p}_0=-g/u_0$ we determine directly from auxiliary table and is also transferred to basic form. In the diagram of the basic form of the values, transferred from the column of the

precomputations of auxiliary form, emphasized.

For calculation $\dot{y}_0$ value u_0 in line (11) and value μ_0, placed in line (12), let us multiply and transfer result into basic form.

We compute E_0 for the torque/moment of the descent of rocket from guides. Bearing in mind that

$$E_0 = \frac{S\varrho_{0N}}{2m_0}\,\frac{\varrho_{0N}}{\varrho_{0N}}\,c_x(M_0)\,v_0,$$

the calculation is carried out in lines with (20) through (26).

For calculation $\dot{u}_0$, first let us determine D_0 in lines with (27) according to (33). Since

$$\dot{u}_0 = u_0\,\frac{m_0}{m_0}\,(D_0 - E_0),$$

that calculation let us conduct, using lines with (34) on (38). Value $\dot{u}_0$ let us transfer into basic form. Initial conditions and the calculated in the column of precomputations data it is brought in into basic form's zero line. We begin calculation in the first approximation, in auxiliary form's zero column. Column let us fill downward.

We find difference $\Delta u_0 = h_t \dot{u}_0 = 0{,}5\cdot 113{,}4 = 56{,}7$ and is brought in it into basic form. In basic form we find $u_1 = u_0 + \Delta u_0 = 42{,}14 + 56{,}70 = 98{,}84$. On the

auxiliary table g/u=f(u) we obtain $\dot{p}_1$, it is brought in into the basic form $\dot{p}_1$=-0.09920 and we find the difference

$$\Delta\dot{p}_0=\dot{p}_1-\dot{p}_0=-0{,}09920-(-0{,}2328)=0{,}1336.$$

In auxiliary form (line 6, 7, 9 and 10) we compute:

$$\Delta p_0=h_t\left(\dot{p}_0+\frac{1}{2}\Delta\dot{p}_0\right)=0{,}5(-0{,}2328+0{,}6680)=-0{,}0830.$$

it is brought in into basic form and we obtain

$$p_1=p_0+\Delta p_0=0{,}8391-0{,}0830=0{,}7561.$$

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In auxiliary form we compute

$$\dot{y}_1=p_1u_1=74{,}73,$$

it is brought in into basic form and we find

$$\Delta\dot{y}_0=\dot{y}_1-\dot{y}_0=74{,}73-35{,}36=39{,}37.$$

In auxiliary form (line 14-18) we compute

$$\Delta y_0=h_t\left(\dot{y}_0+\frac{1}{2}\Delta\dot{y}_0\right)=0{,}5(35{,}36+19{,}68)=27{,}52,$$

it is transferred to basic form and we obtain

$$y_1 = y_0 + \Delta y_0 = 27{,}52.$$

Utilizing the obtained value y_1, in auxiliary form (line from 20 on 38) let us calculate $\dot{u}_1$=122.4, which let us transfer into basic form and will find the difference

$$\Delta \dot{u}_0 = \dot{u}_1 - \dot{u}_0 = 122{,}4 - 113{,}4 = 9{,}0$$

On this, concludes the calculation in the first approximation.

Calculation in the second approach/approximation we conduct simultaneously for zero and first lines. Calculation we begin with the copying of initial conditions and unknown values (for example, P_e/s_0) into basic and auxiliary forms. Further in auxiliary form we compute

$$\Delta u_0 = h_t\left(\dot{u}_0 + \frac{1}{2}\Delta \dot{u}_0\right) = 0{,}5(113{,}4 + 4{,}5) = 58{,}95;$$

$$\Delta u_1 = h_t\left(\dot{u}_1 + \frac{1}{2}\Delta \dot{u}_0\right) = 0{,}5(122{,}4 + 4{,}5) = 63{,}45.$$

The sizes of the elements of formulas $\dot{u}_1$ and $\Delta\dot{u}_0$ we take from the first approximation. In the final stages of calculation in the second approach/approximation (lines from 20 on 38) we compute $\dot{u}_1$ = 122.2 and $\dot{u}_{22}$ = 129.4, which it is transferred to basic form. In basic form we obtain the differences

$$\Delta \dot{u}_0 = 8{,}8; \quad \Delta \dot{u}_1 = 7{,}0; \quad \Delta^2 \dot{u}_0 = -1{,}8.$$

After this we pass to the calculation of the initial values of functions in the third approach/approximation. Calculation in the third approach/approximation we conduct simultaneously for zero, the first and the second of lines. Calculation we begin with the transfer of initial conditions and constant quantities into basic and auxiliary forms.

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We compute in the auxiliary form

$$\Delta u_0 = h_t\left(\dot{u}_0 + \frac{1}{2}\Delta\dot{u}_0 - \frac{1}{12}\Delta^2\dot{u}_0\right) = 0{,}5(113{,}4 + 4{,}40 + 0{,}15) = 59{,}00;$$

$$\Delta u_1 = h_t\left(\dot{u}_1 + \frac{1}{2}\Delta\dot{u}_1 - \frac{1}{12}\Delta^2\dot{u}_0\right) = 0{,}5(122{,}2 + 3{,}50 + 0{,}15) = 62{,}90;$$

$$\Delta u_2 = h_t\left(\dot{u}_2 + \frac{1}{2}\Delta\dot{u}_1 + \frac{5}{12}\Delta^2\dot{u}_0\right) = 0{,}5(129{,}2 + 3{,}50 - 0{,}75) = 66{,}00$$

The values of cell/elements $\dot{u}_0$, $\dot{u}_1$, $\dot{u}_2$, $\Delta\dot{u}_0$, $\Delta\dot{u}_1$, $\Delta^2\dot{u}_0$ we take from the second approach/approximation. Is transferred to the basic form of difference Δu_0, Δu_1, Δu_2 we compute u_1, u_2, u_3. After this, continuing calculation, let us fill the columns of auxiliary and the lines of forms's basic after numbers r=0, 1, 2. At the end of auxiliary form's columns, we again compute values Δu_0, Δu_1, and Δu_2. This calculation is check.

With the observance of equalities $(\Delta u_0)_I \approx (\Delta u_0)_{II}$; $(\Delta u_1)_I \approx (\Delta u_1)_{II}$; $(\Delta u_2)_I \approx (\Delta u_2)_{II}$ within limits of one-two of units of the last/latter sign of approach/approximation it is possible to consider final. Found three of initial value of function make it possible to carry out further numerical integration according to standard schematic.

The common/general/total schematic of solution during the standard growth of lines consists of following.

The first three equations of system (3.76) are solved together, beginning with the leading equation, in this case it is observed the following sequence of calculation.

1. We determine increase $(\Delta u_n)_I$ in the first approximation, on formula of inclined line. Calculations are conducted in the auxiliary form

$$(\Delta u_n)_I = h_t\left(\dot{u}_n + \frac{1}{2}\Delta\dot{u}_{n-1} + \frac{5}{12}\Delta^2\dot{u}_{n-2} + \frac{3}{8}\Delta^3\dot{u}_{n-3}\right).$$

2. In basic form we find

$$(u_{n+1})_I = u_n + (\Delta u_n)_I.$$

3. Using auxiliary table, we determine

$$\dot{p}_{n+1} = -\frac{g}{u_{n+1}}$$

and we increase table of differences by derivative $\dot{p}$ in basic form.

4. On formula of broken line, we determine in auxiliary form

$$\Delta p_n = h_t\left(\dot{p}_n + \frac{1}{2}\Delta\dot{p}_n - \frac{1}{12}\Delta^2\dot{p}_{n-1}\right).$$

5. In basic form we compute

$$p_{n+1} = p_n + \Delta p_n.$$

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6. Utilizing u_{n+1} and p_{n+1}, we determine

$$v_{n+1}=u_{n+1}\sqrt{1+p_{n+1}^{2}}$$ and

7.

$$\dot{y}_{n+1}=p_{n+1}\cdot u_{n+1},$$

after which we increase the table of differences by the derivative $\dot{y}$.

8. On formula of broken line in auxiliary form, we find

$$\Delta y_n=h_t\left(\dot{y}_n+\frac{1}{2}\Delta\dot{y}_n-\frac{1}{12}\Delta^2\dot{y}_{n-1}\right).$$

9. In basic form

$$y_{n+1}=y_n+\Delta y_n.$$

10. Through y_{n+1} and v_{n+1} we find values D_{n+1} and E_{n+1}, entering leading equation.

11. We find

$$\dot{u}_{n+1}=u_{n+1}\frac{m_0}{m_{n+1}}(D_{n+1}-E_{n+1})$$

and we increase table of differences by derivative $\dot{u}$ in

12. We find (Δu_n)min second approach/approximation through formula

of broken line.

13. We amend in basic form $(u_{n+1})_{I}$ on $(u_{n+1})_{II} = u_n + (\Delta u_n)_{II}$.

This ends the solution of three joint equations within the limits of one integration step. The fourth equation is solved separately, after all values u they will already become known. Values x are located for all calculation points in auxiliary form.

The procedure for finding the initial values of functions by approaches is represented in Table 6.4. Systematic calculation with standard connection of the lines it is convenient to follow in digital material of the basic and auxiliary forms of the example being examined (see Tables 6.5 and 6.6). They also give calculations of the characteristics of movement for the moment of the end of the engine's operation.

Let us calculate the coasting trajectory by using the system of differential equations

$$u'_x = -cH_\tau(y)G(v_\tau); \quad p'_x = -\frac{g}{u^2};$$

$$y'_x = p; \quad t'_x = \frac{1}{u}; \quad v_\tau = u\sqrt{1+p^2}\sqrt{\frac{\tau_{0N}}{\tau}}.$$

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Table 6.4. Calculation procedure during successive approximations for determining the initial four values of functions. Basic form of numerical integration.

(1)	№ п/п	t	m	$\dot{y}=pu$: y	Δy	$\dot{y}$	$\Delta\dot{y}$	$\Delta^2 y$	$\dot{p}=\frac{g}{u}$: p	Δp	$\dot{p}$	$\Delta\dot{p}$	$\Delta^2\dot{p}$	$\dot{x}=u$: x	Δx	$\dot{u}=u\frac{m_0}{m}[D-E]$: u	Δu	$\Delta^2 u$	$\dot{u}$	$\Delta\dot{u}$	$\Delta^2\dot{u}$	$\Delta^3\dot{u}$	v
I приближение	0	t_0	m_0	y_0	Δy_0 (9)	$[\dot{y}_0]$	$\Delta\dot{y}_0$		p_0	Δp_0 (5)	$[\dot{p}_0]$	$\Delta\dot{p}_0$		x_0		u_0	Δu_0 (1)		$[\dot{u}_0]$	$\Delta\dot{u}_0$			v_0
	1	$t_1=t_0+h_t$	m_1	y_1 (10)		$\dot{y}_1$ (8)	(7)		p_1 (6)		$\dot{p}_1$ (4)	(3)				u_1 (2)			$\dot{u}_1$ (12) (13)			(11)	v_1
II приближение	2	t_0	m_0	y_0	Δy_0	$[\dot{y}_0]$	$\Delta\dot{y}_0$	$\Delta^2\dot{y}_0$	p_0	Δp_0	$[\dot{p}_0]$	$\Delta\dot{p}_0$	$\Delta^2\dot{p}_0$	x_0		u_0	Δu_0	$\Delta^2 u_0$	$[\dot{u}_0]$	$\Delta\dot{u}_0$	$\Delta^2\dot{u}_0$		v_0
	3	$t_1=t_0+h_t$	m_1	y_1	Δy_1	$\dot{y}_1$	$\Delta\dot{y}_1$		p_1	Δp_1	$\dot{p}_1$	$\Delta\dot{p}_1$		[illegible]		u_1	Δu_1		$\dot{u}_1$	$\Delta\dot{u}_1$			v_1
	4	$t_2=t_0+2h_t$	m_2	y_2		$\dot{y}_2$			p_2		$\dot{p}_2$			[illegible]		u_2			$\dot{u}_2$				v_2

Notes 1. Initial conditions $t_0=0; m_0; x_0; y_0; \theta_0(p_0); v_0(u_0)$ - on form are encircled by small circles.

2. Terms, computed with use of initial differential and auxiliary equations: $\dot{y}_0$, $\dot{p}_0$; $\dot{u}_0$ - on form are included into brackets.

3. By rifleman/pointers is shown sequence of basic form's filling during accomplishing of approach/approximations

I approach/approximation:

$$(1)\ \Delta u_0=h_t\dot{u}_0;\ (2)\ u_1=u_0+\Delta u_0;\ (3)\ \dot{p}_1=-\frac{g}{u_1}\ (^{(1)}\text{на табл.});\ (4)\ \Delta\dot{p}_0=\dot{p}_1-\dot{p}_0;\ (5)\ \Delta p_0=h_t\left(\dot{p}_0+\frac{1}{2}\Delta\dot{p}_0\right)$$

$$(6)\ p_1=p_0+\Delta p_0;\ (7)\ \dot{y}_1=p_1u_1;\ (8)\ \Delta\dot{y}_0=\dot{y}_1-\dot{y}_0;\ (9)\ \Delta y_0=h_t\left(\dot{y}_0+\frac{1}{2}\Delta\dot{y}_0\right);\ (10)\ y_1=y_0+\Delta y_0;\ (11)\ v_1=u_1\sqrt{1+p_1^2}\ (^{(1)}\text{на табл.})$$

$$(12)\ D_1=\frac{P}{m_1v_1};\ E_1=\frac{X}{m_1v_1};\ \dot{u}_1=\frac{1}{\mu}[D_1-E_1]u_1;\ (13)\ \Delta\dot{u}_0=\dot{u}_1-\dot{u}_0$$

Key: (1). on table.

II Approach/approximation:

$$(14)\ \Delta u_0=h_t\left(\dot u_0+\tfrac{1}{2}\Delta\dot u_0\right);\ (16)\ u_1;\ (18)\ \dot p_1\,(\text{(1)});\ (20)\ \dot p_1-\dot p_0=\Delta\dot p_0;\ (22)\ \Delta^2\dot p_0=\Delta\dot p_1-\Delta\dot p_0;\ (23)\ \Delta p_0;\ (25)\ p_1=p_0+\Delta p_0;$$

$$(15)\ \Delta u_1=h_t\left(\dot u_1+\tfrac{1}{2}\Delta\dot u_0\right);\ (17)\ u_2;\ (19)\ \dot p_2\,(\text{(1)});\ (21)\ \dot p_2-\dot p_1=\Delta\dot p_1;\ (24)\ \Delta p_1;\ (26)\ p_2=p_1+\Delta p_1;$$

$$(27)\ \dot y_1;\ (29)\ \Delta\dot y_0=\dot y_1-\dot y_0;\ (31)\ \Delta^2\dot y_0=\Delta\dot y_1-\Delta\dot y_0;\ (32)\ \Delta y_0;\ (34)\ y_1=y_0+\Delta y_0;\ (36)\ v_1;\ (38)\ \dot u_1;\ (40)\ \Delta\dot u_0=\dot u_1-\dot u_0;\ (42)\ \Delta^2\dot u_0=\Delta\dot u_1-\Delta\dot u_0$$

$$(28)\ \dot y_2;\ (30)\ \Delta\dot y_1=\dot y_2-\dot y_1;\ (33)\ \Delta y_1;\ (35)\ y_2=y_1+\Delta y_1;\ (37)\ v_2;\ (39)\ \dot u_2;\ (41)\ \Delta\dot u_1=\dot u_2-\dot u_1;$$

Key: (1). on table.

III Approach/approximation: sequence of filling basic form is similar preceding/previous. Difference is in the fact that is conducted the calculation simultaneously 3-x of lines.

With the normal running of solution (strictly count) are more precisely formulated the first differences, and consequently also very value of function, only for the leading equation.

Key: (1). approach/approximation.

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With independent alternating/variable t the interpolating factor is determined from the formula

$$t_k=\frac{t_k-t_n}{h_t}=\frac{4.6-4.5}{0.5}=0.2,$$

where $t_k=4.6$ s - time of the end/lead of the engine operation:

$h_t=0.5$ s - space of integration for time;

t_n- the calculated moment of time, which precedes t_n.

Abscissa of the end/lead of the active section:

$$x_к = x_n + \xi_к\Delta x_n + \frac{\xi_к(\xi_к-1)}{2}\Delta^2 x_{n-1} = 1564 + 0,2\cdot 375,4 + \frac{0,2(0,2-1)}{2}(375,4-329,1) = 1635 \text{ м}$$

Ordinate of the end/lead of the active section:

$$y_к = y_n + \xi_к\Delta y_n + \frac{\xi_к(\xi_к-1)}{2}\Delta^2 y_{n-1} = 1038 + 0,2\cdot 234,8 + \frac{0,2(0,2-1)}{2}\cdot(234,8-203,4) = 1083 \text{ м}$$

Velocity at the end of the active section: $v_к = v_n + \xi_к\Delta v_n + \frac{\xi_к(\xi_к-1)}{2}\Delta^2 v_{n-1}$

$$= 831,4 + 0,2(940,8-831,4) + \frac{0,2(0,2-1)}{2}[(940,8-831,4)-(831,4-727,6)] = 852,8 \overset{(1)}{\text{м/с}}$$

Key: (1). m/s.

Flight path angle at the end of the active section $p_к = p_n + \xi_к\Delta p_n$

$$+\frac{\xi_к(\xi_к-1)}{2}\cdot\Delta^2 p_{n-1} = 0,6294 + 0,2(-0,006925) + \frac{0,2(0,2-1)}{2}[(-0,006925-(-0,007460)] = 0,6280, \overset{(1)}{\text{или}}\ \theta_к = 32°08'$$

Key: (1). or.

Table 6.5. Example of trajectory calculation (active section). Basic form of numerical integration.

№ строки (1)	t	m	$\dot{y}=pu$					$\dot{p}=-\frac{g}{u}$			
			y	Δy	$\dot{y}$	$\Delta\dot{y}$	$\Delta^2\dot{y}$	p	Δp	$\dot{p}$	$\Delta\dot{p}$
0	0	1174	0	27,52	35,36	39,37		0,8391	−0,0830	−0,2328	0,1336
1	0,5	1231	27,52		74,73			0,7561		−0,09920	
0	0	1174	0	28,03	35,36	41,55	0,94	0,8391	−0,07840	−0,2328	0,1357
1	0,5	1131	28,03	49,04	76,91	42,49		0,7607	−0,03510	−0,09712	0,03748
2	1,0	1088	77,07		119,4			0,7256		−0,05964	
0	0	1174	0	28,05	35,36	41,55	0,54	0,8391	−0,07840	−0,2328	0,1357
1	0,5	1131	28,05	48,95	76,91	42,09	0,11	0,7607	−0,03514	−0,09712	0,03730
2	1,0	1088	77,00	70,05	119,0	42,20	0,8	0,7256	−0,02478	−0,05982	0,01717
3	1,5	1044	147,0	91,40	161,2	43,00	1,8	0,7008	−0,01855	−0,04265	0,00988
4	2,0	1001	238,4	113,2	204,2	44,80	0,3	0,6822	−0,01462	−0,03277	0,00647
5	2,5	957,9	351,6	135,8	249,0	45,10	2,3	0,6676	−0,01196	−0,02630	0,00443
6	3,0	914,6	487,4	158,8	294,1	47,40	2,2	0,6556	−0,01006	−0,02187	0,00333
7	3,5	871,3	646,2	183,0	341,5	49,60	2,0	0,6455	−0,00860	−0,01854	0,00256
8	4,0	828,0	829,2	203,4	391,1	51,60	2,8	0,6369	−0,00746	−0,01598	0,00204
9	4,5	784,7	1038	234,8	442,7	54,40		0,6294	−0,006925	−0,01394	0,00166
10	5,0	741,4	1273		497,1			0,6225		−0,01228	

Note. Performance calculation at the end of the engine operation.

Key: (1). line.

Table 6.5 (cont)

	$\dot{x}=u$		$\dot{u}=u\,\frac{m_0}{m}\,[D-E]$							v
$\Delta^2\dot{p}$	x	Δx	u	Δu	$\Delta^2 u$	$\dot{u}$	$\Delta\dot{u}$	$\Delta^2\dot{u}$	$\Delta^3\dot{u}$	
			42,14	56,70		113,4	9,0			55
			98,84			122,4				123,9
−0,09822			42,14	58,95	4,50	113,4	8,8	−1,8		55
			101,1	63,45		122,2	7,0			127,0
			164,5			129,2				203,2
−0,09840	0	35,66	42,14	59,00	3,90	113,4	8,8	−1,8	0,9	55
−0,02013	35,66	66,15	101,1	62,90	3,25	122,2	7,0	−0,9	1,7	127,0
−0,00729	101,8	98,40	164,0	66,15	3,20	129,2	6,1	0,8	−0,7	202,5
				(66,00)						
−0,00341	200,2	132,3	230,2	69,35	3,50	135,3	6,9	0,1	0,3	282,0
			(230,0)	(69,15)						
−0,00204	332,5	167,8	299,6	72,85	3,60	142,2	7,0	0,4	0,5	362,3
			(299,4)	(73,35)						
−0,00110	500,3	205,2	272,4	76,45	3,90	149,2	7,4	0,9	0	448,3
			(273,0)	(76,25)						
−0,00077	705,5	244,3	448,8	80,35	4,35	156,6	8,3	0,9	0,3	536,1
			(448,6)	(80,30)						
−0,00052	949,8	258,6	529,2	84,70	4,90	164,9	9,2	1,2	1,0	629,6
			(529,1)	(84,80)						
−0,00038	1235	329,1	613,9	89,60	5,70	174,1	10,4	2,2		727,6
			(614,0)	(89,55)						
	1564	375,4	703,5	95,30		184,5	12,6			831,4
			(703,4)	(95,15)						
	1940		798,8			197,1				940,8
			(798,6)							

Note. In brackets are given the results of calculation, obtained in the first approximation.

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Table 6.6. Example of trajectory calculation (active section). Auxiliary form of numerical integration.

(1) № строки	(2) Расчетная величина	(3) Предварительный расчет	$n=0$	$n=0$	$n=1$	$n=0$	$n=1$
0	$\dot{u}_n$		113,4	113,4	122,4	113,4	122,2
1	$\frac{1}{2}\Delta\dot{u}_{n-1}$			4,500	4,500	4,400	3,500
2	$\frac{5}{12}\Delta^2\dot{u}_{n-2}$					0,1500	0,1500
3	$\frac{3}{8}\Delta^3\dot{u}_{n-3}$						
4	$\sum_n$		113,4	117,9	126,9	118,0	125,8
5	$(\Delta u_n)_t = h_t \sum_n{}^*$		56,70	58,95	63,45	59,00	62,90
6	$\dot{\rho}_n$		−0,2328	−0,2328	−0,09712	−0,2328	−0,09712
7	$\frac{1}{2}\Delta\dot{\rho}_n$		0,05680	$0,6785\cdot10^{-1}$	0,01874	0,06785	0,01865
8	$-\frac{1}{12}\Delta^2\dot{\rho}_{n-1}$			$0,8185\cdot10^{-2}$	$0,8185\cdot10^{-2}$	$0,8200\cdot10^{-2}$	$0,8200\cdot10^{-2}$
9	$\sum_n$		−0,1660	−0,1568	−0,07020	−0,1568	−0,07027
10	$\Delta\rho_n = h_t \sum_n{}^*$		−0,0830	−0,07840	−0,03510	−0,07840	−0,03514
11	u_{n+1}	42,14	98,81	101,1	164,5	101,1	164,0
12	ρ_{n+1}	0,8391	0,7561	0,7607	0,7256	0,7607	0,7256
13	$\dot{y}_{n+1} = (11)\cdot(12)^*$	35,36	74,73	76,91	119,4	76,91	119,0
14	$\dot{y}_n$		35,36	35,36	76,91	35,36	76,91
15	$\frac{1}{2}\Delta\dot{y}_n$		19,68	20,78	21,24	20,78	21,04
16	$-\frac{1}{12}\Delta^2\dot{y}_{n-1}$			−0,07833	−0,07833	−0,04500	−0,04500
17	$\sum_n$		55,04	56,03	98,07	56,10	97,90
18	$\Delta y_n = h_t \cdot \sum_n{}^*$		27,52	28,03	49,04	28,05	48,95
19	$\sqrt{1+\rho_{n+1}^2}$	1,305	1,254	1,255	1,235	1,256	1,235
20	$v_{n+1} = (11)\cdot(19)^*$	55,00	123,9	127,0	203,2	127,0	202,5
21	$\frac{\varrho_{n+1}}{\varrho_{0N}} = f(y_{n+1})$	1,0000	0,9971	0,9970	0,9918	0,9970	0,9918
22	$a_{n+1} = f(y_{n+1})$	341,5	341,4	341,4	341,2	341,4	341,2
23	$M_{n+1} = \frac{(20)}{(22)}$	0,1610	0,3629	0,3720	0,5955	0,3720	0,5934
24	$c_x(M_{n+1})$	0,30	0,30	0,30	0,32	0,30	0,31

$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$	$n=8$	$n=9$
129,2	135,3	142,2	149,2	155,6	164,9	174,1	184,5
3,500	3,050	3,450	3,500	3,700	4,150	4,600	5,200
−0,7500	−0,3750	0,6667	0,01167	0,1667	0,3750	0,3750	0,5000
	0,3375	0,4250	−0,2025	0,1125	0,1875	0	0,1125
132,0	138,3	146,[illegible]	152,5	160,6	169,[illegible]	179,1	190,3
66,00	69,15	73,35	76,25	80,30	84,60	89,55	95,15
−0,05[illegible]82	−0,04265	−0,03277	−0,02630	−0,02187	−0,01854	−0,01598	−0,01394
$0,8585\cdot10^{-2}$	$0,4945\cdot10^{-2}$	$0,3235\cdot10^{-2}$	$0,2215\cdot10^{-2}$	$0,1665\cdot10^{-2}$	$0,1280\cdot10^{-2}$	$0,1020\cdot10^{-2}$	0,[illegible]$\cdot10^{-3}$
$0,1678\cdot10^{-2}$	$0,6075\cdot10^{-3}$	$0,2842\cdot10^{-3}$	$0,1700\cdot10^{-3}$	$0,9166\cdot10^{-4}$	$0,6417\cdot10^{-4}$	$0,4335\cdot10^{-4}$	0,[illegible]167$\cdot10^{-4}$
−0,01955	−0,03710	−0,02925	−0,02392	−0,02011	−0,01720	−0,01492	−0,01385
−0,02478	−0,01855	−0,1462	−0,01196	−0,01005	$-0,8600\cdot10^{-2}$	$0,7460\cdot10^{-2}$	$-0,6925\cdot10^{-2}$
230,0	299,4	373,0	448,60	529,1	614,0	703,4	798,6
0,7008	0,6822	0,6676	0,6555	0,6455	0,6369	0,6294	0,6225
161,2	204,2	249,0	294,1	341,5	391,1	442,7	497,1
119,0	161,2	204,2	249,0	294,1	341,5	391,1	442,7
21,10	21,50	22,40	22,55*	23,70	24,80	25,80	27,20
$-0,9167\cdot10^{-2}$	−0,03667	−0,1500	−0,02500	−0,1917	−0,1830	−0,1667	−0,2333
140,1	182,6	225,4	271,5	317,6	366,1	416,7	469,7
70,05	91,40	113,2	135,80	155,8	183,0	208,4	234,8
1,226	1,210	1,202	1,195	1,190	1,185	1,182	1,178
282,0	362,3	448,3	53[illegible],1	629,6	727,6	831,4	940,8
0,9843	0,9747	0,9628	0,9488	0,9326	0,9141	0,8935	0,8707
341,0	340,6	340,2	339,5	339,1	338,4	337,6	336,7
0,8270	1,064	1,318	1,579	1,857	2,150	2,463	2,794
0,58	0,86	0,79	0,71	0,62	0,53	0,43	0,30

Note. The values, marked in auxiliary form by symbol *, are transferred to basic form.

Key: (1). line. (2). Calculated value. (3). Precomputation.

1 № строки	2 Расчетная величина	3 Предварительный расчет	$n=0$	$n=0$	$n=1$	$n=0$	$n=1$
25	$\frac{S v_{0N}}{2m_0}$	$0{,}1576 \cdot 10^{-4}$	$0{,}1576 \cdot 10^{-4}$	$0{,}1576 \cdot 10^{-4}$	$0.1576 \cdot 10^{-4}$	$0.1576 \cdot 10^{-4}$	$0.1576 \cdot 10^{-4}$
26	$E = (20) \cdot (21) \cdot (24) \cdot (25)$	$0.2600 \cdot 10^{-3}$	$0.5841 \cdot 10^{-3}$	$0{,}5986 \cdot 10^{-3}$	$0{,}1016 \cdot 10^{-2}$	$0.5986 \cdot 10^{-3}$	$0{,}9822 \cdot 10^{-3}$
27	$\pi(y_{n+1})$	1.0000	0.9968	0,9967	0.9910	0,9967	0,9910
28	$1-\pi(y_{n+1})$	0.0000	0.0032	0.0033	0,0090	0,0033	0,0090
29	$\frac{S_a p_{0N}}{m_0}$	16.15	16,15	16,15	16,15	16.15	16,15
30	$\frac{S_a p_{0N}}{m_0}[1-\pi(y_{n+1})] = (28) \cdot (29)$	0,0000	0.05168	0,05330	0.1454	0.05330	0.1454
31	$P_0 . m_0$	148,0	148,0	148.0	148.0	148.0	148.0
32	$(30)+(31)$	148.0	148,0	148.0	148.1	148.0	148.1
33	$D_{n+1} = \frac{(32)}{(20)}$	2.691	1.195	1.165	0,7288	1.165	0,7314
34	$(D-E)_{n+1} = (33)-(26)$	2.691	1.194	1,164	0,7278	1.164	0.7304
35	u_{n+1}	42.14	98,84	101,1	164.5	101.1	164,0
36	m_0	1174	1174	1174	1174	1174	1174
37	$m_{n+1} = m_0 - \lvert \dot{m} \rvert \, t$	1174	1131	1131	1088	1131	1088
38	$\dot{u}_{n+1} = \frac{(34)\,(35)\,(36)^*}{(37)}$	113.4	122,4	122,2	129,2	122.2	129,2
39	$\dot{u}_n$						
40	$\frac{1}{2} \Delta \dot{u}_n$						
41	$-\frac{1}{12} \Delta^2 \dot{u}_{n-1}$						
42	$\sum_n$						
43	$(\Delta u_n)_{11} = h_t \sum_n^*$						
44	u_n					42,14	101,1
45	$\frac{1}{2} \Delta u_n$					29.50	31,45
46	$-\frac{1}{12} \Delta^2 u_{n-1}$					−0.3250	−0,2708
47	$\sum_n$					71,82	132,3
48	$\Delta r_n = h_t \sum_n^*$					35,66	66,15

$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$	$n=8$	$n=9$
$0.1576\cdot10^{-4}$	$0.1576\cdot10^{-4}$	$0.1576\cdot10^{-4}$	$0.1576\cdot10^{-4}$	$0,1576\cdot10^{-4}$	$0.1576\cdot10^{-4}$	$0.1576\cdot10^{-4}$	$0.1576\cdot10^{-4}$
$0.2537\cdot10^{-2}$	$0,4786\cdot10^{-2}$	$0,5374\cdot10^{-2}$	$0,5692\cdot10^{-2}$	$0.5737\cdot10^{-2}$	$0,5555\cdot10^{-2}$	$0,5034\cdot10^{-2}$	$0,3875\cdot10^{-2}$
0.9829	0.9723	0.9593	0.9438	0.9261	0.9062	0,8834	0,8583
0.0171	0,0277	0.0407	0.0562	0.0739	0,0938	0,1166	0.1417
16,15	16.15	16,15	16,15	16,15	16,15	16,15	16.15
0,2762	0,4474	0.6573	0.9076	1,193	1.515	1,883	2,288
148,0	148,0	148,0	148,0	148,0	148.0	148,0	148,0
148,3	148,4	148,7	148,9	149.2	149.5	149,9	150,3
0,5259	0,4096	0.3317	0.2777	0,2370	0,2055	0,1803	0.1598
0,5234	0,4048	0,3263	0,2720	0,2313	0.2000	0,1753	0,1559
230,0	299,4	373,0	448.6	529,1	614.0	703,4	798,6
1174	1174	1174	1174	1174	1174	1174	1174
1044	1001	957.9	914,6	871,3	828.0	784.7	741,4
135,3	142,2	149.2	156,6	164.9	174.1	184.5	197.1
129,2	135,3	142,2	149.2	156,6	164.9	174.1	184.5
3,050	3,450	3,500	3,700	4,150	4,600	5,200	6,300
0,07500	−0,06667	−0,008333	−0,03333	−0.07500	−0.07500	−0.1000	−0,1833
132,3	138,7	145.7	152,9	160,7	169.4	179,2	190,6
66,15	69,35	72,85	76,45	80,35	84.70	89,60	95.30
164.0	230,2	299,6	372,4	448,8	529.2	613,9	703,5
33,08	34,68	36,42	38,22	40,18	42,35	44,80	47,65
−0,2667	−0,2917	−0,3000	−0,3250	−0,3625	−0,4083	−0,4750	−0,4750
196,8	264,6	335,7	410,3	488,6	571,1	658,2	750.7
98,40	132,3	167,8	205,2	244,3	285,6	329,1	375.4

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Table 6.7. Example of trajectory calculation (passive section). Basic form of numerical integration.

(i) № строки	x	$y'_x = p$					$p'_x = -\frac{g}{u^2}$			
		y	Δy	p	Δp	$\Delta^2 p$	$p'_x \cdot 10^5$	$\Delta p'_x \cdot 10^5$	$\Delta^2 p'_x \cdot 10^5$	u
0	1635	1083	311.5	0.6280	−0.01000		−1,882	−0.235		722,1
1	2135	1394		0.6180			−2,117			680,7
0	1635	1083	311.6	0.6280	−0.009965	−0.001195	−1,882	−0.226	−0,025	722.1
1	2135	1395	306.2	0.6180	−0.01116		−2.108	−0.251		682.1
2	2635	1701		0.6068			−2,359			644,9
0	1635	1083	311.6	0,6280	−0.009965	−0,001195	−1.882	−0.226	−0,028	722,1
1	2135	1395	306.2	0.6180	−0.01116	−0.001330	−2.108	−0.254	−0.021	682,1
2	2635	1701	300.3	0.6068	−0.01249	−0.001460	−2,362	−0.275	−0.025	644.7
3	3135	2001	293.7	0,5943	−0 01395	−0.001620	−2,637	−0.310	−0,028	609.8 (609.9)
4	3635	2295	286.0	0.5804	−0,01557	−0,001780	−2.947	−0.338	−0,039	577,1 (577,0)
5	4135	2581	278,2	0.5648	−0.01735	−0,001990	−3,285	−0,377	−0,041	546,4 (546,5)
6	4635	2859	269,0	0 5174	−0.01934	−0.002190	−3,662	−0.418	−0,042	517,6 (517,6)
7	5135	3128	253.8	0.5281	−0.02153	−0,002410	−4,080	−0.460	−0.040	490,5 (490,5)
8	5635	3387	247.4	0.5066	−0.02394	−0,002540	−4,540	−0.500	−0,060	465,0 (465,0)
9	6135	3634	234,8	0.4827	−0,02858	−0.00290	−5,010	−0.550	−0,040	441,1 (441,1)
10	6635	3869	220.8	0.4561	−0.02948	−0,00316	−5,600	−0,600	−0.070	418,6 (418,7)
11	7135	4090	205.2	0.4266	−0.03264	−0,00349	−6,200	−0,670	−0,050	397,6 (397,5)
12	7635	4295	188.1	0.3940	−0.03613	−0,00371	−6,870	−0.720	−0,040	377,9 (378,0)
13	8135	4483	169.2	0.3579	−0,03984	−0,00390	−7,590	−0.700	−0.040	359,6 (359,6)
14	8635	4652	148.4	0,3181	−0,04374	−0,00397	−8,350	−0,800	0,020	342.8 (342,8)
15	9135	4800	125,4	0,2744	−0,04771	−0,00369	−9.150	−0.780	0,090	327,6 (327,5)
16	9635	4925	100,6	0,2267	−0,05140	−0,00330	−9.930	−0.690	0,070	314,4 (314,4)
17	10135	5025		0.1753			−10,62			303,7 (304,0)
14	8635	4652	77,00	0.3181	−0.02140	−0.00090	−8,350	−0,399	−0,002	342,8
15	8885	4729	71.00	0,2957	−0,02230	−0,00105	−8,749	−0,401	0,011	335,0
16	9135	4800	66,00	0,2744	−0,02335	−0,00098	−9,150	−0,390	0,006	327,6
17	9385	4866	59,00	0.2510	−0,02434	−0,00094	−9,510	−0,390	0.030	320,7
18	9535	4925	53,52	0,2267	−0,02528	−0,00090	−9,930	−0,360	0,010	314,4
19	9885	4980	47,10	0,2014	−0,02618	−0,00095	−10,29	−0,35	0,04	308,7 (308,7)
20	10136	5027	40,45	0,1752	−0,02700	−0,00072	−10,64	−0,31	0,04	303,7 (303,7)

DOC = [redacted] PAGE ~~21~~ 422

	$u'_\tau = -cH_\tau(y)G(v_\tau)$				$t'_x = \frac{1}{u}$					v
Δu	$u'_\tau \cdot 10^3$	$\Delta u'_\tau \cdot 10^3$	$\Delta^2 u'_\tau \cdot 10^3$	$\Delta^3 u'_\tau \cdot 10^3$	t	Δt	$t'_x \cdot 10^3$	$\Delta t'_x \cdot 10^3$	$\Delta^2 t'_x \cdot 10^3$	
−41,38	−82,77	5,52								852,8
	−77,25									800,5
−40,00	−82,77	5,46	−0,29							852,8
−37,24	−77,31	5,17								802,1
	−72,14									753,9
−40,01	−82,77	5,46	−0,29	−0,28	4,600	0,7125	1,385	0,081	0,004	852,8
−37,35	−77,31	5,17	−0,57	0,22	5,312	0,7540	1,466	0,085	0,004	802,1
−34,90 (−34,84)	−72,14	4,60	−0,35	−0,01	6,066	0,7975	1,551	0,089	0,004	753,6
−32,67 (−32,79)	−67,54	4,25	−0,36	0,05	6,864	0,8430	1,640	0,093	0,004	709,3
−30,66 (−30,59)	−63,29	3,89	−0,31	0,05	7,707	0,8905	1,733	0,097	0,005	667,0
−28,79 (−28,80)	−59,40	3,58	−0,26	0,05	8,598	0,9405	1,830	0,102	0,005	627,9
−27,07 (−27,07)	−55,82	3,32	−0,20	0,12	9,538	0,9925	1,932	0,107	0,005	590,1
−25,46 (−25,46)	−52,50	3,12	−0,08	−0,09	10,53	1,048	2,039	0,112	0,004	554,8
−23,92 (−23,94)	−49,38	3,04	−0,17	0,10	11,58	1,104	2,151	0,116	0,005	521,3
−22,45 (−22,40)	−46,34	2,87	−0,07	−0,03	12,68	1,164	2,267	0,122	0,004	489,6
−21,03 (−21,07)	−43,47	2,80	−0,10	0,26	13,85	1,226	2,389	0,126	0,005	460,2
−19,66 (−19,63)	−40,67	2,70	0,16	−0,03	15,07	1,290	2,515	0,132	0,002	432,1
−18,28 (−18,34)	−37,97	2,86	0,13	0,42	16,36	1,357	2,647	0,134	0,004	405,7
−16,81 (−16,76)	−35,11	2,99	0,55	0,60	17,72	1,424	2,781	0,136	0,000	381,9
−15,20 (−15,29)	−32,12	3,54	1,15	−0,88	19,14	1,492	2,917	0,136	−0,008	359,6
−13,16 (−13,21)	−28,58	4,69	0,27	−0,11	20,64	1,558	3,053	0,128		339,6
−10,72 (−10,42)	−23,89	4,96	0,16	−3,52	22,19		3,181			322,3
	−18,93									305,6
−7,800	−32,12	1,66	0,22	0,09						
−7,400	−30,46	1,88	0,31	0,00						
−6,900	−28,58	2,19	0,31	−0,16						
−6,300	−26,39	2,50	0,15	−0,23						
−5,655 (−5,637)	−23,89	2,65	−0,08	0,31	22,19	0,8025	3,181	0,058	−0,005	322,3
−5,022 (−5,012)	−21,38	2,87	0,23	−0,98	22,99	0,8165	3,239	0,053	−0,004	314,9
−4,359 (−4,410)	−18,81	2,80	−0,75	0,11	23,81	0,8292	3,292	0,049	−0,007	308,3

№ строки	x	$y'_x = p$					$p'_x = -\frac{g}{u^2}$			
		y	Δy	p	Δp	$\Delta^2 p$	$p'_x \cdot 10^5$	$\Delta p'_x \cdot 10^6$	$\Delta^2 p'_x \cdot 10^6$	u
21	10385	5057	33,40	0.1452	−0.02772	−0.00068	−10.95	−0.27	0,00	299.3 (299,3)
22	10635	5101	26,60	0,1205	−0.02840	−0.00062	−11,22	−0.27	0,04	295.6 (295,7)
23	10885	5128	19.41	0,09210	−0,02902	−0.00058	−11.49	−0,23	−0.02	292.3 (292.2)
24	11135	5147	12,08	0,06308	−0.02960	−0,00060	−11,72	−0,25	0.03	289.3 (289.3)
25	11385	5159	4,608	0.03348	−0.03020	−0.00058	−11.97	−0,22	−0,03	286,4 (286.3)
26	11635	5164	−3.015	0,00280	−0.03078	−0.00062	−12.19	−0,25	0.02	283.6 (283.7)
27	11885	5161	−10.79	−0,02750	−0.03140	−0.00058	−12.44	−0.23	−0,02	280.9 (280,8)
28	12135	5150	−18.71	−0.05890	−0.03198	−0.00062	−12,67	−0.25	0,01	278,2 (278,3)
29	12385	5131	−26.78	−0.09088	−0,03260	−0.00062	−12,92	−0.24	−0,02	275.6 (275.5)
30	12635	5104	−35,00	−0.1235	−0.03322	−0.00063	−13.16	−0.26	0.01	273.0 (273.0)
31	12885	5069	−43.40	−0,1557	−0.03385	−0,00065	−13.42	−0.25	−0.02	270.4 (270,4)
32	13135	5026	−51.95	−0,19[illegible]	−0,03450		−13.67	−0.27		267,9 (267.9)
33	13385	4974		−0.2251			−13,94			265,4 (265,3)
27	11885	5161	−30	−0.02750	−0.06338	−0.00244	−12.44	−0.48	−0,02	280.9
28	12385	5131	−62	−0.09038	−0,06582	−0,00258	−12,92	−0.50	−0,02	275.6
29	12885	5069	−95	−0.1557	−0,06840	−0,00265	−13,42	−0,52	−0.03	270.4
30	13385	4974	−130.2	−0.2251	−0,07105	−0,00290	−13,94	−0,55	−0.05	265.3
31	13885	4844	−166.4	−0.2962	−0.07395	−0,00310	−14,49	−0,60	−0,04	260.2 (260,2)
32	14385	4678	−204.2	−0.3702	−0,07705	−0,00335	−15,09	−0.64	−0,08	255.0 (255,0)
33	14885	4474	−243,6	−0,4172	−0,08040	−0.00385	−15,73	−0.72	−0,09	249.7 (249,7)
34	15385	4230	−284,7	−0.5276	−0.08425	−0,00390	−16,45	−0.81	−0,11	244,2 (244,2)
35	15885	3975	−327,8	−0,6118	−0,09815	−0,00540	−17,25	−0,92	−0,16	238,4 (238,4)
36	16385	3607	−373,2	−0,7000	−0,09355	−0,00585	−18,18	−1,03	−0,19	232,3 (232,3)
37	16885	3234	−421.4	−0,7996	−0,09940	−0.00700	−19,26	−1,27	−0,28	225.7 (225,7)
38	17385	2813	−472,8	−0,8930	−0,1034	−0.0086	−20,53	−1.55	−0,36	218,6 (218,6)
39	17885	2340	−528,0	−0,9994	−0,1150	−0.0107	−22,08	−1,91	−0,46	210,8 (210,8)
40	18385	1812	−588,0	−1.114	−0,1257	−0,0137	−23,99	−2,37	−0,79	202.2 (202.2)
41	18885	1224	−649,0	−1,240	−0,1394	−0.0178	−26,36	−3,16	−0,79	192,8 (192,9)
42	19385	575	−728,0	−1.379	−0,1572		−29,52	−3,96		182,4 (182,3)
43	19885	−153		−1.536			−33,47			171,2

	$u'_x = -cH_\tau(\mu)G(v_\varphi)$				$t'_x = \frac{1}{u}$					v
Δu	$u'_x \cdot 10^3$	$\Delta u'_x \cdot 10^3$	$\Delta^2 u'_x \cdot 10^3$	$\Delta^3 u'_x \cdot 10^3$	t	Δt	$t'_x \cdot 10^3$	$\Delta t'_x \cdot 10^3$	$\Delta t'_x \cdot 10^3$	
−3.730 (−3.400)	−16,01	2,05	−0,64	−0,18	21,64	0.8405	3.341	0,042	−0.004	302.3
−3,300 (−3.405)	−13.96	1.41	−0.82	0.75	25.48	0.8505	3.383	0,038	−0,002	297,8
−3,048 (−3,018)	−12,55	0.59	−0.07	−0.11	26.33	0,8598	3.421	0.036	−0.001	293.4
−2.922 (−3.018)	−11.9[illegible]	0.52	−0.18	0.17	27.19	0,8688	3.457	0.035	−0.001	289.9
−2.815 (−2.775)	−11.44	0,34	−0.01	−0.14	28,05	0,8772	3,492	0.034	0.000	286.3
−2.732 (−2.762)	−11,10	0.33	−0,15	0.18	28,94	0,8858	3.526	0.034	0.000	283.7
−2,668 (−2.635)	−10,77	0.18	0.03	−0.11	29.82	0,8942	3.5[illegible]0	0.0[illegible]4	0.000	280.8
−2,622 (−2.655)	−10,59	0.21	−0.11	0.13	30,72	0.9028	3,594	0,034	0,001	278.9
−2.580 (−2.550)	−10.38	0.10	−0.02	−0.05	31,62	0,9112	3,628	0,035	0,000	276.6
−2.5[illegible]0 (−2,582)	−10.28	0,08	−0.07	0,08	32,53	0,9200	3,663	0.035	0 000	274.9
−2.548 (−2.530)	−10,20	0,01	0,01		33,45	0,9288	3,698	0,035	0,002	27[illegible].6
−2.545 (−2.558)	−10.19	0,02			34,[illegible]8	0,9378	3,733	0,037		272.7
	−10,17				35,32		3.770			271.9
−5.3	−10.77	0,39	−0,21	0,06						
−5.2	−10.38	0.18	−0.15	0.01						
−5.1	−10,20	0.04	−0.14	0,03						
−5,105 (−5.100)	−10,17	−0,11	−0.0[illegible]	−0.12	35,32	1.903	3,770	0.073	0.00[illegible]	271.9
−5.180 (−5.195)	−10.28	−0.17	−0.18	0.05	37.22	1.941	3,843	0.079	0,004	271.4
−5.305 (−5.2[illegible]5)	−10.45	−0,35	−0.12	0.00	39,16	1.982	3.922	0.083	0.007	271.8
−5.510 (−5.545)	−10,60	−0,47	−0.12	−0,07	41.14	2.024	4.005	0,090	0,010	273,4
−5.780 (−5.765)	−11.27	−0.59	−0.19	0.02	43.17	2.072	4,095	0,100	0.010	276.2
−6.115 (−6.100)	−11.83	−0,78	−0,17	−0.12	45.24	2.124	4.195	0.110	0,016	279.4
−6.550 (−6.570)	−12.64	−0,95	−0,29	0.02	47.36	2,184	4.305	0.126	0,017	283.6
−7.095 (−7.065)	−13,59	−1,24	−0.27	0,23	49.55	2.250	4.431	0.143	0,027	288.0
−7.780 (−7,810)	−14,83	−1,51	−0,04	−0,30	51.80	2,328	4.574	0,170	0.031	293.1
−8.550 (−8.600)	−16,34	−1.55	−0,34	0,35	54.12	2,420	4,744	0.201	0.047	298.3
−9.405 (−9.500)	−17,89	−1.89	0.01		56,54	2.543	4,945	0,248	0,042	302.7
−10.36 (−10.49)	−19.78	−1.88			59.08	2,666	5.193	0.290	0.068	307,3
−11.23	−21,66				61,74	2,828	5.483	0,358		310.5
					64.57		5.841			313.8

Note. In brackets are given the results of calculation, obtained in

the first approximation.

Key: (1). line.

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The here leading equation is the first. Initial conditions for a passive section correspond to the finite values of the cell/elements of trajectory, determined in the preceding/previous calculation.

$$x_к = x_0 = 1635 \text{ м}; \quad \theta_к = \theta_0 = 32°08';$$
$$y_к = y_0 = 1083 \text{ м}; \quad v_к = v_0 = 852{,}8 \overset{(1)}{\text{м/с}};$$
$$\mathrm{tg}\,\theta_к = \mathrm{tg}\,\theta_0 = p_к = 0{,}6280.$$

Key: (1). m/s.

Furthermore, on passive section the weight of projectile $Q_к = 686{,}46$ n, factor of form i_{43}=1.45 and the greatest diameter of nose cone d = 196 mm. It is obvious that the dimensionality of the values, entering the last/latter three equations, are identical for a technical system and for the system of SI. The dimensionality of the first equation is also identical for both systems u_r s⁻¹. Ballistic coefficient c the function of air resistance $G(v_r)$ have dimensionality in the system of SI

$$c \overset{(1)}{\text{м}^2/\text{н}} \overset{(1a)}{\text{и}} G(v_r) \overset{(2)}{\text{н/м}^2 \text{с}}, a$$

Key: (1). m²/N (1a). And. (2). N/m² s.

In the technical system c m²/kg and $G(v_\tau)$ kg/m² s. In last/latter dimensionality are comprised the corresponding tables for $G(v_\tau)$ weight of rocket on the passive section of problems Q_{KH} and

$$c=\frac{id^2}{Q_K}\,10^3\ \overset{(1)}{\text{м}^2/\text{н}}.$$

Key: (1). m²/N.

During the use of the existing tables $G(v_\tau)$ it is necessary to take into the calculation

$$G(v_\tau)_{CH}=9{,}8066\,G(v_\tau)_{\text{теб}}.$$

Since $v_\tau=v\,\frac{a_{0N}}{a}$, that dimensionality aaaa coincides for a technical system and the system of SI. If during calculation the ballistic coefficient is calculated in system of practical units, then value $G(v_\tau)$ is taken directly from table. The space of integration we take $h_x=500$ m. The calculation of the original values of functions with approach/approximations and the order of the growth of lines are visible from basic design sheet (table 6.7).

Beginning from 14 lines (with v = 359.6 m/s) we decrease the space of integration 2 times, using Bessel's interpolation formula $y_{n+\frac{1}{2}}=\frac{y_{n+1}+y_n}{2}-\frac{1}{16}(\Delta^2 y_n+\Delta^2 y_{n-1})$. After the passage of the interval of the velocities, close to the speed of sound, beginning with 27 line (with v = 280.8 m/s) space we increase 2 times. Calculation of the cell/elements of motion in peak of the trajectory and at impact point is given in an example.

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1. Motion characteristics in peak of the trajectory.

The interpolating factor is determined from following formulas. In the first approximation,

$$\xi_{sI} = -\frac{p_n}{\Delta p_n} = -\frac{0{,}00328}{(-0{,}03078)} = 0{,}1066,$$

where p_n - last/latter tabular value $\mathrm{tg}\,\theta_n$ to apex/vertex. In the second approach/approximation

$$\xi_s \approx \xi_{sII} = \xi_{sI} - \frac{\xi_{sI}(\xi_{sI}-1)}{2} \cdot \frac{\Delta^2 p_{n-1}}{\Delta p_n} =$$

$$= 0{,}1066 - \frac{0{,}1066\,(0{,}1066-1)}{2} \cdot \frac{(-0{,}00058)}{(-0{,}03078)} = 0{,}1075.$$

Abscissa of peak of the trajectory during integration with independent alternating/variable x

$$x_s = x_n + \xi_s h_x = 11\,635 + 0{,}1075 \cdot 250 = 11\,662 \text{ м}; \; x_n = 11\,635 \text{ м}; \; h_x$$

=250 m (space of integration for x).

Ordinate of peak of the trajectory:

$$y_s = y_n + \xi_s \Delta y_n + \frac{\xi_s(\xi_s-1)}{2}\Delta^2 y_{n-1} =$$

$$= 5164 + 0{,}1075\,(-3{,}015) + \frac{0{,}1075\,(0{,}1075-1)}{2}[(-3{,}015) - 4{,}608] =$$

$$= 5164 \text{ м}.$$

Time of the achievement of peak of the trajectory

$$t_s = t_n + \xi_s \Delta t_n + \frac{\xi_s(\xi_s-1)}{2}\Delta^2 t_{n-1} =$$

$$= 28{,}94 + 0{,}1075 \cdot 0{,}8858 + \frac{0{,}1075\,(0{,}1075-1)}{2} \cdot [0{,}8858 - 0{,}8772] =$$

$$= 29{,}03 \text{ с}.$$

Key: (1). s.

Velocity in peak of the trajectory:

$$v_S=v_n+\xi_S\Delta v_n+\frac{\xi_S(\xi_S-1)}{2}\Delta^2 v_{n-1}=$$
$$=283{,}7+0{,}1075(283{,}7-286{,}3)+\frac{0{,}1075(0{,}1075-1)}{2}\times$$
$$\times([283{,}7-286{,}3]-[286{,}3-289{,}9])=283{,}4\ \overset{(1)}{\text{м/с}}.$$

Key: (1). m/s.

II. Motion characteristics at impact point.

Interpolating factor in the first approximation:

$$\xi_{C1}=\frac{y_n}{\Delta y_n}=-\frac{575}{(-728)}=0{,}7898.$$

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In the second approach/approximation:

$$\xi_C\simeq\xi_{CII}=\xi_{C1}-\frac{\xi_{C1}(\xi_{C1}-1)}{2}\cdot\frac{\Delta^2 y_{n-1}}{\Delta y_n}=$$
$$=0{,}7898-\frac{0{,}7898(0{,}7898-1)}{2}\cdot\frac{(-728)-(-649)}{(-728)}=0{,}7988.$$

Flying range during integration with independent alternating/variable x:

$$x_C=x_n+\xi_C h_x=19\,385+0{,}7988\cdot 500=19\,734\ \text{м}.$$

Flight path angle at impact point:

$$p_C=p_n+\xi_C\Delta p_n+\frac{\xi_C(\xi_C-1)}{2}\cdot\Delta^2 p_{n-1}=$$
$$=(-1{,}379)+0{,}7988(-0{,}1572)-\frac{0{,}7988(0{,}7988-1)}{2}\times$$
$$\times(-0{,}0178)=-1{,}532,\quad \overset{(1)}{\text{или}}\quad \theta_C=56°52'.$$

Key: (1). or.

Complete flight time:

$$t_C=t_n+\xi_C\Delta t_n+\frac{\xi_C(\xi_C-1)}{2}\Delta^2 t_{n-1}=$$
$$=61,74+0,7988\cdot 2,828+\frac{0,7988(0,7988-1)}{2}\times$$
$$\times(2,828-2,666)=63,99\overset{(1)}{c}.$$

Key: (1). s.

Velocity in impact point:

$$v_C=v_n+\xi_C\Delta v_n+\frac{\xi_C(\xi_C-1)}{2}\Delta^2 v_{n-1}=$$
$$=310,5+0,7988(310,5-307,3)+\frac{0,7988(0,7988-1)}{2}\times$$
$$\times[(310,5-307,3)-(307,3-302,7)]=313,2\overset{(1)}{\text{м/с}}.$$

Key: (1). m/s.

1.6. Numerical differentiation.

Obtaining derivatives of the functions, assigned tabular, is realized by methods of numerical differentiation. In this case, usually is differentiated the interpolating function, comprised with the constant space of argument. Since for independent variable during the compilation of the standard interpolating functions is taken ξ, then

$$y'_x=\frac{dy}{dx}=\frac{dy}{d\xi}\frac{d\xi}{dx}=\frac{1}{h_x}y'_\xi.$$

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For the horizontal line of consecutive cell/elements, the

interpolating function takes the form

$$y=y_n+\xi\Delta y_n+\frac{\xi(\xi-1)}{2}\Delta^2 y_n+\frac{\xi(\xi-1)(\xi-2)}{6}\Delta^3 y_n. \quad (6.22)$$

After differentiation we will obtain

$$y'_x=\frac{1}{h_x}\left[\Delta y_n+\frac{1}{2}(2\xi-1)\Delta^2 y_n+\frac{1}{6}(3\xi^2-6\xi+2)\Delta^3 y_n\right]. \quad (6.23)$$

a the second derivative

$$y''_x=\frac{1}{h_x^2}\left[\Delta^2 y_n+(\xi-1)\Delta^3 y_n\right]. \quad (6.24)$$

After accepting for the fixed point $\xi=\xi_n=0$, we will obtain:

$$\left.\begin{aligned} y'_{xn}&=\frac{1}{h_x}\left(\Delta y_n-\frac{1}{2}\Delta^2 y_n+\frac{1}{3}\Delta^3 y_n\right);\\ y''_{xn}&=\frac{1}{h_x^2}(\Delta^2 y_n-\Delta^3 y_n).\end{aligned}\right\} \quad (6.25)$$

For the broken line of form we will obtain:

$$y'_x=\frac{1}{h_x}\left[\Delta y_{xn}+\frac{1}{2}(2\xi-1)\Delta^2 y_{n-1}+\frac{1}{6}(3\xi^2-1)\Delta^3 y_{n-1}\right]. \quad (6.26)$$

At the fixed point of "n"

$$y'_{xn}=\frac{1}{h_x}\left(\Delta y_n-\frac{1}{2}\Delta^2 y_{n-1}-\frac{1}{6}\Delta^3 y_{n-1}\right). \quad (6.27)$$

Taking into account a difference in the first and second orders, we will obtain from (6.27) the formula, used in the theory of corrections (chapter XI),

$$y'_n=\frac{y_{n+1}-y_{n-1}}{2h_x}. \quad (6.28)$$

1.7. Method of the numerical solution of the spatial problem of guidance.

As the basic system of differential equations, which describes the three-dimensional/space controlled flight with guidance to the rapidly moved target/purposes, let us take system (4.23). Let us

examine the order of the numerical solution of this system. The values, which correspond to calculation point in the trajectory in question, let us supply with index i, to the preceding/previous point - (i-1), and following - (i+1).

Let us establish the values, which determine the position of plane of interception for i-th point in the trajectory. From (4.30) it follows

$$r_{r\,i}=\sqrt{(x_{u\,i}-x_{p\,i})^2+(z_{u\,i}-z_{p\,i})^2}.$$

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On the basis of (4.33), we find the sum of the angles

$$(\Psi_0+\nu)_i=\operatorname{arc\,tg}\frac{z_{u\,i}-z_{p\,i}}{x_{u\,i}-x_{p\,i}};$$

on the basis of (4.29), let us find

$$\mu_0=\operatorname{arc\,tg}\frac{y_{u\,i}-y_{p\,i}}{r_r}.$$

The angle of the slope of the instantaneous plane of interception to horizontal plane let us determine from the relationship/ratio

$$\chi_i=\operatorname{arc\,tg}\frac{\operatorname{tg}\lambda_i+\operatorname{tg}\mu_i\cos\alpha_{k\,i}}{\sin\alpha_{k\,i}\cos\nu_i}.$$

The angle between the intersection of the plane of interception with horizontal plane (line dl) and the selected reference direction Cx let us define as angular difference

$$\Psi_{0i}=(\Psi_0+\nu)_i-\nu_i.$$

The coordinates of target and rocket in the instantaneous plane

of interception let us find from (4.35), but the position of the line of sighting will be determined by angle γ_i, on the basis of (4.37)

$$x_{\text{ц}\,\chi_i}=y_{\text{ц}\,i}\,\frac{\cos\nu_i}{\operatorname{tg}\mu_i};\qquad y_{\text{ц}\,\chi_i}=\frac{y_{\text{ц}\,i}}{\sin\chi_i};$$

$$x_{\text{р}\,\chi_i}=y_{\text{р}\,i}\,\frac{\cos\nu_i}{\operatorname{tg}\mu_i};\qquad y_{\text{р}\,\chi_i}=\frac{y_{\text{р}\,i}}{\sin\chi_i};$$

$$\gamma_i=\operatorname{arc\,tg}\frac{y_{\text{ц}\,\chi_i}-y_{\text{р}\,\chi_i}}{x_{\text{ц}\,\chi_i}-x_{\text{р}\,\chi_i}}.$$

Further calculation depends on the selection of guidance method. For example, for the group of the methods, united by the sign/criterion of the orientation of velocity vector relative to the line of sighting, it is necessary to use formulas for value determination of lead angle. For the method of pursuit with advance/prevention at fixed-lead angle, it is necessary to take $\alpha_{\text{р}}=\alpha_{\text{р0}}=\text{const}$ and then

$$\varphi_i=\gamma_i+\alpha_{\text{р0}}.$$

For the method of half approach, it is necessary to use the formula, which determines one-half angle, which corresponds to the condition of ideal advance/prevention.

During the calculation of the missile targeting according to pursuit curve the obtained previously formulas can be converted. In this case $\alpha_{\text{р}}=0$, the velocity vector of rocket must coincide with the direction of the line of sighting, $\varphi_i=\gamma_i$; $\mu_i=\theta_i$ and $\xi_i=\nu_i$. It is obvious that $\operatorname{tg}\theta_i=\operatorname{tg}\mu_i$ must be determined according to formula (4.29), and $\operatorname{tg}\xi_i=\operatorname{tg}\nu_i$ is determined from formula (4.32).

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With three-dimensional/space constant bearing guidance it is necessary to ensure in the process of guidance the invariability of angles μ_i and $(\Psi_0+\nu)_i$; with this line of sighting will remain parallel to itself. Then

$$\operatorname{tg}(\Psi_0+\nu)_i=\operatorname{tg}(\Psi_0+\nu)_0=\text{const};$$
$$r_{r\,i}=(x_{u\,i}-x_{p\,i})\sqrt{1+\operatorname{tg}^2(\Psi_0+\nu)_0}.$$

During calculation $\operatorname{tg}\nu_i$ and $\operatorname{tg}\chi_i$ and in further calculations it is necessary to take

$$\mu_i=\mu_0=\operatorname{arc\,tg}\frac{y_{u0}-y_{p0}}{(x_{u0}-x_{p0})\sqrt{1+\operatorname{tg}^2(\Psi_0+\nu)_0}}.$$

Lead angle is located from the condition of the ideal advance/prevention

$$\sin\alpha_{p\,i}=\frac{v_{u\,i}}{v_{p\,i}}\sin\alpha_{u\,i}.$$

With guidance according to the method of coincidence, the angles of sighting and rocket must be equal, also must be equal the azimuths of target/purpose and rocket or angles $A_{xpi}=A_{xui}$. The direction of the line of sighting must pass through the origin and in this case

$$\mu_i=\varepsilon_{p\,i}=\varepsilon_{u\,i}=\operatorname{arc\,tg}\frac{y_{u\,i}}{x_{u\,i}}$$

or

$$\sin\mu_i=\frac{y_{u\,i}}{r_{u\,i}}.$$

Respectively $\Psi_0=0$ and $\nu_i=\operatorname{arc\,tg}\frac{z_{u\,i}}{x_{u\,i}}$.

With any motion characteristics of target/purpose, the

instantaneous plane of interception and the line of its intersection with horizontal plane must pass through the origin, but

$$x_{p\chi_i}=x_{pi};\quad x_{u\chi_i}=x_{ui}.$$

Lead angle in the instantaneous plane of interception is determined from formula (4.5).

With the method of proportional approach, the lead angle directly is not calculated, but is determined derivative of lead angle from formula (4.15). Factor of proportionality a is selected, on the basis of requirement to obtain toward the end of the guidance of the condition of encounter with target, most favorable for a specific case. An increase in the lead angle $\Delta\alpha_{pi}$ is calculated by numerical integration, after which

$$\alpha_{p(i+1)}=\alpha_{pi}+\Delta\alpha_{pi}.$$

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The initial value of lead angle α_0 can be taken either on the basis of guidance method or three points, or on the basis of the method of half approach.

After the determination of the lead angle and angle φ_i the angle of arrival let us find from (4.24), angle ξ_i from (4.26) and the angle of rotation of trajectory Ψ_i from (4.27):

$$\sin\theta_i=\sin\chi_i\sin\varphi_i;\quad \sin\xi_i=\operatorname{tg}\theta_i\operatorname{ctg}\chi_i;\quad \Psi_i=\Psi_{0i}+\xi_i.$$

The elevation of the velocity of rocket is equal to

$$w_{pi}=v_{pi}\sin\theta_i.$$

Increments of coordinates Δy_{pi} is located by numerical integration. If we, for example, with difference method of integration use the horizontal or broken table rows of finite differences, then let us have

$$\Delta y_{pi}=h_i\left(w_{pi}+\frac{1}{2}\Delta w_{pi}-\frac{1}{12}\Delta^2 w_{p(i-1)}\right),$$

after which increase Δx_{pi} and Δz_{pi} will find from (4.39) and (4.40).

$$\Delta x_{pi}\approx\Delta y_{pi}\frac{\cos\Psi_i}{\operatorname{tg}\theta_i};\quad \Delta z_{pi}\approx\Delta y_{pi}\frac{\sin\Psi_i}{\operatorname{tg}\theta_i}.$$

The coordinates of position of the center of mass of rocket for point i+1 will be equal to

$$x_{p(i+1)}=x_{pi}+\Delta x_{pi};\quad y_{p(i+1)}=y_{pi}+\Delta y_{pi};\quad z_{p(i+1)}=z_{pi}+\Delta z_{pi}.$$

A change in the velocity of the center of mass of rocket is located by the method of successive approximations, since for the direct use of equation (3.37) insufficient data for determining the drag of X (there are no angles of attack and slip - α_i and β_i). In the first approximation, we find drag to one of the methods of aerodynamics, on the basis of the functional dependence

$$X_{i1}=f(y_{p(i-1)};\ v_{p(i-1)};\ M_{i-1};\ Re_{i-1};\ \alpha_{i-1};\ \beta_{i-1}\ \overset{(1)}{\text{и т. д.}}),$$

Key: (1). and so forth.

where α_{i-1} and β_{i-1} etc. correspond to preceding/previous point in the trajectory.

For the beginning of calculations, the angles of attack and slip must be selected according to initial conditions. The value of derivative $\dot{v}_p$ at i-th point in the first approximation, will be equal

$$\dot{v}_{p\,i1} = \frac{P_i - X_{i1}}{m_i} - g\sin\theta_i.$$

A velocity increment let us find by numerical integration for the formulas of the horizontal or broken line of the lines

$$(\Delta v_{p\,i})_1 = h_i\left(\dot{v}_{p\,i} + \frac{1}{2}\Delta\dot{v}_{p\,i} - \frac{1}{12}\Delta^2\dot{v}_{p\,(i-1)}\right),$$

after which

$$(v_{p\,(i+1)})_1 = v_{p\,i} + (\Delta v_{p\,i})_1.$$

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The angles of attack and slip α_i and β_i are located from the formulas through $\dot{v}_{p\,i}$ and derivatives $\dot{\theta}_i$ and $\dot{\Psi}_i$, which are determined by the method of numerical differentiation. Using procedure described above, along the values of the motion characteristics of target/purpose for i+1 point $x_{\text{ц}(i+1)}$, $y_{\text{ц}(i+1)}$, $v_{\text{ц}(i+1)}$, μ_{i+1}, $\alpha_{\text{к}(i+1)}$ and along obtained values $x_{p(i+1)}$; $y_{p(i+1)}$; $z_{p(i+1)}$; $v_{p(i+1)}$ we calculate θ_{i+1} and Ψ_{i+1}. After carrying them into the table of differences, let us find

$$\Delta\theta_i = \theta_{i+1} - \theta_i; \quad \Delta\Psi_i = \Psi_{i+1} - \Psi_i$$

and let us increase table. On the formulas of numerical differentiation taking into account the third differences, we will obtain

$$\dot{\theta}_i = \frac{1}{h_i}\left(\Delta\theta_i - \frac{1}{2}\Delta^2\theta_{i-1} - \frac{1}{6}\Delta^3\theta_{i-1}\right);$$

$$\dot{\Psi}_i = \frac{1}{h_i}\left(\Delta\Psi_i - \frac{1}{2}\Delta^2\Psi_{i-1} - \frac{1}{6}\Delta^3\Psi_{i-1}\right).$$

Balance angle is approximately equal to

$$\alpha_{6\,i}=\frac{m_i}{P_i+Y_i^{\alpha}}(v_i\dot{\theta}_i+g\cos\theta_i),$$

a angle of slip

$$\beta_{6\,i}=\frac{m_i v_i\cos\theta_i\dot{\Psi}_i}{P_i+Z_i^{\beta}}.$$

Now it is possible to calculate In the second approach/approximation drag data

$$X_{i\,\mathrm{II}}=f(y_{p\,i};\ v_{p\,i};\ M_i;\ Re_i;\ \alpha_{6i};\ \beta_{6i})$$

and

$$\dot{v}_{p\,i\,\mathrm{II}}=\frac{P_i-X_{i\,\mathrm{II}}}{m_i}-g\sin\theta_i.$$

A velocity increment $(\Delta v_{pi})_{\mathrm{II}}$ and the refined velocity of rocket, which corresponds i+1 to the moment of time, we find in the form

$$(v_{p\,(i+1)})_{\mathrm{II}}=v_{p\,i}+(\Delta v_{p\,i})_{\mathrm{II}}.$$

The difference between $(v_{p\,(i+1)})_{\mathrm{I}}$ and $(v_{p\,(i+1)})_{\mathrm{II}}$, as a rule, is small and it is possible not to repeat the conversion of all values, corresponding to i+1 to point. For further calculations one should use the refined value $(v_{p(i+1)})_{\mathrm{II}}$.

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1.8. Numerical integration of the differential equations of external ballistics by Runge - Kutta's method.

As the basis of Runge - Kutta's method, is placed the expansion of the unknown function y = f(x) about its known point $(x_n y_n)$ in power series in terms of argument $(x-x_n)$:

$$y=y_n+(x-x_n)y'_n+\frac{(x-x_n)^2}{2!}y''_n+\ldots+\frac{(x-x_n)^m}{m!}y_n^{(m)}. \quad (6.29)$$

Expansion makes it possible to obtain the value of function $y(x_n+h_x)$ in terms of the known initial value of function $y(x_n)$ and on space h_x. The unknown subsequent value of function is equal

$$y_{n+1}=y_n+\Delta y_n.$$

Basic calculation formula for determination Δy_n takes the form

$$\Delta y_n\approx\frac{1}{6}(k_1+2k_2+2k_3+k_4), \quad (6.30)$$

where

$$\left.\begin{aligned}
k_1&=h_x f(x_n;\ y_n);\\
k_2&=h_x f\left(x_n+\frac{1}{2}h_x;\quad y_n+\frac{1}{2}k_1\right);\\
k_3&=h_x f\left(x_n+\frac{1}{2}h_x;\quad y_n+\frac{1}{2}k_2\right);\\
k_4&=h_x f(x_n+h_x;\quad y_n+k_3).
\end{aligned}\right\} \quad (6.31)$$

As an example let us give the schematic of the solution of system of equations (3.60)

$$\dot v=\frac{P-X}{m}-g\sin\theta;\quad \theta(t)\approx\theta_{\text{пр}}(t);\quad \dot y=v\sin\theta;\quad \dot x=v\cos\theta.$$

Right part one equation of the written system there is a function of three variable parameters - velocity, time and flight altitude, since a change in the angle θ in flight time by predetermined program. Following two equations represent by themselves the velocity function and time. It is possible to write

$$\dot v=f_1(v,y,t);\quad \theta=f_2(t);\quad \dot y=f_3(v,t);\quad \dot x=f_4(v,t). \quad (6.32)$$

Let us designate the terms of common/general/total formula (6.30) for the first equation through k_i, for the third equation through l_i and for the fourth through m_i. Together must be integrated the first and third equations. The order of the calculation of cell/elements n+1 of line by Runge - Kutta's method on basic formula (6.30) is represented in table (6.8). By rifleman/pointers it is shown sequence accomplishing the process/operations of count for the equations, solved together.

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The application/use of common/general/total formula (6.30) with the calculation of coefficients k_i in (6.31) requires the fourfold substitution of the corresponding values of argument and functions into fundamental differential equation. This leads to an increase in the volume of calculations during the growth of lines in comparison with difference with method. Method is insensitive to errors and does not contain means for a control/check in the process of calculations. The large advantage of method is the uniformity of calculations for all points. Method does not require initial approaches and, therefore, can have one program for entire range of integration by ETsVM. In constructing a program and ETsVM, can render/show more convenient following formula for Δy_n

$$\Delta y_n \approx \frac{1}{6}(k_1 + 3k_2 + k_3 + k_4), \qquad (6.33)$$

where

$$k_1 = h_x f(x_n;\ y_n);$$
$$k_2 = h_x f\left(x_n + \frac{1}{2} h_x;\quad y_n + \frac{1}{2} k_1\right);$$
$$k_3 = h_x f\left(x_n + \frac{1}{2} h_x;\quad y_n - \frac{1}{2} k_1 + k_2\right);$$
$$k_4 = h_x f\left(x_n + h_x;\ y_n + \frac{1}{2} k_2 + \frac{1}{2} k_3\right).$$

Table 6.8.

(1) Вычисление элементов $n+1$ строки		
$k_1 = h_t f_1(v_n; y_n; t_n)$	$l_1 = h_t f_2(v_n; t_n)$	$m_1 = h_t f_3(v_n; t_n)$
$k_2 = h_t f_1\left(v_n + \frac{k_1}{2}; y_n + \frac{l_1}{2}; t_n + \frac{h_t}{2}\right)$	$l_2 = h_t f_2\left(v_n + \frac{k_1}{2}; t_n + \frac{h_t}{2}\right)$	$m_2 = h_t f_3\left(v_n + \frac{k_1}{2}; t_n + \frac{h_t}{2}\right)$
$k_3 = h_t f_1\left(v_n + \frac{k_2}{2}; y_n + \frac{l_2}{2}; t_n + \frac{h_t}{2}\right)$	$l_3 = h_t f_2\left(v_n + \frac{k_2}{2}; t_n + \frac{h_t}{2}\right)$	$m_3 = h_t f_3\left(v_n + \frac{k_2}{2}; t_n + \frac{h_t}{2}\right)$
$k_4 = h_t f_1(v_n + k_3; y_n + l_3; t_n + h_t)$	$l_4 = h_t f_2(v_n + k_3; t_n + h_t)$	$m_4 = h_t f_4(v_n + k_3; t_n + h_t)$
$\Delta v_n = \frac{k_1 + 2k_2 + 2k_3 + k_4}{6}$	$\Delta y_n = \frac{l_1 + 2l_2 + 2l_3 + l_4}{6}$	$\Delta x_n = \frac{m_1 + 2m_2 + 2m_3 + m_4}{6}$
$v_{n+1} = v_n + \Delta v_n$	$y_{n+1} = y_n + \Delta y_n$	$x_{n+1} = x_n + \Delta x_n$

Key: (1). Calculation of cell/elements n+1 of line.

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§ 2. Application/use of electronic digital computers for the solution of the equations of external ballistics.

2.1. Reduction of the equations of external ballistics to the form, convenient for programming and compilation of algorithms.

The preparation of the systems of equations of external ballistics for the compilation of detailed algorithm and programming consists in two basic operations. The first of them the transformation of the differential second order equations into the system of equations of the first order. In accordance with the rule

cf a decrease in the order of derivative, each second order equation must be replaced by two first-order equations. Second process/operation - this grouping and early calculation of the entering the equations constant values.

Let us examine the compilation of the algorithm of the solution of simple system (3.76). The basic system of differential equations takes the form

$$\frac{du}{dt}=\frac{1}{\mu}(D-E)u;\quad \frac{dp}{dt}=-\frac{g}{u};\quad \frac{dy}{dt}=up;\quad \frac{dx}{dt}=u.$$

Here all the equations - the first order and system is suitable for further work. Auxiliary formulas under the assumption about the constancy of fuel consumption per second take the form

$$\mu=\frac{m}{m_0};\ m=m_0-|\dot{m}|t;\ v=u\sqrt{1-p^2};$$

$$D=\frac{1}{v}\left\{\frac{P_0}{m_0}+\frac{S_a p_{0N}}{m_0}\left[1-\pi(y)\right]\right\};$$

$$E=\frac{S\varrho_{0N}}{2m_0}\,\frac{\varrho}{\varrho_{0N}}\,vc_x(M).$$

Let us isolate constants for the calculation of value and let us designate them

$$c_1=\frac{P_0}{m_0};\quad c_2=\frac{S_a p_{0N}}{m_0};\quad c_3=\frac{S\varrho_{0N}}{2m_0}.$$

Furthermore,

$$\mu=\frac{m}{m_0}=\frac{m_0-|\dot{m}|t}{m_0}=1-\frac{|\dot{m}|}{m_0}t;\quad c_4=\frac{|\dot{m}|}{m_0}.$$

Algorithm must contain limitation on integration limits of system. In limitation in question system they can be superimposed on time, on velocity or on μ by the inequalities

$$t_0\leqslant t\leqslant t_к;\quad v_0\leqslant v\leqslant v_к;\quad \mu_0>\mu>\mu_к.$$

Limitations on coordinates usually are not conducted.

One of the written inequalities must be used for an introduction into algorithm and program of jump if not operation. After the termination of cycle, for example, when $\mu=\mu_к$, the jump if not operation must ensure passage to calculation with $\mu=\mu_к=\text{const}$ and $D = 0$. In this sense the preferable systems of equations, which allow with the observance of other conditions of mass calculation for the active and passive trajectory phases without a change in the basic system of equations and, consequently, also programs for ETsVM.

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In the amalgamated form the algorithm of the solution of the system in question will be written as follows:

$$\left.\begin{aligned} &\frac{du}{dt}=\frac{1}{\mu}(D-E)u;\quad \mu_0\geqslant\mu\geqslant\mu_к\quad \frac{dp}{dt}=-\frac{g}{u};\\ &\frac{dy}{dt}=up;\quad \frac{dx}{dt}=u;\quad \frac{dt}{dt}=1.\end{aligned}\right\}\qquad (6.34)$$

Auxiliary formulas take the form

$$\left.\begin{aligned} &v=u\sqrt{1+p^2};\quad D=\frac{1}{v}\{c_1+c_2[1-\pi(y)]\};\\ &E=c_3\frac{\varrho}{\varrho_{0N}}vc_x\left(\frac{v}{a}\right);\quad \mu=1-c_4t.\end{aligned}\right\}\qquad (6.35)$$

In the second equation of basic system for relatively small trajectories g = const. Depending on the number of significant

digits, taken in calculations, one should take g=9.80665; 9.8066; 9.807 and 9.81 m/s².

During calculation E, the function $c_x(M)$ can be utilized either independently or in products $vc_x\left(\frac{v}{a}\right) = f_1(v)$ and $v^2 c_x\left(\frac{v}{a}\right) = f_2(v)$. In the latter case

$$E = c_3 \frac{\varrho}{\varrho_{0N}} \frac{v^2 c_x\left(\frac{v}{a}\right)}{v}. \qquad (6.36)$$

Functions $f_1(v)$ and $f_2(v)$ change more smoothly than $c_x\left(\frac{v}{a}\right)$; they just as $c_x\left(\frac{v}{a}\right)$, they can be calculated previously.

It is most frequently during calculations necessary to use the values of functions $c_x(M)$, $f_1(v_\tau)$ and $f_2(v_\tau)$, those obtained for terrestrial conditions at the speed of sound a_{0N}. In these cases it is necessary during integral use $c_x(M)$ to define M as $\frac{v_\tau}{a_{0N}}$, where v_τ they find from (2.102):

$$v_\tau = v\sqrt{\frac{\tau_{0N}}{\tau}}.$$

If is previously comprised function $f_1(v_\tau)$ when a_{0N}, then

$$f_1(v) = vc_x\left(\frac{v}{a}\right) = vc_x\left(\frac{v_\tau}{a_{0N}}\right) = v_\tau c_x\left(\frac{v_\tau}{a_{0N}}\right)\sqrt{\frac{\tau}{\tau_{0N}}} = f_1(v_\tau)\sqrt{\frac{\tau}{\tau_{0N}}}$$

and then

$$E = c_3 \frac{\varrho}{\varrho_{0N}} c_x\left(\frac{v_\tau}{a_{0N}}\right) v_\tau \sqrt{\frac{\tau}{\tau_{0N}}} = c_3 \frac{\varrho}{\varrho_{0N}} f_1(v_\tau)\sqrt{\frac{\tau}{\tau_{0N}}}. \qquad (6.37)$$

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If is precomputed function $f_2(v_\tau)$, then

$$f_2(v) = v^2 c_x\left(\frac{v}{a}\right) = v^2 c_x\left(\frac{v_\tau}{a_{0N}}\right) = v_\tau^2 c_x\left(\frac{v_\tau}{a_{0N}}\right)\cdot\frac{\tau}{\tau_{0N}} =$$

$$= f_2(v_\tau)\frac{\tau}{\tau_{0N}}$$

and then

$$E=c_3\frac{\varrho}{\varrho_{0N}}v_\tau^2\frac{\tau}{\tau_{0N}}\frac{c_x\left(\frac{v_\tau}{a_{0N}}\right)}{v}=c_3\frac{\varrho}{\varrho_{0N}}f_2(v_\tau)\frac{1}{v_\tau}\sqrt{\frac{\tau}{\tau_{0N}}}. \quad (6.38)$$

The compilation of the part of the common/general/total algorithm, which concerns the solution of system of equations by Runge - Kutta's method, let us observe, after writing the algorithm of the solution of system (3.76) at the first two spaces of integration. During the writing of algorithm, let us designate

$$k=h_t\frac{1}{\mu}(D-E)u;\quad l=-h_t\frac{g}{u};\quad m=h_t up;\quad n=h_t u. \quad (6.39)$$

The variable values, which relate to 0-01 and 1-01 calculation points, let us supply with the double indices the first of which corresponds to the number of the computed point, the second - to an index of coefficient in Runge - Kutta's formula. For example, values for a zero point will have the indices: D_{01} - for determination k_{01}, D_{02} - for determination k_{02}; D_{03} and D_{04} they will enter into formulas for k_{03} and k_{04}. The algorithm of solution let us carry in table 6.9 and 6.10. Numerical solutions by the written algorithm for the first and second space of integration are utilized for the adjustment of program and control/check of machine count.

If is utilized $f_1(v_\tau)$, the calculation of coefficients in Runge

- Kutta's method will be several other, than in table 6.9 and 6.10. For example, for determining the coefficient k_{12} order of calculation will be following:

$$\left.\begin{aligned}
v_{12}&=\left(u_1+\frac{k_{11}}{2}\right)\sqrt{1+\left(p_1+\frac{l_{11}}{2}\right)^2};\\
v_{\tau 12}&=v_{12}\sqrt{\frac{\tau_{0N}}{\tau\left(y_1+\frac{m_{11}}{2}\right)}};\\
D_{12}&=\frac{1}{v_{12}}\left\{c_1+c_2'\left[1-\pi\left(y_1+\frac{m_{11}}{2}\right)\right]\right\};\\
f_1(v_\tau)&=v_{\tau_{12}}c_x\left(\frac{v_{\tau_{12}}}{a_{0N}}\right);\\
E_{12}&=c_3\frac{\varrho\left(y_1+\frac{m_{11}}{2}\right)}{\varrho_{0N}}\sqrt{\frac{\tau\left(y_1+\frac{m_{11}}{2}\right)}{\tau_{0N}}}f_1(v_\tau);
\end{aligned}\right\}\quad(6.40)$$

$$\left.\begin{aligned}
\frac{1}{\mu_{12}}&=\frac{1}{1-c_4\left(t_1+\frac{h_t}{2}\right)};\\
k_{12}&=h_t\frac{1}{\mu_{12}}(D_{12}-E_{12})\left(u_1+\frac{k_{11}}{2}\right).
\end{aligned}\right\}\quad(6.40)$$

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During the use of function $f_2(v_\tau)$ the formula for calculation E_{12} will take this form:

$$E_{12}=c_3\frac{\varrho\left(y_1+\frac{m_{11}}{2}\right)}{\varrho_{0N}}\sqrt{\frac{\tau\left(y_1+\frac{m_{11}}{2}\right)}{\tau_{0N}}}f_2(v_\tau)\cdot\frac{1}{v_{\tau_{12}}}\qquad(6.41)$$

Formulas for determination $v_{12}, v_{\tau 12}, D_{12}$ and coefficient k_{12} will remain without change.

The fourth differential equation of system (3.76) is solved separately in the following order:

for the first space of integration (zero point)

$$\left.\begin{aligned} & n_{01}=h_t u_0;\ n_{02}=h_t\left(u_0+\frac{k_{01}}{2}\right); \\ & n_{03}=h_t\left(u_0+\frac{k_{02}}{2}\right);\quad n_{04}=h_t(u_0+k_{03}); \\ & \Delta x_0=\frac{1}{6}(n_{01}+2n_{02}+2n_{03}+n_{04});\quad x_1=x_0+\Delta x_0; \end{aligned}\right| \quad (6.42)$$

For the second space of integration (first point)

$$\left.\begin{aligned} & n_{11}=h_t u_1;\ n_{12}=h_t\left(u_1+\frac{k_{11}}{2}\right);\quad n_{13}=h_t\left(u_1+\frac{k_{12}}{2}\right); \\ & n_{13}=h_t\left(u_1+\frac{k_{12}}{2}\right);\ n_{14}=h_t(u_1+k_{13}); \\ & \Delta x_1=\frac{1}{6}(n_{11}+2n_{12}+2n_{13}+n_{14}); \\ & x_2=x_1+\Delta x_1. \end{aligned}\right| \quad (6.43)$$

The system of equations, which describes together the motion of the center of mass of rocket and oscillations relative to the center of mass, is considerably more complex. We will obtain a similar system in the form, suitable for programming. Utilizing (3.37) and (3.41), the equation of the rotary motion of axis of rocket relative to axis Oz_1 and kinematic equations, let us write the system, which describes the flat/plane axial motion of the rocket

$$m\dot{v}=P-X-mg\sin\theta;$$
$$mv\dot{\theta}=(P+Y^{\alpha})\alpha-mg\cos\theta;$$
$$J_{z_1}\ddot{\vartheta}=\sum M_{z_1};\quad \dot{y}=v\sin\theta;\quad \dot{x}=v\cos\theta;\quad \alpha=\vartheta-\theta.$$

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Utilizing the dependences, obtained into §§ 3, 4 and 5 chapters II, let us discover the values of the parameters, entering the last/latter system

for the first space of integration (zero point)

$$\left.\begin{array}{l} n_{01}=h_t u_0;\ n_{02}=h_t\left(u_0+\frac{k_{01}}{2}\right);\\ n_{03}=h_t\left(u_0+\frac{k_{02}}{2}\right);\quad n_{04}=h_t(u_0+k_{03});\\ \Delta x_0=\frac{1}{6}(n_{01}+2n_{02}+2n_{03}+n_{04});\quad x_1=x_0+\Delta x_0; \end{array}\right| \quad (6.42)$$

For the second space of integration (first point)

$$\left.\begin{array}{l} n_{11}=h_t u_1;\ n_{12}=h_t\left(u_1+\frac{k_{11}}{2}\right);\quad n_{13}=h_t\left(u_1+\frac{k_{12}}{2}\right);\\ n_{13}=h_t\left(u_1+\frac{k_{12}}{2}\right);\ n_{14}=h_t(u_1+k_{13});\\ \Delta x_1=\frac{1}{6}(n_{11}+2n_{12}+2n_{13}+n_{14});\\ x_2=x_1+\Delta x_1. \end{array}\right| \quad (6.43)$$

The system of equations, which describes together the motion of the center of mass of rocket and oscillations relative to the center of mass, is considerably more complex. We will obtain a similar system in the form, suitable for programming. Utilizing (3.37) and (3.41), the equation of the rotary motion of axis of rocket relative to axis Oz_1 and kinematic equations, let us write the system, which describes the flat/plane axial motion of the rocket

$$m\dot{v}=P-X-mg\sin\theta;$$

$$mv\dot{\theta}=(P+Y^{\alpha})\alpha-mg\cos\theta;$$

$$J_{z_1}\ddot{\vartheta}=\sum M_{z_1};\quad \dot{y}=v\sin\theta;\quad \dot{x}=v\cos\theta;\quad \alpha=\vartheta-\theta.$$

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Utilizing the dependences, obtained into §§ 3, 4 and 5 chapters II, let us discover the values of the parameters, entering the last/latter system

$$
\left.\begin{aligned}
m\dot{v} &= P_0 + S_a p_{0N}[1-\pi(y)] - \\
&-\frac{1}{2} S \varrho_{0N} H(y) v^2 c_x(M) - mg\sin\theta; \\
mv\dot{\theta} &= \{P_0 + S_a p_{0N}[1-\pi(y)]\}\alpha + \\
&+\frac{1}{2} S \varrho_{0N} H(y) v^2 c_y^\alpha \alpha - mg\cos\theta; \\
J_{z_1}\ddot{\vartheta} &= -S\frac{\varrho_{0N} H(y)}{2} v^2 l\,|m_{z_1}^\alpha|\,\alpha - \\
&-S\frac{\varrho_{0N} H(y)}{2} v l^2\,|m_{z_1}^{\omega_{z_1}}|\,\dot{\vartheta}; \\
\dot{y} &= v\cos\theta; \quad \dot{x} = v\sin\theta; \quad \alpha = \vartheta - \theta.
\end{aligned}\right| \qquad (6.44)
$$

For short trajectories it is possible to substantially simplify equations, after accepting functions $\pi(y) \approx H(y) \approx 1$. In this case, the error will not virtually influence the final results of calculation. Further, after using the method of a decrease in the order of derivative, let us replace the momental equation, which represents by itself the differential second order equation, with the system of two first-order equations. Taking into account this we will obtain the following system of equations in which for simplicity of recording it is accepted that $c_x(M) = c_x$, $P = P_0$:

$$
\left.\begin{aligned}
\dot{v} &= \frac{P}{m} - \frac{1}{2m} S\varrho_{0N} v^2 c_x - g\sin\theta; \\
\dot{\theta} &= \frac{P\alpha}{mv} + \frac{1}{2m} S\varrho_{0N} v c_y^\alpha \alpha - \frac{g}{v}\cos\theta; \\
\dot{\omega}_{z_1} &= -\frac{1}{J_{z_1}}\left[\frac{S\varrho_{0N} v^2}{2} l\,|m_{z_1}^\alpha|\,\alpha + \frac{S\varrho_{0N} v}{2} l^2\,|m_{z_1}^{\omega_{z_1}}|\,\omega_{z_1}\right]; \\
\dot{\vartheta} &= \omega_{z_1}; \quad \dot{y} = v\sin\theta; \quad \dot{x} = v\cos\theta; \quad \alpha = \vartheta - \theta.
\end{aligned}\right| \qquad (6.45)
$$

Further transformations are connected with the

exception/elimination of the process/operations of calculating the relations (or products) of the predetermined constants, and also with accomplishing of the process/operations above different-scale values. Let us write two first equations of system (6.45) in the form

$$\dot{v}=\frac{m_0}{m}\left[\frac{P}{m_0}-\frac{1}{2m_0}S\varrho_{0N}v^2c_x\right]-g\sin\theta;$$

$$\dot{\theta}=\frac{m_0}{m}\left[\frac{P\alpha}{m_0 v}+\frac{1}{2m_0}S\varrho_{0N}vc_y^{\alpha}\alpha\right]-\frac{g}{v}\cos\theta.$$

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Ratio m_0/m can be converted as follows:

$$\frac{m_0}{m}=\frac{m_0}{m_0-|\dot{m}|t}=\frac{1}{1-\frac{|\dot{m}|}{m_0}t}=\frac{1}{1+k_1t},$$

where $k_1=-\frac{|\dot{m}|}{m_0}$.

In the case when is known the initial value of the moment of inertia $J_{z_1 0}$ and value $J_{z_1 k}$ corresponding to the end/lead of the active section, and the law of a change of it is accepted linear, then

$$J_{z_1}=J_{z_1 0}-\frac{\Delta J_{z_1}}{t_k}t=J_{z_1 0}\left(1-\frac{\Delta J_{z_1}}{J_{z_1 0}t_k}t\right)=J_{z_1 0}(1-k_2t),$$

where

$$\Delta J_{z_1}=J_{z_1 0}-J_{z_1 k},\quad k_2=\frac{\Delta J_{z_1}}{J_{z_1 0}t_k}.$$

Thus,

$$\frac{1}{J_{z_1}}=\frac{1}{J_{z_1 0}(1-k_2t)}.$$

After replacing further the constant values

$$\frac{P}{m_0}=c_1;\quad \frac{S\varrho_{0N}}{2m_0}=c_2;\quad \frac{S\varrho_{0N}l}{2J_{z_1 0}}=c_3;\quad \frac{S\varrho_{0N}l^2}{2J_{z_1 0}}=c_4$$

with their designations, let us lead system (6.44) to the following form:

$$\begin{aligned}
\dot{v} &= \frac{1}{1+k_1 t}(c_1 - c_2 v^2 c_x) - g\sin\theta;\\
\dot{\theta} &= \frac{1}{1+k_1 t}\left(\frac{c_1\alpha}{v} + c_2 v c_y^{\alpha}\alpha\right) - \frac{g}{v}\cos\theta;\\
\dot{\omega}_{z_1} &= \frac{1}{k_2 t - 1}(c_3 v^2 |m_{z_1}^{\alpha}|\alpha + c_4 v |m_{z_1}^{\omega_{z_1}}|\omega_{z_1});\\
\dot{\vartheta} &= \omega_{z_1};\quad \dot{y} = v\sin\theta;\quad \dot{x} = v\cos\theta;\quad \alpha = \vartheta - \theta.
\end{aligned} \qquad (6.46)$$

In the given form the system is more convenient for programming in comparison with the preceding/previous forms of its recording (6.44), (6.45).

The system of differential equations for an inactive leg with independent alternating/variable x is somewhat simpler than examined higher, and consequently, is simpler the algorithm of its solution by Runge - Kutta's method. Let us take system (5.8) and let us discover value of E:

$$\frac{du}{dx} = -cH_{\tau}(y)G(v_{\tau});\quad \frac{dp}{dx} = -\frac{g}{u^2};\quad \frac{dy}{dx} = p;\quad \frac{dt}{dx} = \frac{1}{u}.$$

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Auxiliary formulas two

$$c = \frac{id^2}{Q_{\kappa}}10^3;\quad v_{\tau} = u\sqrt{1+p^2}\sqrt{\frac{\tau_{0N}}{\tau}}.$$

During the writing of algorithm, let us designate

$$\begin{aligned}
k &= -h_x cH_{\tau}(y)G(v_{\tau});\\
l &= h_x\left(-\frac{g}{u^2}\right);\quad m = h_x p;\quad n = \frac{h_x}{u}.
\end{aligned} \qquad (6.47)$$

As before the variable values k, l, m, n let us supply with the double indices the first of which corresponds to the number of the computed point, the second - to an index of coefficient in Runge - Kutta's formula. The algorithm of solution let us carry in table 6.11 and 6.12. The fourth equation can be solved separately in the following order:

a) for the first space of integration (zero point)

$$\left.\begin{aligned} n_{01}&=h_x\frac{1}{u_0};\quad n_{02}=h_x\left(\frac{1}{u_0+\frac{k_{01}}{2}}\right);\\ n_{03}&=h_x\left(\frac{1}{u_0+\frac{k_{02}}{2}}\right);\quad n_{04}=h_x\left(\frac{1}{u_0+k_{03}}\right);\\ \Delta t_0&=\frac{1}{6}(n_{01}+2n_{02}+2n_{03}+n_{04});\\ t_1&=t_0+\Delta t_0;\end{aligned}\right| \qquad (6.48)$$

b) for the second space of integration (first point)

$$\left.\begin{aligned} n_{11}&=h_x\frac{1}{u_1};\quad n_{12}=h_x\left(\frac{1}{u_1+\frac{k_{11}}{2}}\right);\\ n_{13}&=h_x\left(\frac{1}{u_1+\frac{k_{12}}{2}}\right);\quad n_{14}=h_x\left(\frac{1}{u_1+k_{13}}\right);\\ \Delta t_1&=\frac{1}{6}(n_{11}+2n_{12}+2n_{13}+n_{14});\\ t_2&=t_1+\Delta t_1.\end{aligned}\right| \qquad (6.49)$$

2.2. Preparation of initial data.

The initial data, which determine the solution of different problems of external ballistics, can be broken into three large groups. To the first group one should relate the initial design-weight parameters of rocket or projectile, to the second - the initial conditions of motion, also, to the third - the value, the function and the parameters, determined by the model of the Earth and atmosphere accepted.

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Table 6.9. Recording of the algorithm of solution at the first space of integration. Powered flight trajectory.

(1) Решаемое уравнение		
$\frac{du}{dt} = \frac{1}{\mu}(D-E)u$	$\frac{dp}{dt} = -\frac{g}{u}$	$\frac{dy}{dt} = up$
$v_{01} = u_0\sqrt{1+p_0^2}$ $D_{01} = \frac{1}{v_{01}}\{c_1 + c_2[1-\pi(y_0)]\}$ $E_{01} = c_3\frac{\varrho(y_0)}{\varrho_{0N}} v_{01}c_x\left[\frac{v_{01}}{a(y_0)}\right]$ $\frac{1}{\mu_{01}} = \frac{1}{1-c_4t_0} = 1$ $k_{01} = h_t\frac{1}{\mu_{01}}[D_{01}-E_{01}]u_0$	$l_{01} = h_t \times$ $\times\left(-\frac{g}{u_0}\right)$	$m_{01} = h_t u_0 p_0$
$v_{02} = \left(u_0+\frac{k_{01}}{2}\right)\sqrt{1+\left(p_0+\frac{l_{01}}{2}\right)^2}$ $D_{02} = \frac{1}{v_{02}}\left\{c_1+c_2\left[1-\pi\left(y_0+\frac{m_{01}}{2}\right)\right]\right\}$ $E_{02} = c_3\frac{\varrho\left(y_0+\frac{m_{01}}{2}\right)}{\varrho_{0N}}\times$ $\times v_{02}c_x\left[\frac{v_{02}}{a\left(y_0+\frac{m_{01}}{2}\right)}\right]$ $\frac{1}{\mu_{02}} = \frac{1}{1-c_4\left(t_0+\frac{h_t}{2}\right)}$ $k_{02} = h_t\frac{1}{\mu_{02}}[D_{02}-E_{02}]\left(u_0+\frac{k_{01}}{2}\right)$	$l_{02} = h_t \times$ $\times\left(-\frac{g}{u_0+\frac{k_{01}}{2}}\right)$	$m_{02} = h_t \times$ $\times\left(u_0+\frac{k_{01}}{2}\right)\times$ $\times\left(p_0+\frac{l_{01}}{2}\right)$

Key: (1). Equation being solved.

(1) Решаемое уравнение		
$\frac{du}{dt} = \frac{1}{\mu}(D-E)u$	$\frac{dp}{dt} = -\frac{g}{u}$	$\frac{dy}{dt} = up$
$v_{03} = \left(u_0 + \frac{k_{02}}{2}\right)\sqrt{1+\left(p_0+\frac{l_{02}}{2}\right)^2}$ $D_{03} = \frac{1}{v_{03}}\left\{c_1 + c_2\left[1-\pi\left(y_0+\frac{m_{02}}{2}\right)\right]\right\}$ $E_{03} = c_3\frac{\varrho\left(y_0+\frac{m_{02}}{2}\right)}{\varrho_{0N}}v_{03}c_x\left[\frac{v_{03}}{a\left(y_0+\frac{m_{02}}{2}\right)}\right]$ $\frac{1}{\mu_{03}} = \frac{1}{1-c_4\left(t_0+\frac{h_t}{2}\right)}$ $k_{03} = h_t\frac{1}{\mu_{03}}[D_{03}-E_{03}]\left(u_0+\frac{k_{02}}{2}\right)$	$l_{03} = h_t \times \left[-\frac{g}{u_0+\frac{k_{02}}{2}}\right]$	$m_{03} = h_t \times \left(u_0+\frac{k_{02}}{2}\right)\times \left(p_0+\frac{l_{02}}{2}\right)$
$v_{04} = (u_0+k_{03})\sqrt{1+(p_0+l_{03})^2}$ $D_{04} = \frac{1}{v_{04}}\{c_1+c_2[1-\pi(y_0+m_{03})]\}$ $E_{04} = c_3\frac{\varrho(y_0+m_{03})}{\varrho_{0N}}\cdot v_{04}c_x\left[\frac{v_{04}}{a(y_0+m_{03})}\right]$ $\frac{1}{\mu_{04}} = \frac{1}{1-c_4(t_0+h_t)}$ $k_{04} = h_t\frac{1}{\mu_{04}}[D_{04}-E_{04}](u_0+k_{03})$	$l_{04} = h_t \times \left(-\frac{g}{u_0+k_{03}}\right)$	$m_{04} = h_t \times (u_0+k_{03}) \times (p_0+l_{03})$
$\Delta u_0 = \frac{1}{6}(k_{01}+2k_{02}+2k_{03}+k_{04})$ $u_1 = u_0 + \Delta u_0$	$\Delta p_0 = \frac{1}{6} \times (l_{01}+2l_{02}+2l_{03}+l_{04})$ $p_1 = p_0 + \Delta p_0$	$\Delta y_0 = \frac{1}{6}(m_{01}+2m_{02}+2m_{03}+m_{04})$ $y_1 = y_0 + \Delta y_0$

Key: (1). Solved equation.

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Table 6.10. Recording of the algorithm of solution at the second space of integration. Powered flight trajectory.

(1) Решаемое уравнение		
$\frac{du}{dt}=\frac{1}{\mu}(D-E)u$	$\frac{dp}{dt}=-\frac{g}{u}$	$\frac{dy}{dt}=up$
$v_{11}=u_1\sqrt{1+p_1^2}$ $D_{11}=\frac{1}{v_{11}}\{c_1+c_2[1-\pi(y_1)]\}$ $E_{11}=c_3\frac{\varrho(y_1)}{\varrho_{0N}}v_{11}c_x\left[\frac{v_{11}}{a(y_1)}\right]$ $\frac{1}{\mu_{11}}=\frac{1}{1-c_4t_1}$ $k_{11}=h_t\frac{1}{\mu_{11}}[D_{11}-E_{11}]u_1$	$l_{11}=h_t\left(-\frac{g}{u_1}\right)$	$m_{11}=h_tu_1p_1$
$v_{12}=\left(u_1+\frac{k_{11}}{2}\right)\sqrt{1+\left(p_1+\frac{l_{11}}{2}\right)^2}$ $D_{12}=\frac{1}{v_{12}}\left\{c_1+c_2\left[1-\pi\left(y_1+\frac{m_{11}}{2}\right)\right]\right\}$ $E_{12}=c_3\frac{\varrho\left(y_1+\frac{m_{11}}{2}\right)}{\varrho_{0N}}\times$ $\times v_{12}c_x\left[\frac{v_{12}}{a\left(y_1+\frac{m_{11}}{2}\right)}\right]$ $\frac{1}{\mu_{12}}=\frac{1}{1-c_4\left(t_1+\frac{h_t}{2}\right)}$ $k_{12}=h_t\frac{1}{\mu_{12}}[D_{12}-E_{12}]\left(u_1+\frac{k_{11}}{2}\right)$	$l_{12}=h_t\times$ $\times\left(-\frac{g}{u_1+\frac{k_{11}}{2}}\right)$	$m_{12}=h_t\times$ $\times\left(u_1+\frac{k_{11}}{2}\right)\times$ $\times\left(p_1+\frac{l_{11}}{2}\right)$

(1) Решаемое уравнение		
$\frac{du}{dt} = \frac{1}{\mu}(D-E)u$	$\frac{dp}{dt} = -\frac{g}{u}$	$\frac{dy}{dt} = up$
$v_{13} = \left(u_1 + \frac{k_{12}}{2}\right)\sqrt{1 + \left(p_1 + \frac{l_{12}}{2}\right)^2}$ $D_{13} = \frac{1}{v_{13}}\left\{c_1 + c_2\left[1 - \pi\left(y_1 + \frac{m_{12}}{2}\right)\right]\right\}$ $E_{13} = c_3 \frac{\varrho\left(y_1 + \frac{m_{12}}{2}\right)}{\varrho_{0N}}$ $\times v_{13} c_x \left[\frac{v_{13}}{a\left(y_1 + \frac{m_{12}}{2}\right)}\right]$ $\frac{1}{\mu_{13}} = \frac{1}{1 - c_4\left(t_1 + \frac{h_t}{2}\right)}$ $k_{13} = h_t \frac{1}{\mu_{13}}[D_{13} - E_{13}]\left(u_1 + \frac{k_{12}}{2}\right)$	$l_{13} = h_t \times$ $\times\left(-\frac{g}{u_1 + \frac{k_{12}}{2}}\right)$	$m_{13} = h_t \times$ $\times\left(u_1 + \frac{k_{12}}{2}\right) \times$ $\times\left(p_1 + \frac{l_{12}}{2}\right)$
$v_{14} = (u_1 + k_{13})\sqrt{1 + (p_1 + l_{13})^2}$ $D_{14} = \frac{1}{v_{14}}\{c_1 + c_2[1 - \pi(y_1 + m_{13})]\}$ $E_{14} = c_3 \frac{\varrho(y_1 + m_{13})}{\varrho_{0N}} \times$ $\times v_{14} c_x \left[\frac{v_{14}}{a(y_1 + m_{13})}\right]$ $\frac{1}{\mu_{14}} = \frac{1}{1 - c_4(t_1 + h_t)}$ $k_{14} = h_t \frac{1}{\mu_{14}}[D_{14} - E_{14}](u_1 + k_{13})$	$l_{14} = h_t \times$ $\times\left(-\frac{g}{u_1 + k_{13}}\right)$	$m_{13} = h_t \times$ $\times (u_1 + k_{13})$ $\times (p_1 + l_{13})$
$\Delta u_1 = \frac{1}{6}(k_{11} + 2k_{12} + 2k_{13} + k_{14})$ $u_2 = u_1 + \Delta u_1$	$\Delta p_1 = \frac{1}{6} \times$ $\times (l_{11} + 2l_{12} +$ $+ 2l_{13} + l_{14})$ $p_2 = p_1 + \Delta p_1$	$\Delta y_1 = \frac{1}{6} \times$ $\times (m_{11} + 2m_{12} +$ $+ 2m_{13} + m_{14})$ $y_2 = y_1 + \Delta y_1$

Key: (1). Solved equation.

$= J_2(v_\tau)\,\frac{\quad}{\tau_{0N}}$

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Table 6.11. Recording of the algorithm of solution at the first space of integration. Inactive leg.

(1) Решаемое уравнение		
$\frac{du}{dx}=-cH_\tau(y)G(v_\tau)$	$\frac{dp}{dx}=-\frac{g}{u^2}$	$\frac{dy}{dx}=p$
$v_{01}=u_0\sqrt{1+p_0^2}$ $v_{\tau01}=v_{01}\sqrt{\frac{\tau_{0N}}{\tau(y_0)}}$ $k_{01}=-h_x cH_\tau(y_0)G(v_{\tau01})$	$l_{01}=h_x\times$ $\times\left(-\frac{g}{u_0^2}\right)$	$m_{01}=h_x p_0$
$v_{02}=\left(u_0+\frac{k_{01}}{2}\right)\sqrt{1+\left(p_0+\frac{l_{01}}{2}\right)^2}$ $v_{\tau02}=v_{02}\sqrt{\frac{\tau_{0N}}{\tau\left(y_0+\frac{m_{01}}{2}\right)}}$ $k_{02}=-h_x cH_\tau\left(y_0+\frac{m_{01}}{2}\right)G(v_{\tau02})$	$l_{02}=h_x\times$ $\times\left[-\frac{g}{\left(u_0+\frac{k_{01}}{2}\right)^2}\right]$	$m_{02}=h_x\times$ $\times\left(p_0+\frac{l_{01}}{2}\right)$
$v_{03}=\left(u_0+\frac{k_{02}}{2}\right)\sqrt{1+\left(p_0+\frac{l_{02}}{2}\right)^2}$ $v_{\tau03}=v_{03}\sqrt{\frac{\tau_{0N}}{\tau\left(y_0+\frac{m_{02}}{2}\right)}}$ $k_{03}=-h_x cH_\tau\left(y_0+\frac{m_{02}}{2}\right)G(v_{\tau03})$	$l_{03}=h_x\times$ $\times\left[-\frac{g}{\left(u_0+\frac{k_{02}}{2}\right)^2}\right]$	$m_{03}=h_x\times$ $\times\left(p_0+\frac{l_{02}}{2}\right)$
$v_{04}=(u_0+k_{03})\sqrt{1+(p_0+l_{03})^2}$ $v_{\tau04}=v_{04}\sqrt{\frac{\tau_{0N}}{\tau(y_0+m_{03})}}$ $k_{04}=-h_x cH_\tau(y_0+m_{03})G(v_{\tau04})$	$l_{04}=h_x\times$ $\times\left[-\frac{g}{(u_0+k_{03})^2}\right]$	$m_{04}=h_x\times$ $\times(p_0+l_{03})$
$\Delta u_0=\frac{1}{6}(k_{01}+2k_{02}+2k_{03}+k_{04})$ $u_1=u_0+\Delta u_0$	$\Delta p_0=\frac{1}{6}\times$ $\times(l_{01}+2l_{02}+$ $+2l_{03}+l_{04})$ $p_1=p_0+\Delta p_0$	$\Delta y_0=\frac{1}{6}\times$ $\times(m_{01}+2m_{02}+$ $+2m_{03}+m_{04})$ $y_1=y_0+\Delta y_0$

Key: (1). Solved equation.

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Table 6.12. Recording of the algorithm of solution at the second space of integration. Inactive leg.

(1) Решаемое уравнение		
$\frac{du}{dx} = -cH_\tau(y)\,G(v_\tau)$	$\frac{dp}{dx} = -\frac{g}{u^2}$	$\frac{dy}{dx} = p$
$v_{11} = u_1\sqrt{1+p_1^2}$ $v_{\tau 11} = v_{11}\sqrt{\frac{\tau_{0N}}{\tau(y_1)}}$ $k_{11} = -h_x c H_\tau(y_1)\,G(v_{\tau 11})$	$l_{11} = h_x\left(-\frac{g}{u_1^2}\right)$	$m_{11} = h_x p_1$
$v_{12} = \left(u_1 + \frac{k_{11}}{2}\right)\sqrt{1+\left(p_1+\frac{l_{11}}{2}\right)^2}$ $v_{\tau 12} = v_{12}\sqrt{\frac{\tau_{0N}}{\tau\left(y_1+\frac{m_{11}}{2}\right)}}$ $k_{12} = -h_x c H_\tau\left(y_1+\frac{m_{11}}{2}\right)G(v_{\tau 12})$	$l_{12} = h_x \times$ $\times\left[-\frac{g}{\left(u_1+\frac{k_{11}}{2}\right)^2}\right]$	$m_{12} = h_x \times$ $\times\left(p_1+\frac{l_{11}}{2}\right)$
$v_{13} = \left(u_1 + \frac{k_{12}}{2}\right)\sqrt{1+\left(p_1+\frac{l_{12}}{2}\right)^2}$ $v_{\tau 13} = v_{13}\sqrt{\frac{\tau_{0N}}{\tau\left(y_1+\frac{m_{12}}{2}\right)}}$ $k_{13} = -h_x c H_\tau\left(y_1+\frac{m_{12}}{2}\right)G(v_{\tau 13})$	$l_{13} = h_x \times$ $\times\left[-\frac{g}{\left(u_1+\frac{k_{12}}{2}\right)^2}\right]$	$m_{13} = h_x\left(p_1+\frac{l_{12}}{2}\right)$
$v_{14} = (u_1 + k_{13})\sqrt{1+(p_1+l_{13})^2}$ $v_{\tau 14} = v_{14}\sqrt{\frac{\tau_{0N}}{\tau(y_1+m_{13})}}$ $k_{14} = -h_x c H_\tau(y_1+m_{13})\,G(v_{\tau 14})$	$l_{14} = h_x \times$ $\times\left[-\frac{g}{(u_1+k_{13})^2}\right]$	$m_{14} = h_x(p_1+l_{13})$
$\Delta u_1 = \frac{1}{6}[k_{11}+2k_{12}+2k_{13}+k_{14}]$ $u_2 = u_1 + \Delta u_1$	$\Delta p_1 = \frac{1}{6}\times$ $\times(l_{11}+2l_{12}+$ $+2l_{13}+l_{14})$ $p_2 = p_1 + \Delta p_1$	$\Delta y_1 = \frac{1}{6}\times$ $\times(m_{11}+2m_{12}+$ $+2m_{13}+m_{14})$ $y_2 = y_1 + \Delta y_1$

Key: (1). Solved equation.

The first and second groups of initial conditions are mutually connected and are determined by the type of ballistic task. For example, during the solution of the first (straight line) problem of external ballistics for the unguided rocket on powered flight trajectory it is necessary to know: initial mass - m_0, thrust P_0 and its change in height/altitude or the flow rate per second of mass -$|\dot{m}|$ the nozzle exit area - S_a, the significant dimension of rocekt (TRS) - bore or the area of midsection, aerodynamic coefficients - depending on the formulation of the problem, the operating time of engine - t_k, the initial parameters of motion - v_0, θ_0, x_0, y_0. For the calculation of the inactive leg and trajectory of artillery shell, it suffices to know weight Q_k, size/dimensions, aerodynamic characteristics and the initial conditions of motion. During the generalized solutions of ballistic problems for the projectiles of terrestrial artillery, is sufficient to know the ballistic coefficient of c and the initial conditions of motion v_0 and θ_0. As a rule, in the initial stages of the design of initial data proves to be insufficiently for complete performance calculation of motion. For example, for a ground-based rocket system can be at first assigned only weight head (combat) of part, maximum and minimum firing distance. I this case the missing for calculations values are determined in the process of ballistic design.

The third group of initial data is necessary during the solution of any ballistic task and their concrete/specific/actual enumeration depends on the assumptions, accepted with comparison of differential equations of motion.

Upon sufficiently strict setting the potential of the force of gravity and the acceleration of gravity can be determined by formulas (2.15) and (2.21). Values $a, \frac{\rho}{\rho_{0N}}, \pi(y), H(y)$ are the functions of height/altitude (coordinate y).

Aerodynamic characteristics are criterial and functional dependences (§ 4, chapter II). The named functions are determined by the state of the atmosphere and by the form of flight vehicle, are assigned they in the form of curve/graphs or tables. Since the tables of the numerical values of function and its curve/graphs can be made up one according to other, the similar functions accepted to call tabular. Tabular functions are introduced in ETsVM or in the form of the approximating polynomials or directly by table.

Programming with the use of the tabular assigned functions has some specific special feature/peculiarities. Prior to the beginning of calculations, the table of the values of function is introduced

into storage unit.

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During storage of the tabular assigned function in the "memory" of machine in main program, is included the subprogram for the selection of the corresponding value of function with the aid of interpolation. If table completely cannot be placed in the "memory" of machine, then it "coagulate", i.e., they enlarge tabular space. Between the assemblies of interpolation, tabular function is determined from the approximating polynomials. Consequently, into the "memory" of machine together with the tabular values of function must be introduced the coefficients of the approximating polynomials. This method, besides the large storage capacity, requires the considerable expenditures of machine time.

Our opinion the preference should return to the method of the introduction of tabular functions with the aid of the previously composed approximating polynomials. During the application/use of this method the interval of the variation in the independent variable they divide/mark off on several finite intervals. For each of them, they replace tabular function by certain polynomial, close in its form to function on the section in question. The degree of each polynomial and its coefficients are selected so that the values of

polynomials would differ from the appropriate values of function for the value, not exceeding the maximally permissible error in approximation. The important condition of the correctness of the assignment to function as a whole during the use of principle of piecewise approximation is equality the values of function and its first-order derivative at the end-points of two adjacent intervals. With the approximation of functions in the tasks of the theory of flight, usually are utilized the polynomials whose degree does not exceed the third. The polynomial of this form has the following form of recording:

$$y = y_0 - (y_0 - y_1)\frac{(x_0 - x)^2(3x_1 - x_0 - 2x)}{(x_1 - x_0)^3} - \left[\left(\frac{dy}{dx}\right)_0 (x_1 - x) + \left(\frac{dy}{dx}\right)_1 (x_0 - x)\right]\frac{(x_0 - x)(x_1 - x)}{(x_1 - x_0)^2}. \quad (6.50)$$

where y_0 and y_1 - value of function at the end/leads of the section; $(dy/dx)_0$ and $(dy/dx)_1$ - derivatives at the same points.

If the immediate determination of the values of derived at end/leads cuts is impossible, the calculation of derivatives at end-points can be made with the aid of the method of numerical differentiation.

The coefficients of the approximating polynomial are calculated previously even in the process of the preparation of task for solution by ETsVM. During the application/use of a method of the approximating polynomials instead of the introduction into the

"memory" of the machine of large in volume tables and programs for calculating the values of functions, it is necessary to introduce only small tables of the coefficients of the polynomials indicated.

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The jump if not operation, on which is realized the selection of the corresponding groups of these coefficients for calculating function value within the limits of the concrete/specific/actual interval of independent variable, is included in the main program of the solution of problem.

§ 3. Use of the electronic analog computers for the solution of the problems of the external ballistics.

The electronic analog computers continuously reproduce the physical process, which interests researcher, in the form of voltages. This analogy is reached by the fact that the process, which occurs in electrical circuit, collected in machine, is described by the same differential equations, as examine/considered. Change in different electrical values, in that of numbers and voltages, to easy write and to obtain in the form of curve/graphs. The interpretation of curve/graphs with the application/use of corresponding scales makes it possible to establish/install not only qualitative picture

of phenomenon, but also the numerical values of different characteristics of process.

Analog computers are structural type models. They represent by themselves the totality of the separate units each of which is intended for accomplishing of one or several elementary mathematical process/operations.

Function generators of analog computers they are:

- units of operational amplifiers - BOU;

- units of constant coefficients (scale dividers) - MD;

- units of variable coefficients - BPK;

- functional boxes - FB;

- units of product (multiplication) - BU.

Let us examine briefly the designation/purpose of each of the units indicated.

The units of operational amplifiers occupy in machine the

central place. Their basic cell/elements are three-stage (as a rule) dc amplifiers (UPT) with high amplification factor. BOU fulfill the process/operations of algebraic summation, integrations, differentiations and inversions (multiplication by minus unit).

During the solution of problems, frequently appears the need for the multiplication of functions by certain constant coefficient of k < 1. This process/operation is realized with the aid of two- or the 3-decade dividers of voltage (units of constant coefficients).

The units of variable coefficients provide the possibility of obtaining the dependence

$$U_{\text{вых}}=ka(t)\,U_{\text{вх}},$$

where $U_{\text{вых}}$ - output voltage;

$U_{\text{вх}}$ voltage input;

k - constant coefficient;

$a(t)$ - a variable coefficient.

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The realizable with the aid of the unit of variable coefficients function is divided according to argument into several (to 100) intervals and after its stepped approximation and passage to machine variables is unsoldered on the panel of the unit in question. Figure 6.2 depicts an example of this reorganization of function.

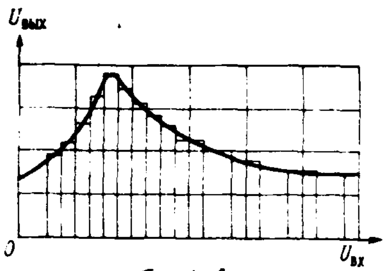
Functional boxes make it possible to assign the known previously dependence y=f(x). Wide application found the function generators, made on diode schematics. Curved (Fig. 6.3), subject to reproduction with the aid of functional box, is approximated by the linear cuts, which have the specific angle of the slope with respect to the axis of abscissas, and the obtained linear cuttings off of function then they are reproduced with the aid of diode cell/elements.

Finally, the units of product are intended for accomplishing of the process/operations of multiplication (division). The product of two functions at the output of unit (BU) is represented in the form

$$U_z = \frac{U_x U_y}{100}.$$

During the solution of problem, separate units are connected in the specific sequence by special switching system. A sequence of connection, and also a quantity of units depend on character and complexity of the process being investigated.

All analog computers are divided into two class: linear and nonlinear. In the composition of linear machines, enter only the units (BOU), (MD) and (BPK). Nonlinear machines, besides the usual cell/elements of linear models, have also units, which make it possible to reproduce different nonlinear functions and to make nonlinear process/operations - units (FE) and (BU). Giving to linear machine nonlinear units, it is possible also on usual linear model to solve nonlinear differential equations.



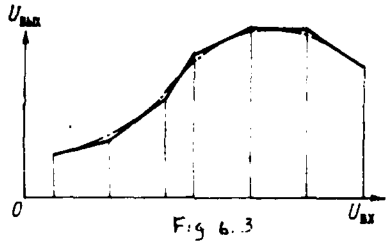


Fig. 6.2. Stepped approximation of function.

Fig. 6.3. Approximation of functions with the aid of linear cuts.

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Without possessing of high accuracy (error in the calculation for the majority of analog computers 3-10c/c), analog computers make it possible to reveal the qualitative picture of phenomenon. They are irreplaceable during the investigations, connected with the study of the effect of different perturbation factors, which act on rocket in flight (see Chapter XI). The electronic analog computers can work in natural time scale, which provides the possibility of their use together with real controlled systems whose work does not yield to precise analytical description (autopilot, control-drive, etc.).

The study of spatial motion of the guided missile in complete setting (with least possible number of assumptions) in analog

computers represents by itself complex problem. In a series of the cases, it is not possible to solve two-dimensional problem within the limits of the assumptions, generally accepted during numerical integration. This is explained by the fact that the aerodynamic forces and torque/moments nonlinear depend on many variables - α, β, v, etc. The known from the mathematical theory of electron analogues methods, which make it possible to solve problems with the functions of many variables, also are very difficult during practical performance. Therefore the solution of the complex problems of the theory of flight it is more expedient to carry out in digital electronic computers. On models are solved the problems with the supplementary assumptions, determined by the special feature/peculiarities of the concrete/specific/actual machine, planned for use.

3.1. Determining the motion characteristics of the center of mass of the unguided rocket on powered flight trajectory.

Let us take already known to us system (3.76), which describes the flat/plane axial motion of the center of mass of the rocket:

$$\frac{du}{dt}=\frac{m_0}{m}(D-E)u; \qquad \frac{dp}{dt}=-\frac{g}{u};$$

$$\frac{dy}{dt}=up; \qquad \frac{dx}{dt}=u,$$

where

$$D=\frac{1}{v}\left\{\frac{P_0}{m_0}+\frac{S_a p_{0N}}{m_0}\left[1-\pi(y)\right]\right\};$$

$$E=cH(y)G(v_\tau).$$

Assuming subsequently to utilize for solution simplified linear simulator ~~lmu~~ LMU-1, we decrease a number of nonlinearity in equations and, consequently, also a quantity of nonlinear units in schematic.

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For the concrete definition of the solved task as an example, let us conduct calculation in connection with the following hypothetical initial conditions

$$d=1{,}8\ \text{м};\ m_0=960\ \text{кг}\cdot\text{с}^2/\text{м}^{(1)};\ |\dot{m}|=16\ \text{кг}\cdot\text{с/м}^{(2)};\ t_к=20\ \text{с}^{(3)};$$
$$P_0=34\,000\ \text{кг}^{(4)};\ S_a=0{,}51\ \text{м}^2;\ x_0=2100\ \text{м};\ y_0=2200\ \text{м};$$
$$\theta_0=50°;\ i_{43}=1{,}2.$$

Key: (1). kg•s²/m. (2). kg•s/m. (3). s. (4). kg.

Taking into account that under conditions of an example dependence m_0/m (t) bears the weakly expressed curvilinear character, let us replace it with the linear function μ(t), presented in Fig. 6.4.

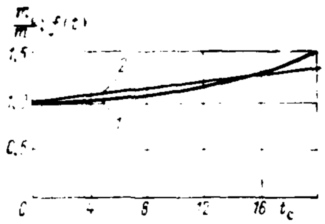
Product (D-E) u is the function of many variables. Taking into account possibility ~~lmu~~ LMU-1 and the qualitative character of investigation, let us present it simply as function only of of one value and will designate Φ(t). Being oriented toward the data of the

trajectories, close in the basic parameters, let us accept for calculating the function $\phi(u)$

$$H(y)\approx H(y_{cp})\approx 0{,}5;\quad \pi(y)\approx\pi(y_{cp})\approx 0{,}5.$$

The neutral flight path angle let us accept $\theta\approx\theta_{cp}\approx 43°31'$. Drag let us calculate with the use of the standard function G(v) within the limits of the expected change in the horizontal projection of speed 300-860 m/s. Plotted function $\phi(u) = (D-E)u$ and that approximate it in unit BE broken line are shown on Fig. 6.5. Thus previously let us calculate and approximate function $f(u)=g/u$, the curve/graph by which is shown on Fig. 6.6. Taking into account the introduced simplifications and designations, basic system (3.76) will take the form

$$\frac{du}{dt}=f(t)\varphi(u);\quad \frac{dp}{dt}=-f(u);\quad \frac{dy}{dt}=up;\quad \frac{dx}{dt}=u. \quad (6.51)$$



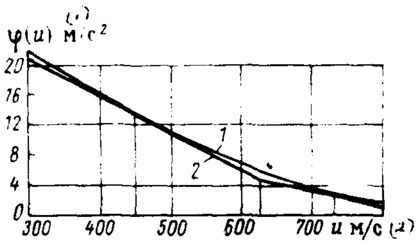


Fig. 6.4. The approximation of time function m_0/m (t): 1 - calculated curve; 2 - linear approximating dependence.

Fig. 6.5. Approximation of function $\phi(u)$: 1 - calculated curve; 2 - broken line approximating.

Key: (1). m/s². (2). s.

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In accordance with the principle of the operation of simulator, the differential equations, which describe the electrical process, which occurs in it during simulation, must be the same as the differential equations, which describe real process. Let us write machine system of equations:

$$\left.\begin{array}{ll} \frac{dU_u}{d\tau}=k_1U_{f(t)}U_{\varphi(u)}; & \frac{dU_p}{d\tau}=-k_2U_{f(u)}; \\ \frac{dU_y}{d\tau}=k_3U_uU_p; & \frac{dU_x}{d\tau}=k_4U_u. \end{array}\right| \quad (6.52)$$

Here U_u, $U_{f(t)}$, $U_{\varphi(u)}$, $U_{f(u)}$, U_p, U_y, U_x - the voltages, which characterize on the appropriate scale of the functions, marked in index; k_1, k_2, k_3 and

k_4 - generalized scale factors.

The parameters of real process let us connect with voltages through the scales

$$\left.\begin{array}{ll} U_u=\mu_u u; & U_{f(u)}=\mu_{f(u)} f(u); \\ U_{f(t)}=\mu_{f(t)} f(t); & U_y=\mu_y y; \\ U_{\varphi(u)}=\mu_{\varphi(u)}\varphi(u); & U_x=\mu_x x; \\ U_p=\mu_p p; & \tau=\mu_t t. \end{array}\right| \quad (6.53)$$

Substituting (6.53) and (6.52) and grouping all scale factors in the right sides of the equations, we will obtain the generalized coefficients, which consider the relationship/ratios of the scales

$$\left.\begin{array}{ll} k_1=\dfrac{\mu_u}{\mu_t\mu_{f(t)}\mu_{\varphi(u)}}; & k_2=\dfrac{\mu_p}{\mu_t\mu_{f(u)}}; \\ k_3=\dfrac{\mu_y}{\mu_t\mu_u\mu_1}; & k_4=\dfrac{\mu_x}{\mu_t\mu_u}. \end{array}\right| \quad (6.54)$$

On the basis of the operating voltage of amplifiers, equal to 100 V, the initial conditions of task and expected changes in the values, entering the systems of equations (6.52), moreover the following values of scale factors: $\mu_t=1$ V/s (problem is solved on the full scale of time); $\mu_u=0.1$ V•s/m; $\mu_p=50$ V; $\mu_{f(t)}=1$ V; $\mu_{f(u)}=250$ V•s/1; $\mu_{\varphi(u)}=$ V•s²/m; $\mu_x=\mu_y=0.001$ V/m or 1 V/km.

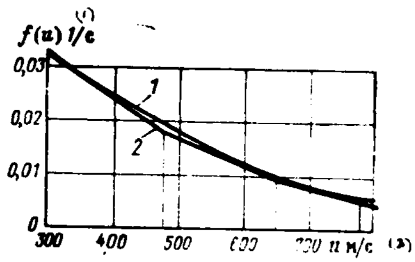

Fig. 6.6. The approximating function f(u): 1 - calculated curve; 2 - broken line approximating.

Key: (1). 1/s. (2). m/s.

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The generalized dimensionless coefficients in system (6.52) in this case will be equal to: k_1=0.1; k_2=0.2; k_3=0.0002; k_4=0.01.

The system of equations, converted to set in machine, takes the following form:

$$\left.\begin{array}{ll} \frac{dU_u}{dt}=0,1U_{f(t)}U_{\varphi(u)}; & \frac{dU_p}{dt}=-0,2U_{f(u)}; \\ \frac{dU_y}{dt}=0,0002U_uU_p; & \frac{dU_x}{dt}=0,01U_u. \end{array}\right\} \quad (6.55)$$

The solution of the obtained system of equations can be realized in standard linear analog computer LMU-1, by which must be given four nonlinear units (2 units FE and 2 EU). The block diagram which must be collected for the solution of problem, is represented in Fig. ~~to~~ 6.7.

Nonlinear units (FE and BU) are depicted in the diagram by rectangles, the units of constant coefficients are depicted by small circles and the units of operational amplifiers are depicted in the form of triangles with the indicator of the made by this unit process/operation. For example, mark shows that the operational amplifier works as integrating unit; mark - as inverting unit; mark - as summing unit, etc. Numbers on triangles correspond to the numbers of operational amplifiers in the setting field of machine.

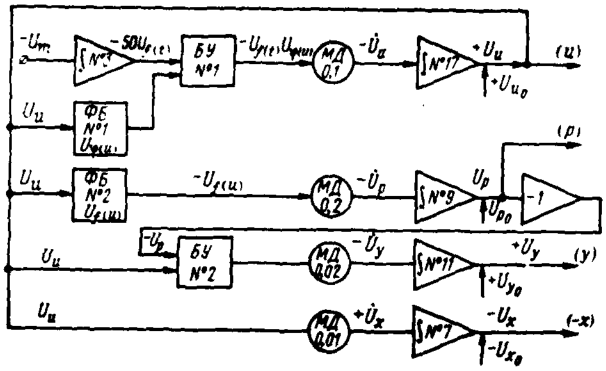

Fig. 6.7. Block diagram of the set of task for machine solution.

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For an example of the value of the removed from machine voltages in question are placed in table by 6.13. There are placed the values of motion characteristics, converted along voltages taking into account scale of coefficients. According to the results of the solution of problem, are constructed the curve/graphs (Fig. 6.8).

Table 6.13.

№ БОУ	№ 17		№ 9		№ 11		№ 7	
t, c (1)	U_u, в (2)	u, м/с (3)	U_p, в (2)	p	U_y, в (2)	y, м	U_x, в (2)	x, м
0	32	320	60	1,20	2,2	2200	2,1	2100
5	44	440	55	1,10	3,8	3800	3,4	3400
10	56	560	50	1,00	5,9	5900	5,0	5000
15	66	660	47	0,94	8,2	8200	7,2	7200
20	74	740	44	0,88	11,5	11500	10,0	10000

Key: (1). s. (2). V. (3). m/s.

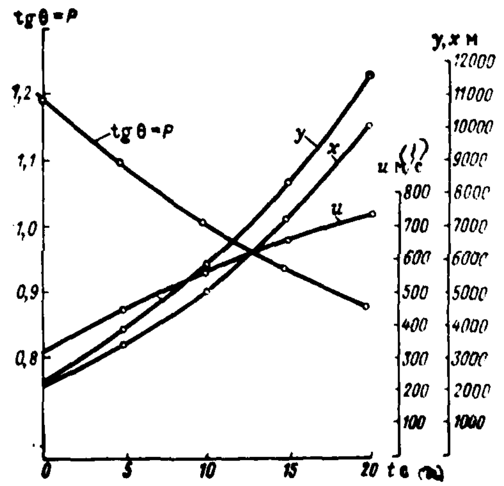

Fig. 6.8. Curve/graphs of results of machine solution.

Key: (1). m/s. (2). s.

3.2. Study of the motion of the longitudinal axis of the fin-stabilized projectile of constant mass relative to the center of mass.

Let us examine the motion of projectile relative to the center of mass in vertical plane on inactive leg, considering that the motion characteristics of the center of mass are known.

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Let the center of mass move rectilinearly with small change in altitude so that mass air density can be accepted as constant, and the flight path angle to the horizon θ - equal relative to the center of mass it is possible to write in the form

$$J_{z_1}\ddot{\vartheta}=M_{\text{в}}+M_{\text{ст}}+M_{\text{д}}, \qquad (6.56)$$

where, besides known stabilizing moment ($M_{\text{ст}}$) and damping moment ($M_{\text{д}}$), is introduced another moment of perturbing forces $M_{\text{в}}$. Let us assume that this torque/moment acts during small time interval. During the action of projectile, similar torque/moment always can be by the consequence of any of the short-term acting reason, for example, for wind gust. Utilizing formulas (2.82) and (2.86), let us write:

$$M_{\text{ст}}=-S\frac{\varrho v^2}{2}l\,|m^{\alpha}_{z_1}|\,\alpha;\quad M_{\text{д}}=-S\varrho v l^2\,|m_{\text{д}}|\,\dot{\alpha}.$$

Let us designate

$$a_1=\frac{S\varrho l^2\,|m_{\text{д}}|}{J_{z_1}}\ 1/\text{м};\quad a_2=\frac{S\varrho l\,|m^{\alpha}_{z_1}|}{2J_{z_1}}\ 1/\text{м}^2;$$

$$a_3=\frac{M_{\text{в}}}{J_{z_1}}\ 1/\overset{(1)}{\text{сек}^2};\quad f_1(t)=v;\quad f_2(t)=v^2.$$

Key: (1) 1/s².

Since in our case $\theta=0$, then $\vartheta=\alpha$ and $\dot{\vartheta}=\dot{\alpha}$. Taking into account resulting expressions for torque/moments and designations, equation (6.56) will take the form

$$\ddot{\vartheta}+a_1 f_1(t)\dot{\vartheta}+a_2 f_2(t)\vartheta=a_3.$$

As a result we will obtain the linear differential second order equation with variable coefficients and constant right side, which describes the process of oscillations. It cannot be solved in our case through elementary functions, but it can be solved in analog computer.

For convenience in the solution, let us replace the obtained second order equation with equivalent system of two first-order equations, bearing in mind that $\dot{\vartheta}=\omega_z$, where ω_z - angular velocity of the plane oscillation of axis of rocket relative to the center of mass:

$$\dot{\omega}_z+a_1 f_1(t)\omega_z+a_2 f_2(t)\vartheta=a_3;\quad \dot{\vartheta}=\omega_z. \qquad (6.57)$$

Page 243. In the case in question the system of differential equations, which describes the process, realized by electron analogue, must take this form:

$$\frac{dU_{\omega_z}}{d\tau}+k_1 U_{f_1(t)}U_{\omega_z}+k_2 U_{f_2(t)}U_{\vartheta}=k_3,$$
$$\frac{dU_{\vartheta}}{d\tau}=k_4 U_{\omega_z}, \qquad (6.58)$$

where U_{ω_z}, $U_{f_1(t)}$, $U_{f_2(t)}$, U_{ϑ} - value of the voltages, characteristic on the

appropriate scale values ω_z, $f_1(t)$, $f_2(t)$, θ.

k_1, k_2, k_3, k_4 - coefficients, which consider the relationship/ratios of scales.

Let us present voltages through real values and the corresponding scale factors:

$$U_{\omega_z}=\mu_{\omega_z}\omega_z;\quad U_{f_1(t)}=\mu_{f_1(t)}f_1(t);$$
$$U_{f_2(t)}=\mu_{f_2(t)}f_2(t);\quad U_\theta=\mu_\theta\theta;\quad \tau=\mu_t t,$$

where μ_{ω_z} V·s/1; $\mu_{f_1(t)}$ V·s/s; $\mu_{f_2(t)}$ V·s/s²;

μ_θ V/1 - corresponding scales on voltages;

μ_t V/s - time scale during passage from real process to that been simulated.

The written transfer formulas let us introduce into system of equations and let us divide the first equation on $\frac{\mu_{\omega_z}}{\mu_t}$, and the second on $\frac{\mu_\theta}{\mu_t}$:

$$\frac{d\omega_z}{dt}+k_1\mu_{f_1(t)}\mu_t f_1(t)\,\omega_z+\frac{k_2\mu_{f_2(t)}\mu_\theta\mu_t f_2(t)\,\theta}{\mu_{\omega_z}}=k_3\,\frac{\mu_t}{\mu_{\omega_z}};$$
$$\frac{d\theta}{dt}=k_4\,\frac{\mu_{\omega_z}\mu_t}{\mu_\theta}\,\omega_z. \qquad (6.59)$$

So that equation (6.59) would be the same, as equation (6.57), it is necessary to calculate the coefficients through the selected scales. It is necessary to ensure the equalities

$$k_1=\frac{a_1}{\mu_{f_1(t)}\mu_t}\ 1/B^2;\qquad k_2=\frac{a_2\mu_{\omega_z}}{\mu_{f_2(t)}\mu_\vartheta\mu_t}\ 1/B^2;$$

$$k_3=\frac{a_3\mu_{\omega_z}}{\mu_t}\ 1/1;\qquad k_4=\frac{\mu_\vartheta}{\mu_{\omega_z}\mu_t}\ 1/B.$$

Key: (1). $1/V^2$. (2). 1/V.

The numerical values of scales are selected on the basis of the special feature/peculiarities of machine so that operating output potentials of amplifiers would not exceed ±100 V.

Having additionally designated

$$U_{f_{11}(t)}=k_1U_{f_1(t)};\quad U_{f_{21}(t)}=k_2U_{f_2(t)}$$

and after replacing in formula (6.58) infinitesimal values by low final, we will obtain the system of equations, directly compose/collected in the machine

$$\frac{\Delta U_{\omega_z}}{\Delta\tau}+U_{f_{11}(t)}U_{\omega_z}+U_{f_{21}(t)}U_\vartheta=k_3;$$

$$\frac{\Delta U_\vartheta}{\Delta\tau}=k_4U_{\omega_z}. \qquad (6.60)$$

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System of equations is very simple and can be solved in any small analog computer, for example, on widespread Soviet small universal model MN-7.

The block diagram which must be selected for the solution of the

described problem, is given to Fig. 6.9. Independent variable during solution is time; the voltage, proportional to time, is introduced into schematic from amplifier No 8. Complete output potential from amplifier U_8 at the end of the process must not exceed 100 V; therefore it is necessary previously at least to tentatively know the duration of the expected process for the selection of time scale during simulation.

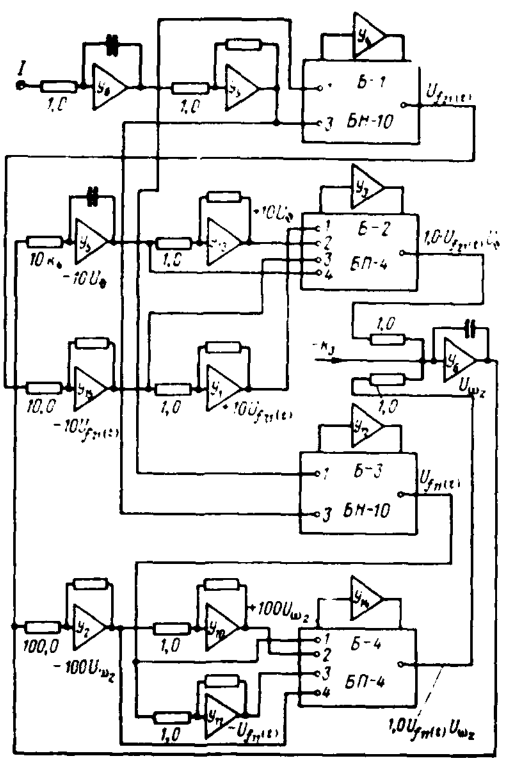

Fig. 6.9. Block diagram, comprised on model MN-7, for the solution of the problem of the oscillations of axis of rocket.

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Let, for example, the duration of process 1 s, a μ_t =100 V/s, then 0.01 from real process will correspond the voltage in 1 v; 0.02 s - 2 v and so forth. Through 1 with voltage will be equally to 100 v. After amplifier U_8 the voltage is inverted by amplifier U_9 and is

supplied to the units of nonlinearity B-1 and B-3, providing obtaining functions $U_{f_1(t)}$ and $U_{f_2(t)}$. From the outputs of the units of multiplication B-2 and B-4 into schematic, enter voltages $U_{f_1(t)} \cdot U_\theta$ and $U_{f_2(t)} U_{\omega_z}$. In this case, it is necessary to remember that the construction of the unit of multiplication is such, that during the supplying to its entry of two factors at output it is obtained the product one hundred times less. For obtaining the complete value of product, it is necessary to increase factors so that the product from the coefficients of an increase would be equal to 100. In the diagram (Fig. 6.9) to unit B-2 are supplied voltages with 10 U_θ and 10 $U_{f_1(t)}$, while to unit B-4, - voltage 100 U_{ω_z} and $U_{f_2(t)}$. From the units of the multiplication of product $U_{f_1(t)} U_\theta$ and $U_{f_2(t)} U_{\omega_z}$ they are transferred to the integrating amplifier U_6. Here through the time relay is supplied the voltage, which corresponds to the perturbation torque/moment and determined by the right side of equation (6.60), i.e., with value k_3. The time relay works in programmed conditions/mode depending on the character of the action of torque/moment.

Voltage U_{ω_z} from the output of amplifier U_6 is supplied in two directions: in the unit of multiplication B-4 for obtaining product $U_{f_2(t)} U_{\omega_z}$ and in the integrating amplifier U_5 whose output voltage is equal to $-10\,U_\theta$. Subsequently voltage $-10\,U_\theta$ after inversion is supplied as factor to the unit of multiplication B-2.

Amplifiers U_3 and U_4, U_{12} and U_{14} provide the work of nonlinear units in accordance with the construction of machine itself. The remaining amplifiers, shown in the diagram (Fig. 6.9) are special not not stipulated, are intended for the inversion of voltages.

The value that removed from amplifier U_5 of the voltage, which corresponds to the expressed to scale value ϑ, is record/written either with the aid of electron oscillograph and photo attachment or with the aid of loop oscillograph.

Figures 6.10 shows the example of recording $U_\vartheta=f(t)$. In an example the voltage, which corresponds to the perturbation torque/moment and determined by value k_3, was switched off at the moment of the first maximum curved $U_\vartheta=f(t)$. As can be seen from Fig. 6.10, after the break-down of the perturbation torque/moment of oscillating the axis of rocket, rapidly they attenuate. After deciphering recording in the scale of ordinates. easy to obtain pitching in the time

$$\vartheta(t)=\frac{\mu_{\text{осц}}U_\vartheta(t)}{\mu_\vartheta},$$

where $\mu_{\text{осц}}$ - scale of oscillogram.

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After the interpretation of oscillogram according to the character of pitching ϑ and in its maximum value, it is possible to

judge stability of motion of object and the correctness of established/installed during aerodynamic design of coefficients m_z and m_z^{α} placed during solution on model in a_1 and a_2. The solution of the more complex problems of the theory of flight, it is logical, will be provided by more complex block diagrams. Used in this case models must possess greater possibilities than the described by us simplest model.

During the simulation of the complex phenomena, connected with the flight of the guided missiles, it is not always possible to describe sufficiently reliably the work of the separate links of control or the work of any instrument with mathematical dependences, by accurately reflecting the physical picture of phenomenon. In this case is utilized the property of electron analogues to work in combination with the real equipment, included in the common/general/total block diagram of the solution of problem on model.

During the use of electron analogues, it is necessary to remember that the solution of the various kinds of tasks is accompanied by the errors, sometimes sufficiently considerable. Therefore, solving any problem of the theory of flight, it is necessary to rate/estimate the accuracy of the obtained solution. The methods of estimation of error in the implementation of different

dynamic systems in analog computers are described in specialized literature.

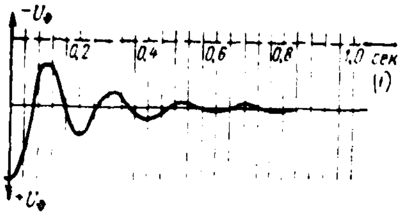

Fig. 6.10. Example of the recording of solution regarding pitch angle on electron analogue.

Key: (1). s.

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Chapter VII

ANALYTICAL AND THE TABULAR METHODS OF THE SOLUTION OF THE PROBLEMS OF EXTERNAL BALLISTICS.

The examined in chapter VI numerical methods make it possible to integrate any of the given above systems of differential equations with the necessary accuracy. At the same time these methods are very laborious and more expedient entire to utilize them for conducting precise ballistic calculations in the final stages of the design of artillery or missile complex. In the initial stages of the design of requirement for accuracy to ballistic calculations, it is below. For similar calculations widely are applied different analytical and tabular methods of the solution of the problems of external ballistics.

The analysis of systems of equations, which describe the motion of rockets and projectiles in the dense layers of the atmosphere, shows that completely these systems cannot be solved analytically. Even protozoan of these systems - system of equations (5.10) with argument θ, in which together must be integrated only two equations,

it cannot be without simplifications solved in quadratures, since it includes the tabular function F(v) and in it it cannot be divided alternating/variable u and 6. Consequently, for the possibility of obtaining the simplified in comparison with numerical methods solutions it is necessary to respectively simplify the systems of differential equations.

Depending on practical advisability in addition to the already examined simplifications (separation of motion into longitudinal and lateral, forward/progressive and rotary) are applied these or other the assumptions. For example, during the trajectory calculation of the ballistic missiles, intended for a firing to very long range, for the larger part of the trajectory it is possible not to consider the air resistance, but in this case, is necessary the account to the variability of gravitational force. For the relatively small distances when virtually entire/all trajectory passes in the dense layers of the atmosphere, it is possible to take $\bar{g}$=const, but the disregard of the air resistance leads to large errors.

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Depending on the character of the adopted assumptions analytical methods of the solution of the problems of external ballistics can be divided into four basic groups.

To the first group one should relate the methods of the solution of systems of equations in which the terms, which consider resistance of medium, are lowered.

To the second group let us relate the methods, in which the air resistance is considered in equations in the form of any analytic function, which reflects the dependence between the air resistance and the velocity of the motion of the center of mass. In similar solutions, as a rule, are considered only one to two aerodynamic characteristics, for example, only drag or drag and lift, communication/connection between which is given in the form of special function - the so-called polar of flight vehicle, for which is selected analytical expression.

To the third group let us relate the methods, instituted on the artificial transformations of fundamental differential equations of motion, which make it possible to divide variables. However, separation of variables not always does lead to the solutions in final form, which make it possible to have numerical result. In many instances it proves to be necessary the nontaken integrals to represent in the form of tables or curve/graphs. Similitude is not always sufficiently strict and requires the introduction of the

matching coefficients and auxiliary tables.

The four-group includes the methods of the solutions, in which previously is assigned the form of the function, which determines a change in one or the other action characteristic. For example, can be assigned the form of the dependence of the velocity of the motion of the center of mass of flight vehicle on time v(t). Last/latter methods require the supplementary assignment of the law of the resistance of medium or conducting successive approximations during the numerical calculation of the definite integrals, which contain functions from the air resistance.

Each of the methods let us use for the specific flight conditions of projectile, which approach the assumptions, accepted during the compilation of differential equations of motion. For example, neglect of the air resistance is applicable during the investigation of the flight of projectile in the rarefied layers of the atmosphere or with very low velocities of action.

§ 1. Parabolic theory.

During the motion of the projectile of constant mass in conditional plane-parallel gravitational field with $\bar{g}=\bar{g}_0$=const and the absence of the air resistance, the trajectory of projectile is

described by the system of differential equations (5.14)

$$\frac{d^2x}{dt^2}=0 \text{ and } \frac{d^2y}{dt^2}=-g,$$

solution of which is determined by the initial characteristics of flight v_0 and θ_0.

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Consecutively twice integrating, we will obtain

$$\dot{x}=c_1; \qquad x=c_3+c_1t;$$
$$\dot{y}=c_2-gt; \qquad y=c_4+c_2t-\frac{gt^2}{2}.$$

From expression $\dot{x}=v\cos\theta=c_1$ it follows that the motion of projectile in question is characterized by constancy along the trajectory of the horizontal projection of speed.

On the basis of the initial conditions $t=0$, $y_0=0$, $x_0=0$, we find for integration constants the following values:

$$c_3=c_4=0; \quad c_1=\dot{x}_0=v_0\cos\theta_0; \quad c_2=\dot{y}_0=v_0\sin\theta_0.$$

Substituting the value of constants in expressions for x and y, we will obtain the equation of trajectory in the parametric form

$$x=v_0\cos\theta_0 t; \quad y=v_0\sin\theta_0 t-\frac{gt^2}{2},$$

or, after the exception/elimination of parameter t by substitution $t=\frac{x}{v_0\cos\theta_0}$ in equation for y, in the form

$$y=x\,\mathrm{tg}\,\theta_0-\frac{gx^2}{2v_0^2\cos^2\theta_0}. \qquad (7.1)$$

The comparison of equation (7.1) with the general view of equation curved of the second order

$$Ax^2+2Bxy+Cy^2+2Dx+2Ey+F=0$$

shows that the discriminant of equation (7.1) $B^2-AC=0$ and, therefore, curve, described by equation (7.1), is parabola. Based on this, the method of solution in question frequently calls the parabolic theory of the motion of projectile.

Let us find the relationship/ratios, which make it possible to calculate the motion characteristics of projectile at the arbitrary point of parabolic trajectory. Differentiating (7.1) on x, we obtain dependence for determining the angle of the slope of velocity vector to the horizon

$$y'_x=\operatorname{tg}\theta=\operatorname{tg}\theta_0-\frac{gx}{v_0^2\cos^2\theta_0}. \qquad (7.2)$$

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The velocity of projectile in arbitrary point in the trajectory at height/altitude y let us determine from the equation of the kinetic energies

$$\frac{mv^2}{2}-\frac{mv_0^2}{2}=-mgy,$$

whence it follows

$$v=\sqrt{v_0^2-2gy}. \qquad (7.3)$$

From last/latter formula it is evident that the velocities of projectile in two points, which are located on identical height/altitude on that ascend and on descending the branches of trajectory, are identical.

The time of the motion of projectile is determined by the dependence

$$t=\frac{x}{v_0\cos\theta_0}. \tag{7.4}$$

Let us find the values of the motion characteristics of projectile in impact point C, utilizing the condition that $y_C=0$. In this case, from equations (7.1)-(7.4) we respectively obtain: complete flying range

$$x_C=\frac{v_0^2\sin 2\theta_0}{g}; \tag{7.5}$$

angle of incidence $\operatorname{tg}\theta_C=-\operatorname{tg}\theta_0$; the rate of incidence/drop $v_C=v_0$; the complete flight time

$$t_C=\frac{2v_0\sin\theta_0}{g}. \tag{7.6}$$

The obtained dependences show that for a parabolic trajectory at impact point

$$|\theta_C|=\theta_0 \text{ and } v_C=v_0.$$

Motion characteristics for peak of the trajectory will be located from condition $\operatorname{tg}\theta_S=0$ (or $\theta_S=0$).

In this case, of (7.2) it follows

$$x_S=\frac{v_0^2\sin 2\theta_0}{g}, \tag{7.7}$$

i.e.

$$x_S=\frac{1}{2}x_C.$$

Analogously from (7.4) we obtain

$$t_S=\frac{v_0 \sin \theta_0}{g}, \quad \text{i.e.} \quad t_S=\frac{1}{2}t_C.$$

Since in peak of the trajectory the comprising velocities are equal to

$$\dot{x}_S=v_0 \cos \theta_0 \text{ and } \dot{y}_S=v_0 \sin \theta_0-gt_S=0,$$

that the full speed

$$v_S=\sqrt{\dot{x}_S^2+\dot{y}_S^2}=v_0 \cos \theta_0.$$

Trajectory height y_S let us find, after substituting expression for x_C into fundamental equation (7.1),

$$y_S=\frac{v_0^2 \sin^2 \theta_0}{2g}. \qquad (7.8)$$

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Frequently, multiplying (7.8) by cofactor $\frac{2\cos\theta_0}{2\cos\theta_0}$, is reduced expression for y_S to the form

$$y_S=\frac{x_C}{4} \operatorname{tg} \theta_0. \qquad (7.9)$$

The analysis of the dependences, which determine trajectory elements in apex/vertex and at impact point, shows that the parabolic trajectory is symmetrical curve, moreover its axis of symmetry is ordinate y_S, passing through the middle of complete flying range.

During the approximate computations of trajectories in air frequently the function of density charge with height/altitude takes by average value $H(y) \approx H(y_{cp})$. As the first approximation medium altitude of trajectory can be determined, using the conclusions of the parabolic theory

$$y_{cp} = \frac{1}{x_C} \int_0^{x_C} y\,dx = \frac{1}{x_C} \int_0^{x_C} \left(x \operatorname{tg} \theta_0 - \frac{g x^2}{2 v_0^2 \cos^2 \theta_0} \right) dx.$$

After integration and transformations, we will obtain

$$y_{cp} = \frac{2}{3} y_S. \tag{7.10}$$

Parabolic theory can be applied for performance calculation of the motion of the projectiles of constant mass beyond the limits of the dense layers of the atmosphere (at height/altitudes more than 20 km). During the calculation of the relatively larger trajectory phases, it is necessary to remember that the dependences of parabolic theory are obtained without the account of bending of the earth's surface, the Coriolis acceleration and variability $\bar{g}$, calculation according to them can lead to noticeable errors. So, with distance ~500 km calculation according to the dependences of parabolic theory gives error of approximately 10%.

For the calculation of ground-based trajectories, parabolic theory can be utilized in the case of the low expected firing distances (trajectory calculation of means close combat). With $v_0 < 60$ m/s and $\theta_0 = 45°$ range error during the use of parabolic theory

comprises less than 5o/c.

§2. Elliptical theory.

The theory of the motion of the long range ballistic missiles (BRDD [(БРДД)- long-range ballistic missile]) and of global rockets on inactive leg is a special case of the "two-body problem", set-forth in classical celestial mechanics. External ballistics examines the motion of the body of constant mass in the central gravitational field of the Earth of relatively inertial coordinate system.

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In this case, the trajectories of motion represent by themselves Kepler's elliptical trajectories which are mathematically described by system of equations (5.20), that yields sufficiently to simple analytical solution. In this system the second equation expresses the law of conservation of moment of momentum in central gravitational field. Integrating it, we find the relation between the parameters of the beginning of passive section and the current parameters of the trajectory

$$c_1 = r^2\dot{\gamma} = r_н v_н \cos\theta_н = rv\cos\theta. \qquad (7.1)$$

Here $r_н = r_к$, a $v_н$ and $\theta_н$ - initial parameters of the motion of body in

absolute motion; moreover $\theta_н$ and θ - angles of the slope of vectors $\bar{v}_н$ and $\bar{v}$ to the local horizons (standards to radii $\bar{r}_н$ and $\bar{r}$ respectively).

Dependence (7.11) is called of the integral of areas. The physical sense of the introduced concept consists in the fact that the areas, swept by radius-vector for identical time intervals, are equal (Fig. 7.1).

Integrating first equation (5.20) taking into account the second equation of this system, we will obtain

$$\dot{r}^2+r^2\dot{\lambda}^2=\frac{2K}{r}+c_2, \quad (7.12)$$

whence

$$c_2=v^2-\frac{2K}{r}=v_н^2-\frac{2K}{r_н}. \quad (7.13)$$

Expression (7.13) is called the integral of energy or the integral of kinetic energies.

For two points in the trajectory

$$v_1^2-\frac{2K}{r_1}=v_2^2-\frac{2K}{r_2},$$

and after the multiplication of all terms of equation by m/2 let us have

$$\frac{mv_2^2}{2}=\frac{mv_1^2}{2}+\left(\frac{Km}{r_2}-\frac{Km}{r_1}\right),$$

i.e. kinetic energy of body in the second point is equal to kinetic energy in the first point plus a change of the potential energy of position.

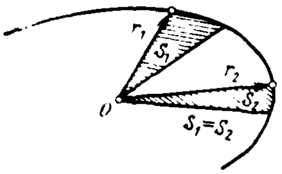

Fig. 7.1. To the concept of the integral of areas.

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Equations of the trajectory of body in central gravitational field can be obtained from equations of motion (5.20) with the exception/elimination of them of time. Substituting in (7.12) $\dot{\gamma}$ from (7.11) and bearing in mind that

$$\dot{r}=\frac{dr}{d\gamma}\cdot\frac{d\gamma}{dt}=\frac{c_1}{r^2}\cdot\frac{dr}{d\gamma},$$

we will obtain

$$d\gamma=\frac{\frac{c_1}{r^2}}{\sqrt{c_2-\frac{c_1^2}{r^2}+\frac{2K}{r}}}\,dr.$$

The final solution of this equation takes the form

$$r=\frac{p}{1-e\cos(\varphi_s-\varphi)}. \qquad (7.14$$

The obtained dependence is the equation of conic section in polar coordinates.

Dependence (7.14):

φ and φ_s - the vectorial angles, calculated off radius vector $\bar{r}_k$.

which determine respectively the position of current point in the trajectory and its apex/vertex;

$p = c_1^2/K$ the - focal parameter of conic section;

$$e=\sqrt{1+c_2\frac{c_1^2}{K^2}} \text{ - eccentricity.} \qquad (7.15)$$

For parameter p frequently utilize other, derivative, the expression

$$p = \varkappa_H r_H \cos^2\theta_H,$$

where $\varkappa_H = \frac{r_H v_H^2}{K}$ - dimensionless relation to the doubled kinetic energy to potential energy at point in the trajectory, which corresponds to the beginning of passive section.

Expression (7.15) after the substitution of values c_1 and c_2 gives formula for determining the current velocity in the function of the basic parameters of the trajectory

$$v^2=\frac{K}{r}\left[1\pm\sqrt{1-\frac{1-e^2}{\cos^2\theta}}\right].$$

For points in the trajectory, arrange/located on the focal axis of the conic section in which $\theta=0$ ($\cos\theta=1$), obtained equation takes the form

$$v^2=\frac{K}{r}(1\pm e). \qquad (7.16)$$

Dependences (7.14) and (7.16) make it possible to investigate

the effect of initial conditions for the form of trajectories and rate of the motion of flight vehicles.

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Let us examine the possible cases.

1 case: e=0.

In this case, of (7.14) it follows that the body will be circled whose equation in polar coordinates has form $r=p=r_n$. Velocity $v_1=\sqrt{K/r}$ is called circular and is the velocity which must be reported to body so that it would become the Earth satellite. In quotient, conditional, case with $r=R_3=6371$ km

$$v_{01}=7{,}906 \text{ km/s}.$$

This velocity is called orbital velocity. Introduced above parameter x_n can be expressed through the value of the circular velocity, which corresponds to radius r_n

$$x_n=\frac{v_n^2}{\frac{K}{r_n}}=\left(\frac{v_n}{v_{n1}}\right)^2.$$

It is clear that to the motion of body along circumference corresponds value $x_n=1$.

2 case: 0<e<1.

Accordingly (7.14) will occur elliptical trajectories. However, are possible two diverse variants, since in formula (7.16) before the value of eccentricity they stand two signs:

a) assuming that $v^2=\frac{K}{r}(1+e)=v_n^2$, then let us have an ellipse whose attract/tightening center coincides with the nearest, with respect to point with a velocity of v_n, focus. For the version in question this point in the trajectory - perigee. The body, driving/moving in perigee with this velocity, will be satellite, since ellipse does not intersect with the attract/tightening body;

b) if $v^2=\frac{K}{r}(1-e)=v_A^2$, then let us have an ellipse whose attract/tightening center coincides with distant focus. Velocity v_A characterizes the point, called the apogee of trajectory. In this case the trajectory can intersect with the attract/tightening body, but it can and not intersect. Boundary value $v_{A.rp}$ easily is determined, since velocity $v_{A.rp}$ must be equal to the apogeal velocity of such ellipse which in perigee will touch the surface of the Earth. Utilizing condition $v_n R_3 = v_{A.rp} r_{A.rp} = c_1$, let us substitute in it v_n and v_A and after transformations we will obtain

$$v_{A.rp}=v_1\sqrt{\frac{2R_3}{r_{A.rp}+R_3}}.$$

If $v_A>v_{A.rp}$ - ellipse it does not intersect with the attract/tightening

body;

$v_A = v_{A.rp}$ - ellipse will touch the attract/tightening body;

$v_A < v_{A.rp}$ - ellipse intersects with the body (this version corresponds to trajectories BBDD).

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For ellipses parameters $\varkappa_H < 2$.

3 case: e=1.

Trajectory is parabola. If we give to rocket parabolic velocity, it will overcome the force of gravity. From the equation of velocity (7.16) follows

$$v_{II} = \sqrt{\frac{2K}{r}} = v_I \sqrt{2};$$

with $r = R_3 = 6371$ km $v_{0II} = v_{0I}\sqrt{2} = 11.180$ km/s is called escape velocity.

To parabolic trajectory corresponds parameter $\varkappa_H = 2$.

4 case: e>1.

Accordingly (7.14) trajectory will be hyperbola; in this case $\varkappa_H > 2$

For the explanation of the examined versions, are given by Fig. 7.2., by Fig. 7.3 and by Fig. 7.4. Figures 7.2 shows the forms of the trajectories of body depending on the value of parameter $\varkappa_{11}$.

Plotted functions $\frac{v}{v_{01}}=(1\pm e)^{\frac{1}{2}}$ from (7.16), depicted in Fig. 7.3, make it possible to set the form of the trajectory of body depending on the relationship/ratio of eccentricity e and of velocity v in point in the trajectory on the focal axis of conic section.

The dependence of the form of the trajectories of the body, distant from attract/tightering center up to distance of r, from the velocity of this body v (with $\bar{v}\perp r$), is shown on Fig. ~~to~~ 7.4.

The introduction of the concept of parabolic velocity makes it possible to give still one interpretation to the term mK/r in expression for the integral of energy.

It is clear that $\frac{mK}{r}=\frac{2K}{r}\cdot\frac{m}{2}=\frac{mv_{11}^2}{2}$ - there is kinetic energy of body at point with a radius of r, necessary for the overcoming of the gravitational field of the Earth.

For constant C_2, analogously it is possible to obtain expression

$c_2 = v_n^2 - v_{nII}^2$, from which it follows that if $c_2 \geq 0$, then $r \to \infty$ and the trajectory of body is the extended curve (hyperbola or parabola); but if $c_2 < 0$, then trajectory - closed curve with a maximum radius of $r_{mp} = r_{A.ap} = \frac{2K}{c_2}$ (ellipse or circumference). Subsequently we will examine the elliptical trajectories for which $c_2 < 0$ and $\varkappa_n < 2$.

The practical application/appendix of the dependences of elliptical theory is connected with approximate solution of following basic tasks.

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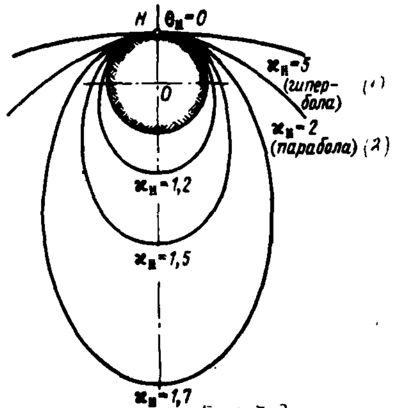

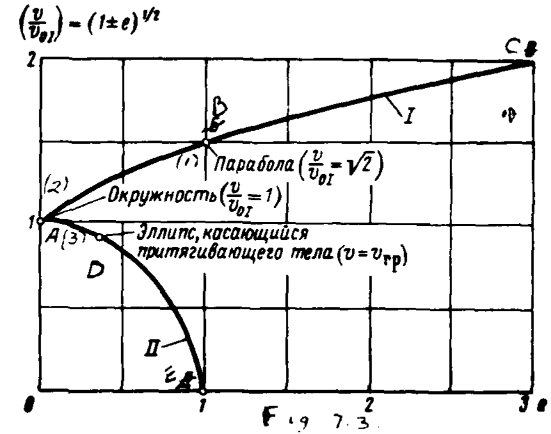

Fig. 7.2. Family of characteristic trajectories of flight at high altitudes.

Key: (1). (hyperbola). (2). (parabcla).

Fig. 7.3. Graph/diagram of dependence v/v_{01} in function of eccentricity e:

AE - corresponds to ellipses, for which attract/tightening body is lccated in near to initial point focus; BC - ccrresponds tc hyperbolas;

$II - v/v_{01} = (1-e)^{1/2}$;

AE - it corresponds to ellipses, fcr which attract/tightening body is lccated in distant from initial point fcous; DE - corresponds tc

ellipses, intersecting with attract/tightening body.

Key: (1). Parabola. (2). Circumference. (3). Ellipse, which concerns attract/tightening body.

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1. To determine parameters of motion at current point in the trajectory along known characteristics in the beginning of passive section.

2. From assigned parameters in the beginning of inactive leg to find value of complete distance.

3. To determine initial velocity $v_н$ for achievement of required flying range at known values $r_н$ and angle $\theta_н$ in the beginning of inactive leg.

4. From assigned flying range and $r_н$ to calculate optimum angle $\theta_{н\,опт}$, at which distance will be reached at minimum initial velocity $v_{н\,min}$.

5. At known values $r_н$ and $v_н$ to find value of angle of maximum range $\theta_{н\,max}$.

For real BRDD the length of the last/latter phase of trajectory - the section of the atmospheric entry - comprises less than 5o/o common/general/total flying range. Therefore assumption about the fact that the rocket moves here just as along elliptical trajectory, gives good first approximation for ranging and evaluations of error. Taking into account this, during further conclusions let us examine the flow chart, presented in Fig. 7.5, by which NTs - this is the elliptical trajectory phase.

In differential equations of motion, let us pass from independent by the variable t of independent variable ϕ.

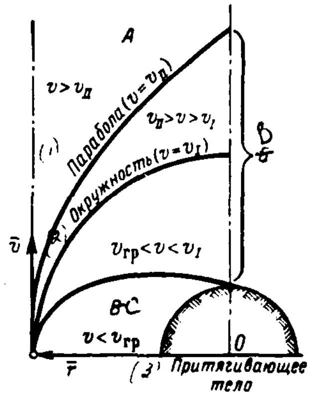

Fig. 7.4.

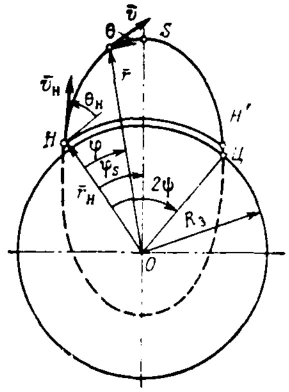

Fig. 7.5.

Fig. 7.4. The fields of possible trajectories: A - field of hyperbolas; B - field of the ellipses, which do not intersect with the attract/tightening body; C - field of the ellipses, which intersect with the attract/tightening body.

Key: (1). Parabola. (2). Circumference. (3). Attract/tightening body.

Fig. 7.5. Simplified diagram of trajectory of long range ballistic missile.

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Transformation takes the form

$$\frac{d}{dt}=\dot{\varphi}\frac{d}{d\varphi}; \quad \frac{d}{dt}=\frac{r_{н}v_{н}\cos\theta_{н}}{r^{2}}\cdot\frac{d}{d\varphi}.$$

Producing one additional replacement $\rho=1/r$, let us write the

first equation of system (5.20) for the case in question as

$$\frac{d^2\varrho}{d\varphi^2}+\varrho=\frac{1}{\varkappa_{н}r_{н}\cos^2\theta_{н}}.$$

Integration constant are located from the initial conditions

$$\varrho_{н}=\frac{1}{r_{н}};\quad \frac{d\varrho_{н}}{d\varphi}=-\frac{1}{r_{н}}\operatorname{tg}\theta_{н}.$$

The solution of the obtained equation is obvious

$$r_{н}\varrho=\frac{1-\cos\varphi}{\varkappa_{н}\cos^2\theta_{н}}+\frac{\cos(\theta_{н}+\varphi)}{\cos\theta_{н}}.$$

Producing reverse/inverse replacement ϱ on 1/r, we will obtain the equation of elliptical trajectory in the form

$$r=\frac{\varkappa_{н}r_{н}\cos^2\theta_{н}}{1-\cos\varphi+\varkappa_{н}\cos\theta_{н}\cos(\theta_{н}+\varphi)}. \tag{7.17}$$

Although this form of recording is more complex in comparison with conventional (7.14), it possesses that advantage that are included parameters $v_{н}$, $\theta_{н}$ and $r_{н}$, which characterize the position and energy of rocket at the end-point, which corresponds to the beginning of the passive phase of flight. Dependence (7.17) together with the given in the preceding/previous section relationship/ratios makes it possible to sufficient simply determine trajectory elements according to known initial data. The next problems also are reduced to the solution of the obtained previously relationship/ratios.

During the determination of the parameters of motion, which correspond to the impact point in the nose cone, it is necessary to keep in mind that target position relative to the beginning inactive leg of rocket is unambiguously determined by angular target range 2ψ. Communication/connection between angular and by the linear distance L

(undertaken over the surface of the Earth). is expressed by the relationship/ratio

$$L = 2R_3\psi, \qquad (7.18)$$

where 2ψ - an angle of the relative attitude of target/purpose in radians. A series of the numerical values of angles 2ψ and of the corresponding to them values of linear flying range L with $R_3 = 6371$ km is given in Table 7.1.

Table 7.1.

2ψ, град (1)	5	8	15	30	60	75	90	120	140	180
L, км	556	890	1668	3336	6672	8340	10008	13344	15566	20016

Key: (1). deg.

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The condition of the hit is the observance of equality $r=R_3$ when $\varphi=2\psi$. After substituting this condition in (7.17), we will obtain the equation of relation of calculated angular distance with the parameters of the beginning of the passive section

$$r_н=\frac{R_3[1-\cos 2\psi+\varkappa_н\cos\theta_н\cos(\theta_н+2\psi)]}{\varkappa_н\cos^2\theta_н}. \qquad (7.19)$$

Dependence (7.19) makes it possible to also solve the inverse problem: to determine the necessary velocity for achievement of the required distance at assigned values $\theta_н$ and $r_н$ [2]:

$$v_н=v_{01}\left[\frac{1-\cos 2\psi}{\frac{r_н}{R_3}\cos^2\theta_н-\cos\theta_н\cos(\theta_н+2\psi)}\right]^{\frac{1}{2}}. \qquad (7.20)$$

With the very long range of flight, it is possible to accept (in the first approximation,) assumption about the instantaneous combustion of the fuel/propellant of rocket ($r_н=R_3$). In this case

$$v_н=v_{01}\left[\frac{1-\cos 2\psi}{\cos^2\theta_н-\cos\theta_н\cos(\theta_н+2\psi)}\right]^{\frac{1}{2}}.$$

The analysis of the obtained simplified equation (Fig. 7.6) makes it possible to draw the conclusion that one and the same range angle 2ψ can be reached with many combinations of values $v_н$ and $\theta_н$ at

the cutoff of engine.

Practically important is the determination of the position of rocket in the elliptical trajectory with respect to elapsing of certain time interval after launching/starting. The first stage of obtaining initial dependence is connected with the integration of the momental equation of momentum along time upon consideration the dependence, adjustable by the equation of trajectory (7.17). Final formula is located as a result of sufficiently complex analytical solution [65] and takes the form

$$t=\frac{r_{\text{н}}}{v_{\text{н}}\cos\theta_{\text{н}}}\left\{\frac{\operatorname{tg}\theta_{\text{н}}(1-\cos\varphi)+(1-\varkappa_{\text{н}})\sin\varphi}{(2-\varkappa_{\text{н}})\left[\frac{1-\cos\varphi}{\varkappa_{\text{н}}\cos^2\theta_{\text{н}}}+\frac{\cos(\theta_{\text{н}}+\varphi)}{\cos\theta_{\text{н}}}\right]}+\right.$$

$$\left.+\frac{2\cos\theta_{\text{н}}}{\varkappa_{\text{н}}\left(\frac{2}{\varkappa_{\text{н}}}-1\right)^{3/2}}\operatorname{arctg}\frac{\left(\frac{2}{\varkappa_{\text{н}}}-1\right)^{\frac{1}{2}}}{\cos\theta_{\text{н}}\operatorname{ctg}\frac{\varphi}{2}-\sin\theta_{\text{н}}}\right\}. \quad (7.21)$$

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Total flying time t_C is calculated with the substitution of value $\varphi=2\psi$ into equation (7.21)

$$t_C=\frac{r_{\text{н}}}{v_{\text{н}}\cos\theta_{\text{н}}}\left\{\frac{\operatorname{tg}\theta_{\text{н}}(1-\cos2\psi)+(1-\varkappa_{\text{н}})\sin2\psi}{(2-\varkappa_{\text{н}})\frac{r_{\text{н}}}{R_3}}+\right.$$

$$\left.+\frac{2\cos\theta_{\text{н}}}{\varkappa_{\text{н}}\left(\frac{2}{\varkappa_{\text{н}}}-1\right)^{\frac{3}{2}}}\operatorname{arctg}\frac{\left(\frac{2}{\varkappa_{\text{н}}}-1\right)^{\frac{1}{2}}}{\cos\theta_{\text{н}}\operatorname{ctg}\psi-\sin\theta_{\text{н}}}\right\}. \quad (7.22)$$

The graphic representation of the dependence of flight time along

ballistic trajectory (when $r_{H}=R_{3}$) on angle θ_{H} of different range angles is represented in Fig. ~~26~~ 7.7.

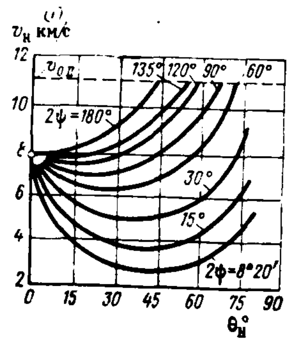

Fig 7.6.

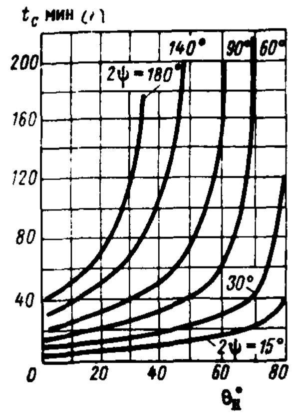

Fig. 7.7

Fig. 7.6. Dependences of the velocity in the beginning of passive section $v_н$ on angle $\theta_н$ for the different values of range angle 2ψ (when $r_н=R_3$)

Key: (1). km/s.

Fig. 7.7. Total flying time in function of angle $\theta_н$ for different range angles (when $r_н=R_3$)

Key: (1). min.

3. Approximate analytical methods of the trajectory calculation of the projectiles of constant mass.

3.1. Methods, which use analytic functions for describing the air

resistance.

The analytical methods of the trajectory calculation of the motion of the projectiles of constant mass in air usually are based on system of equations with independent variable θ (5.10). The greatest difficulties cause the determination of the velocity of the motion of the center of mass of projectile.

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Velocity is found by integrating the hodograph equation (5.9). Let us present it in this form

$$\frac{d(v\cos\theta)}{d\theta}=\frac{Ev^2}{g}, \tag{7.23}$$

where

$$E=\frac{X}{m_0 v}.$$

If we for determining of X use (2.90), then we will obtain

$$E=\frac{\varrho v S}{2m_0}c_x(M); \tag{7.24}$$

if we use (2.95), then let us have

$$E=\frac{c}{v}H(y)F(v). \tag{7.25}$$

in the first case from (7.23)

$$\frac{d(v\cos\theta)}{d\theta}=\frac{\varrho}{\varrho_{0N}}\cdot\frac{\varrho_{0N}v^3}{2m_0 g}Sc_x(M). \tag{7.26}$$

The simultaneous replacement of functions $\frac{\varrho}{\varrho_{0N}}(y)$ and $c_x(M)$ with analytical dependences does not lead to separation of variables in equation (7.26) and does not give solution in final form.

Even if to accept $c_x(M)\approx\tilde{c}_x=\text{const}$, which leads to quadratic dependence for the air resistance

$$X=\frac{S\varrho v^2}{2}\tilde{c}_x, \qquad (7.27)$$

then variables in equation (7.26) nevertheless cannot be divided, since in (7.27) remains function $\varrho(y)$.

If we utilize (7.25), then we will obtain the first equation of system (5.10) in this form

$$\frac{d(v\cos\theta)}{d\theta}=c_1\frac{vF(v)}{g}, \qquad (7.28)$$

where $c_1=cH(y)$.

Investigations show that for the approximative integration of equation (7.28) during the provision for acceptable accuracy of calculation one should take $H(y)\approx H(y_{\text{cp}})$, and the air resistance to describe one of the analytical dependences:

$$F(v)=a+bv^n, \qquad (7.29)$$

$$F(v)=Bv^n, \qquad (7.30)$$

where a, b, B, n - constant experimental coefficients.

Last/latter dependence was proposed by N. V. Mayevskiy and N. A.

Zakudskiy. The values of exponent n for different speed ranges are brought to following table.

v, m/s	from	0	240	295	375	419	550	800
	to	240	295	375	419	550	800	1000
n		2	3	5	3	2	1,70	1,55

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Coefficients B_i for each section were selected so that the function F(v) would be continuous.

Let us substitute (7.29) in (7.28) and will discover derivative on the left side of the equation

$$\frac{d(v\cos\theta)}{d\theta}=\frac{c_1 v}{g}(a+bv^n);$$

$$\frac{dv}{d\theta}=\frac{v}{g\cos\theta}(c_1 a+g\sin\theta)+\frac{c_1 b}{g\cos\theta}v^{n+1}.$$

Let us introduce substitution $v_* v^n=1$, from which by differentiation we will obtain

$$dv=-\frac{dv_*}{nv^{-n-1}}. \qquad (7.31)$$

After replacement and transformations, let us have linear differential first-order equation

$$\frac{dv_*}{d\theta}+nN(\theta)v_*=nc_1M(\theta), \qquad (7.32)$$

where

$$N(\theta)=\frac{a+g\sin\theta}{g\cos\theta};\quad M(\theta)=-\frac{c_1}{g\cos\theta}.$$

In accordance with the general rule of the solution of differential equations of the named type, let us find the particular

solution of the equation

$$\frac{dv_*}{d\theta}+nN(\theta)v_*=0$$

or, by dividing variables, we will attain:

$$\frac{dv_*}{v_*}=-nN(\theta)d\theta. \qquad (7.33)$$

The solution of equation (7.33) takes the form

$$v_*=C^*e^{-n\int N(\theta)d\theta}, \qquad (7.34)$$

where C^*- there is a function θ.

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Let us find C^* when solution (7.34) satisfies equation (7.32). From (7.34) we will obtain

$$\frac{dv_*}{d\theta} = -C^* nN(\theta)\, e^{-n\int N(\theta)d\theta} + e^{-n\int N(\theta)d\theta}\frac{dC^*}{d\theta}.$$

Substituting last/latter equality and expression (7.34) in (7.32), we will obtain

$$e^{-n\int N(\theta)d\theta}\frac{dC^*}{d\theta} = nc_1 M(\theta),$$

whence

$$C^* = nc_1\int e^{n\int N(\theta)d\theta} M(\theta)\,d\theta + k, \tag{7.35}$$

where k - integration constant.

After this the general solution of equation (7.32) will take the form

$$v_* = nc_1 e^{-n\int_{\theta_0}^{\theta} N(\theta)d\theta}\left[\int_{\theta_0}^{\theta} e^{n\int_{\theta_0}^{\theta} N(\theta)\,d\theta} M(\theta)\,d\theta + k\right]. \tag{7.36}$$

From last/latter equality and (7.31) for initial conditions $\theta=\theta_0$:

$v=v_0$ we will obtain

$$k=\frac{1}{v_0^n}.$$

After determination v_* from (7.31) we will obtain also the value of the unknown velocity

$$v=\frac{1}{\sqrt[n]{v_*}}. \qquad (7.37)$$

Let us examine analytical solution, after using for determining the function F(v) formula (7.30). Then from (7.28) we are have

$$\frac{d(v\cos\theta)}{d\theta}=\frac{c_1Bv^{n+1}}{g}. \qquad (7.38)$$

For separation of variables, let us multiply numerator and the denominator of the right side of the preceding/previous equality on $\cos^{n+1}\theta$

$$\frac{d(v\cos\theta)}{(v\cos\theta)^{n+1}}=\frac{c_1B}{g}\,\frac{d\theta}{\cos^{n+1}\theta}.$$

After integration let us have

$$\frac{1}{n}\left[\frac{1}{(v_0\cos\theta_0)^n}-\frac{1}{(v\cos\theta)^n}\right]=\frac{c_1B}{g}\int\limits_{\theta_0}^{\theta}\frac{d\theta}{\cos^{n+1}\theta}. \qquad (7.39)$$

Page 264. The integral of right side can be undertaken in elementary functions. Are known solutions to n=5, given, for example, in [30].

For determining of coordinates and time of motion, we will use

three last/latter equations of system (5.10). From the equation

$$\frac{dy}{d\theta} = -\frac{v^2 \operatorname{tg}\theta}{g}$$

we will obtain

$$y = y_0 + \int_{\theta}^{\theta_0} \frac{v^2}{g} \operatorname{tg}\theta d\theta. \qquad (7.40)$$

From the equation

$$\frac{dx}{d\theta} = -\frac{v^2}{g}$$

we will obtain

$$x = x_0 + \int_{\theta}^{\theta_0} \frac{v^2}{g} d\theta \qquad (7.41)$$

and from the equation

$$\frac{dt}{d\theta} = -\frac{v}{g \cos\theta}$$

we will obtain

$$t = t_0 + \int_{\theta}^{\theta_0} \frac{v}{g} \frac{d\theta}{\cos\theta}. \qquad (7.42)$$

The complexity of formulas for determining velocity (7.36) and (7.37) or (7.39) does not make it possible to take the integrals of right sides (7.40), (7.41) and (7.42) in final form.

Let us examine the solution of the basic problem of external ballistics with the approximation of function F(v) by the one-term quadratic dependence $F(v) = Ev^2$. This solution was called L. Euler's method [63].

In the first equation of system (5.10) let us designate

$b = BcH(y_{cp})$. After this the hodograph equation will take the form

$$\frac{du}{d\theta} = \frac{b}{g} \frac{u^3}{\cos^3\theta}.$$

After integration we will obtain

$$\frac{1}{2u_0^2} - \frac{1}{2u^2} = \frac{b}{g} \int_{\theta_0}^{\theta} \frac{d\theta}{\cos^3\theta}, \qquad (7.43)$$

where $u_0 = v_0 \cos\theta_0$.

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For simplification in further solution and its bringing to tabular form usually is introduced the auxiliary function

$$\varepsilon(\theta) = \int_0^{\theta} \frac{d\theta}{\cos^3\theta},$$

and then

$$\frac{1}{u^2} = \frac{1}{u_0^2} - \frac{2b}{g}[\varepsilon(\theta) - \varepsilon(\theta_0)]. \qquad (7.44)$$

If we the constant values, which depend on initial conditions, group to together and designate

$$\varepsilon(v_0, \theta_0) = \varepsilon(\theta_0) + \frac{g}{2bu_0^2},$$

that we will obtain

$$u^2 = \frac{g}{2b} \frac{1}{\varepsilon(v_0, \theta_0) - \varepsilon(\theta)}. \qquad (7.45)$$

For function $\varepsilon(\theta)$, entering in (7.45), can be obtained the analytical

dependence

$$\varepsilon(\theta)=\int_0^\theta \frac{d\theta}{\cos^3\theta}=\frac{1}{2}\left[\frac{\sin\theta}{\cos^2\theta}+\ln\operatorname{tg}\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right].$$

After determination of u the velocity of projectile let us find from the formula

$$v=\frac{u}{\cos\theta}.$$

Substituting (7.45) in the remaining equations of system (5.10), we will obtain respectively

$$\begin{aligned} dy&=-\frac{1}{2b}\cdot\frac{\operatorname{tg}\theta d\theta}{[\varepsilon(v_0,\ \theta_0)-\varepsilon(\theta)]\cos^2\theta};\\ dx&=-\frac{1}{2b}\cdot\frac{d\theta}{[\varepsilon(v_0,\ \theta_0)-\varepsilon(\theta)]\cos^2\theta};\\ dt&=-\frac{1}{\sqrt{2bg}}\cdot\frac{d\theta}{\sqrt{\varepsilon(v_0,\ \theta_0)-\varepsilon(\theta)}\cos^2\theta}\end{aligned}\qquad(7.46)$$

also, after the integration

$$\begin{aligned} y&=\frac{1}{2b}\int_\theta^{\theta_0}\frac{\operatorname{tg}\theta d\theta}{[\varepsilon(v_0,\ \theta_0)-\varepsilon(\theta)]\cos^2\theta};\\ x&=\frac{1}{2b}\int_\theta^{\theta_0}\frac{d\theta}{[\varepsilon(v_0,\ \theta_0)-\varepsilon(\theta)]\cos^2\theta};\\ t&=\frac{1}{\sqrt{2bg}}\int_\theta^{\theta_0}\frac{d\theta}{\sqrt{\varepsilon(v_0,\ \theta_0)-\varepsilon(\theta)}\cos^2\theta}.\end{aligned}\qquad(7.47)$$

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The integrals, which are contained in the right sides of equations (7.47), in final form are not taken, but they are the functions one and the same values

$$\left.\begin{aligned}2by=\varphi_1(\theta_0,\ bv_0^2,\ \theta);\ 2bx=\varphi_2(\theta_0,\ bv_0^2,\ \theta);\\ \sqrt{2bg}\,t=\varphi_3(\theta_0,\ bv_0^2,\ \theta).\end{aligned}\right\} \quad (7.48)$$

This made it possible to create tables with entries θ_0 and bv_0^2 for the calculation of trajectory elements. On their base were later comprised more convenient for practical calculations tables with entries θ_0 and $2bx_C$.

Tables gives the values of values

$$\frac{bv_0^2}{g};\ \frac{v_0^2}{2gx_C};\ |\theta_C|;\ \frac{v_C}{v_0};\ \frac{t_C\sqrt{g}}{\sqrt{x_C}};\ \frac{y_S}{x_C}.$$

FOOTNOTE [1]. In tables x_C is marked, X, t_C—T and y_s—Y. ENDFOOTNOTE.

The tables are comprised for θ_0 from 15° to 75° at values $2bx_C$ from

0 to (0.90-2.5) depending on angle of departure [9]. The examined method of Euler and table are used for trajectory calculation at the low initial velocities.

3.2. Method of pseudovelocity. Basic and auxiliary functions.

The method of pseudovelocity is related to the third group of analytical methods, since at its basis lie/rests the supplementary transformation of the hodograph equation of velocity (5.9). In hodograph equation for separation of variables, is introduced function F(U), where U - the value, which has the dimensionality of velocity and called pseudovelocity. Value U will be defined according to Fig. by 7.8 as

$$U = \frac{v \cos \theta}{\cos \theta_0}. \tag{7.49}$$

The vector of pseudovelocity is parallel to the vector of the initial velocity and has the same horizontal projection u, as is real of velocity v. Since the simple replacement F(v) by F(U) gives considerable errors, then into hodograph equation are introduced supplementary correction factors.

Substitution takes the form

$$H(y) F(v) \approx kF(U), \tag{7.50}$$

where k - the correction factor, which compensates for error.

For the low trajectories, which are characterized low θ_0 and large initial velocities, angle θ it is small changes along trajectory, but $H(y)\approx 1$. In this case it suffices to accept

$$k=\frac{1}{\cos\theta}. \qquad (7.51)$$

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For anti-aircraft trajectories are also characteristic high initial velocities and the weak change cos θ in trajectory, but $H(y)\neq 1$ and therefore is necessary the substitution of another form -

$$H(y)F(v)\approx H(y_{cp})F(U)\frac{\cos\theta_0}{\cos\theta}, \qquad (7.52)$$

i.e.

$$k=H(y_{cp})\frac{\cos\theta_0}{\cos\theta}. \qquad (7.53)$$

For the mean angles of casting and velocities, characteristic for the trajectories of the projectiles of field artillery pieces, substitution somewhat more complex than preceding/previous

$$H(y)F(v)\approx\beta F(U)\frac{\cos^2\theta_0}{\cos\theta}, \qquad (7.54)$$

i.e.

$$k=\beta\frac{\cos^2\theta_0}{\cos\theta}, \qquad (7.55)$$

where β - supplementary numerical correction factor.

Last/latter replacement is called of the substitution Siacci.

With known pseudovelocity real velocity is determined from equality (7.49):

$$v = \frac{U \cos \theta_0}{\cos \theta}, \qquad (7.56)$$

a the horizontal projection of speed is equal to

$$u = U \cos \theta_0. \qquad (7.57)$$

Let us introduce substitution (7.54) into the hodograph equation of velocity, let us replace $c' = c\beta$ and let us divide variables:

$$\frac{d\theta}{\cos^2 \theta} = \frac{g}{c' \cos^2 \theta_0} \frac{dU}{UF(U)}. \qquad (7.58)$$

After integration let us have

$$\operatorname{tg} \theta - \operatorname{tg} \theta_0 = \frac{g}{c' \cos^2 \theta_0} \int_{U_0}^{U} \frac{dU}{UF(U)}. \qquad (7.59)$$

Integral in right side can be undertaken only numerically.

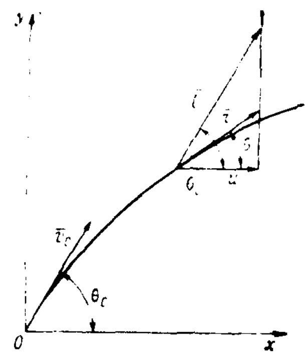

Fig. 7.8. Construction of the vector of pseudovelocity U.

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In practice the trajectory calculation by the method of Siacci is conducted with the use of special tables. For convenience in the compilation of tables and work on them, are introduced to the function

$$J(U)=K_1-\int_{U_H}^{U}\frac{2g\,dU}{UF(U)} \text{ and } J(v_0)=K_1-\int_{U_H}^{v_0}\frac{2g\,dU}{UF(U)},$$

where $v_0=U_0$.

Carrying out replacement in (7.59), we will obtain:

$$\operatorname{tg}\theta=\operatorname{tg}\theta_0-\frac{1}{2c'\cos^2\theta_0}[J(U)-J(v_0)]. \tag{7.60}$$

Let us integrate the remaining equations of system (5.10). In the equation

$$\frac{dt}{d\theta} = -\frac{u}{g}\frac{1}{\cos^2\theta}$$

let us substitute (7.58). After integration let us have

$$t = \frac{1}{c'\cos\theta_0}\int_{v_0}^{U}\frac{dU}{F(U)}.$$

Introducing the function

$$T(U) = K_2 - \int_{U_H}^{U}\frac{dU}{F(U)},$$

we will obtain

$$t = \frac{1}{c'\cos\theta_0}[T(U) - T(v_0)]. \qquad (7.61)$$

Let us take equation

$$\frac{dx}{d\theta} = -\frac{u^2}{g\cos^2\theta}.$$

After dividing in it variables and after substituting (7.58), we will obtain

$$dx = -\frac{UdU}{c'F(U)}. \qquad (7.62)$$

After integration let us have

$$x = -\frac{1}{c'}\int_{v_0}^{U}\frac{UdU}{F(U)}.$$

Introducing the function

$$D(U) = K_3 - \int_{U_H}^{U}\frac{UdU}{F(U)},$$

we will obtain

$$x=\frac{1}{c'}[D(U)-D(v_0)]. \qquad (7.63)$$

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For determining the ordinate of trajectory, let us proceed from the usual equality

$$dy=\operatorname{tg}\theta dx.$$

After substitution into this equality (7.60) and (7.62) and integrations we will obtain

$$y=x\operatorname{tg}\theta_0-\frac{x}{2c'\cos^2\theta_0}\left[\frac{A(U)-A(v_0)}{D(U)-D(v_0)}-J(v_0)\right], \qquad (7.64)$$

where

$$A(U)=K_4-\int_{U_H}^{U}\frac{J(U)UdU}{F(U)}.$$

For determination tgθ, t, x and y from formulas (7.60), (7.61), (7.63) and (7.64) is necessary the application/use of tables of the special functions J(U), T(U), D(U) and A(U), called basic functions. As input value in such tables is received pseudovelocity U. The numerical values of functions depend on the dependence accepted for

the air resistance F(U) and on constant numbers U_{n} and K_{i}, which are selected so that the tabular values of functions would be convenient when conducting of calculations. The tables of basic functions, calculated for a law air resistance of Siacci, are placed in [9, h. III]. The method of pseudovelocity makes it possible to calculate action characteristics at any point in the trajectory, including in apex/vertex and at impact point. For the brevity of writing, basic functions frequently designate only by first letters index of letters it designates the position of the point in question in trajectory [59].

For an impact point, the ordinate of trajectory is equal to zero. Substituting in (7.64) $y_C=0$ and converting, we will obtain:

$$\sin 2\theta_0=\frac{1}{c'}\left[\frac{A_C-A_0}{D_C-D_0}-J_0\right]. \tag{7.65}$$

To isolate in an explicit form from this equation pseudovelocity at impact point U_C is not represented possible. To solve it relatively U_C is possible by selection, being assigned by exemplary/approximate values U_C. After determining U_C, let us determine remaining trajectory elements at impact point

$$\left.\begin{aligned} x_C&=\frac{1}{c'}(D_C-D_0);\quad t_C=\frac{1}{c'\cos\theta_0}(T_C-T_0);\\ \operatorname{tg}\theta_C&=\operatorname{tg}\theta_0-\frac{1}{2c'\cos^2\theta_0}(J_C-J_0).\end{aligned}\right\} \tag{7.66}$$

In peak of the trajectory $\theta_s=0$ and $\operatorname{tg}\theta_s=0$, then from (7.60) after transformation we will obtain:

$$J_s=c'\sin 2\theta_0+J_0. \tag{7.67}$$

Using the tables of basic functions, from J_s let us find D_s, T_s, A_s; from initial conditions with $U_0=v_0$ let us find D_0, T_0 and A_0, after which from (7.61), (7.63) and (7.64) let us find

$$\begin{aligned}
t_s&=\frac{1}{c'\cos\theta_0}(T_s-T_0);\\
x_s&=\frac{1}{c'}(D_s-D_0);\\
y_s&=x_s\operatorname{tg}\theta_0-\frac{x_s}{2c'\cos^2\theta_0}\left(\frac{A_s-A_0}{D_s-D_0}-J_0\right).
\end{aligned} \tag{7.68}$$

The cell/elements of impact point can be found, after avoiding trial-and-error solution of equation (7.65). From equality (7.63) for an impact point $D(U_c)=c'x_c-D(v_0)$ it follows that

$$U_c=f(c'x_c;\ v_0).$$

Taking into account this expression in the brackets from equation (7.65) can be considered as certain function f_0 of these arguments

$$\frac{A(U_c)-A(v_0)}{D(U_c)-D(v_0)}-J(v_0)=f_0(c'x_c;\ v_0)$$

and to write (7.65) in the form

$$c'\sin 2\theta_0=f_0(c'x;\ v_0). \tag{7.69}$$

For $f_0(c'x_C; v_0)$ are calculated the tables.

On initial data it is possible to find $c' \sin 2\theta_0$; through this value and v_0, using tables, is found product $c'x_C$ and further x_C.

Similarly it is possible to obtain some other functions from $c'x_C$ and v_0 [59]:

$$f_1 = \frac{\sin 2\theta_0 v_0^2}{x_C};\quad f_2 = \frac{\operatorname{tg}|\theta_C|}{\operatorname{tg}\theta_0};\quad f_3 = \frac{v_0 \sin\theta_0}{t_C};$$
$$f_4 = \frac{v_0\cos\theta_0}{v_C\cos\theta_C};\quad f_5 = \frac{x_S}{x_C};\qquad f_6 = \frac{y_S}{x_C\operatorname{tg}\theta_0}. \tag{7.70}$$

Functions f_0-f_6 are called the auxiliary functions of Siacci. These functions, besides f_5 and f_6, are suitable for calculating the action characteristics at any point in the trajectory. In this case product $c'x$ does not have an index.

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For example, for instantaneous value of x coordinate y will be determined according to following formula [59]:

$$y = x\operatorname{tg}\theta_0\left[1 - \frac{f_0(c'x; v_0)}{f_0(c'x_C; v_0)}\right]. \tag{7.71}$$

Artificial transformation of the hodograph equation of velocity with the replacement of function F(v) by F(L) gives good on accuracy results during trajectory calculation, in which cosθ it is little affected along trajectory, and it is possible to accept $H(y)\approx 1$. For short low trajectories (at high initial velocities and the low angles of departure), when it is possible to accept $\cos\theta\approx\cos\theta_0\approx 1$, it is obvious, that $\beta=1$, $c'=c$ and $U\approx v$. Then

$$\left.\begin{aligned} x&=\frac{1}{c}\,[D(v)-D(v_0)];\\ y&=x\operatorname{tg}\theta_0-\frac{x}{2c}\left[\frac{A(v)-A(v_0)}{D(v)-D(v_0)}-J(v_0)\right];\\ t&=\frac{1}{c}\,[T(v)-T(v_0)]. \end{aligned}\right|\qquad (7.72)$$

During the solution of the reverse problem, the angle of departure in this case will be determined from condition $y_C=0$ for the formula

$$\sin 2\theta_0=\frac{1}{c}\left[\frac{A(v)-A(v_0)}{D(v)-D(v_0)}-J(v_0)\right].$$

With an increase in the angle of departure and decrease of the initial velocity the error in determination for trajectory elements at impact point increases. Error compensation is realized by a way of introduction into the calculation of coefficient β. Strictly speaking, for each trajectory element, it is necessary to introduce its correction factor β_x, β_y, β_θ, β_t. When conducting of practical

calculations, use one compensating factor - β_x, that agree the complete distances, designed by the method of pseudovelocity even one of the more precise methods. This coefficient is named main coefficient β.

The table of the values of coefficient β in connection with the law of air resistance of Siacci for small arms and the artillery of the low caliber (with c > 1) is comprised according to intake numbers of θ_0=6°-30° and x_C=1000—7000 m. Coefficient β in the table changes within narrow limits from 0.97 to 1.06.

For medium and high calibers (with c<1) Ya. M. Shapiro it make table with entries c=0.2-1.0, v_0=300-1000 m/s and θ_0=5-60°. The tables are comprised by processing the results of trajectory calculations, carried out by the method of numerical integration [9] and [59]. Depending on velocity and angle of departure, the main coefficient of agreement β substantially changes (from 0.609 to 1.329 for θ_0=60° and from 0.984 to 1.039 for θ_0=5°).

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During the agreement of complete distance, the error in the determination of remaining cell/elements reaches to 5o/o (at θ_0=30°-40°).

During the trajectory calculation of the anti-aircraft shells into the hodograph equation of velocity, it is substituted (7.52). Comparing (7.52) and (7.54), we will obtain $\zeta = \frac{H(y_{cp})}{\cos\theta_0}$.

Designating $c_1 = cH(y_{cp})$ from (7.60), (7.61), (7.63) and (7.64) we will obtain calculation formulas for determination of the motion characteristics of the anti-aircraft shells of the constant mass

$$\left.\begin{aligned} \operatorname{tg}\theta &= \operatorname{tg}\theta_0 - \frac{1}{2c_1\cos\theta_0}\left[J(U) - J(v_0)\right]; \\ t &= \frac{1}{c_1}\left[T(U) - T(v_0)\right]; \quad x = \frac{\cos\theta_0}{c_1}\left[D(U) - D(v_0)\right]; \\ y &= x\operatorname{tg}\theta_0 - \frac{x}{2c_1\cos\theta_0}\left[\frac{A(U) - A(v_0)}{D(U) - D(v)_0} - J(v_0)\right]. \end{aligned}\right| \quad (7.73)$$

During practical calculations determining action characteristics for each calculation point of zenith trajectory is conducted into two approach/approximations. In the first approximation, we take $H(y_{cp}) = 1$ and $c_1 = c$; in the second approach/approximation, after calculation y_i, we determine $H(y_{cp}) = H\left(\frac{y_i}{2}\right)$, we find c_1 and we perform calculation repeatedly.

The application/use of a method of pseudovelocity is possible in principle also for performance calculation of the motion of the body of variable mass, i.e., for the calculation of the powered flight trajectories of the unguided rockets.

The first equation of system (3.82) can be replaced by the equation

$$\frac{du}{d\lambda}=\left[\frac{w_e}{1-\lambda}-\frac{id^2}{Q_{\text{сек}}}10^3H(y)\frac{F(v)}{1-\lambda}\right]\cos\theta. \qquad (7.74)$$

This - the projection of equation of motion on axis ox of the starting Cartesian coordinate system.

If we now accept $Q_{\text{сек}}=\text{const}$, $H(y)=H(y_{\text{cp}})$ and $\theta=\theta_{\text{cp}}$, then it is possible to obtain

$$\frac{d\lambda}{1-\lambda}=\frac{d\left(v\frac{\cos\theta}{\cos\theta_{\text{cp}}}\right)}{w_e[1-kF(v)]}, \qquad (7.75)$$

where in k are included all constant values and a numerical correction factor.

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If we additionally accept

$$v=\frac{u}{\cos\theta}\approx\frac{u}{\cos\theta_{\text{cp}}}$$

and to designate

$$U=\frac{v\cos\theta}{\cos\theta_{\text{cp}}},$$

that after integraticn we will cbtain

$$\ln [illegible] = \frac{1}{a_i}\int_{U_i}^{U}\frac{dU}{1-kF(U)}, \qquad 7.76$$

where U_i - value U for the beginning cf the section of integration.

Similarly and in the remaining equaticns cf system (3.82) variables can be divided, but equaticns are integrated.

Integral in the right side of equality (7.76) in final form is not taken. So cannot be undertaken integrals, also, in the remaining equalities, cbtained during the integraticn cf system (3.82). The numerical calculaticns cf the powered flight trajectories cf the unguided projectiles of variable mass can be carried out by the generalized method of pseudcvelocity, propcsed tc L. B. Komarov, with the use of tables. The tables are calculated cn the basis of standard functicn from the air resistance, kncwn by the name cf the "law cf the resistance of 1943".

Example of the trajectcry calculation cf the projectile of ccnstant mass with the aid cf basic functicns.

Is assigned: d=100 mm, i=0.65, Q=12 kg, v_0=1200 m/s.

To find the velocity of projectile with flat trajectory fire with angle of elevation, by close to zero, on distances - 500, 1000 and 1500 m.

Ballistic coefficient

$$c = \frac{id^2}{Q} 10^3 = \frac{0,65 \cdot (0,1)^2}{12} 10^3 = 0,5417.$$

On first formula (7.72) we will obtain

$$D(v_C) = D(v_0) + cx_C.$$

Value $D(v_0)$ let us find from the tables of basic functions from input value v_0=1200 m/s. Afterward calculation $D(v_C)$ for the assigned distances let us find initial velocity of projectile v_C. The results of calculation are given below:

x_C, м	500	1000	1500
$D(v_0)$	2022	2022	2022
cx_C	270,8	541,7	812,4
$D(v_C)$	2292,8	2563,7	2834,4
v_C, м/с	1123,5	1048,7	975,6

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3.3. Solutions with the assigned analytical dependences for motion characteristics.

In certain cases of calculation, it is possible to previously assume known the form analytic functions, which describe any of the motion characteristics. Most frequently such methods are utilized during the calculation of trajectories of missile targetings to the moved target/purposes. Is well known the method of the trajectory calculation of guidance, proposed by professor I. V. Ostoslavskiy, in whom they accept, that a change in the velocity of the motion of rocket along trajectory is determined by the function of the form

$$v_p = v_{p0} + \dot{v}_p t. \qquad (7.77)$$

In this case, the motion characteristics are calculated several approach/approximations.

It is presented the common/general/total procedure of the solution of problem, being based on previously obtained dependences.

acceleration $\dot{v}_p$ in the first approximation, can be determined from the first equation of system (4.23) for the known initial data

$$\dot{v}_{p1} = \frac{P_0 - X_0}{m_0} - g \sin \theta_0.$$

The initial values of thrust P_0 and of density ρ_0 let us define

as functions of origin coordinate y_0, but balance angle - according to formula (3.66), also with the use of the initial data

$$\alpha_{60}=\frac{m_0}{P_0+q_0Sc_y^{*}}(v_{p0}\dot{\theta}_0+g\cos\theta_0),$$

where $\dot{\theta}_0$ it corresponds to the guidance method accepted.

Drag coefficient let us determine according to formula (2.89)

$$c_x(M_0)=c_{x0}+c_{x_i}(\alpha_{60}).$$

After determination of the drag

$$X_0=\frac{\varrho_0 v_{p0}^2}{2}Sc_x(M_0)$$

and of the initial value of thrust, it is possible to calculate $\dot{v}_{pI}$ and v_{pI} according to (7.77). Besides dependence (7.77), for trajectory calculation it is necessary to have a dependence, which makes it possible to calculate the values of angle θ. For this purpose, are utilized the kinematic equations, which correspond to guidance method.

Let us examine the solution of problem with pursuit guidance. For horizontal rectilinear action of target/purpose with the planar trajectory of the guidance

$$\frac{d\theta}{dt}=\frac{1}{r}v_u\sin\theta. \qquad (7.78)$$

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Dividing variables, we will obtain

$$\frac{d\theta}{\sin\theta} = \frac{v_n}{r}\,dt, \qquad (7.79)$$

whence it follows

$$\operatorname{tg}\frac{\theta}{2} = \operatorname{tg}\frac{\theta_0}{2}\,e^{\int_0^t v_n \frac{dt}{r}}. \qquad (7.80)$$

A change in the distance between the rocket and the target/purpose for the case in question in accordance with (4.8) is equal

$$\dot r = -(v_p + v_n\cos\theta). \qquad (7.81)$$

Integrating last/latter equality, we will obtain in the first approximation,

$$r_1 = r_0 - \int_0^t v_{p1}dt - \int_0^t v_n\cos\theta_0\,dt, \qquad (7.82)$$

after which from (7.80) it is possible to expect $\operatorname{tg}\frac{\theta_1}{2}$ and to refine r_1.

The ordinate of trajectory will be determined from the usual kinematic relationship/ratio

$$y_{p1} = y_{p0} + \int_0^t v_{p1}\sin\theta_1\,dt. \qquad (7.83)$$

For simplification in the solution most frequently is used $v_\text{п}=\text{const}$. For conducting the calculation in the second approach/approximation, are utilized values $v_\text{I}(t)$, $\dot\theta_\text{I}(t)$, $y_{\text{p I}}(t)$ and $r_\text{I}(t)$, obtained in the first approximation. Is calculated balance angle

$$\alpha_{\delta\text{I}}(t)=\frac{m(t)}{P_\text{I}+q_\text{I}Sc_y^\alpha}(v_{\text{pI}}\dot\theta_\text{I}+g\cos\theta_\text{I})$$

and the drag

$$X_\text{I}(t)=\frac{\varrho_\text{I}v_{\text{pI}}^2}{2}Sc_x(\text{M}_\text{I}),$$

where

$$c_x(\text{M}_\text{I})=c_{x\,0\text{I}}+c_{x_i}(\alpha_{\delta\text{I}}).$$

As the entry into curve/graph $c_x(\text{M})$, will serve value $v_{\text{p I}}/a_\text{I}$, where a_I - the speed of sound at height/altitude y_I. Thrust $P_\text{I}(t)$ is calculated also on ordinate y_I. Thus, acceleration in the second approach/approximation will be equal

$$\dot v_{\text{pII}}(t)=\frac{P_\text{I}-X_\text{I}}{m(t)}-g\sin\theta_\text{I}.$$

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As a result of the numerical solution of last/latter

differential equation, we will obtain in the second approach/approximation the dependence of the velocity of rocket on time - $v_{pII}(t)$. The distance between the rocket and the target/purpose in the second approach/approximation will be equal to:

$$r_{II}=r_0-\int_0^t v_{pII}dt-\int_0^t v_{ll}\cos\theta_{l}\,dt.$$

Integrals in last/latter equality also must be undertaken numerically. The slope tangent in the second approach/approximation will be determined by the repeated solution of equation (7.80) with the new data

$$\operatorname{tg}\frac{\theta_{II}}{2}=\operatorname{tg}\frac{\theta_0}{2}e^{v_{ll}\int_0^t\frac{dt}{r_{II}}}.$$

Ordinate of calculation point in the trajectory in the second approach/approximation

$$y_{II}=y_0+\int_0^t v_{pII}\sin\theta_{II}dt.$$

For calculation in the third approach/approximation, are utilized the data, obtained in the second approach/approximation. Calculation points are selected through equal time intervals so as to have within the limits of the calculated phase of expected trajectory of 7-10 points. Kinematic equation for the definition of the abscissas of calculation points is solved independently after they will become known v_{pIII} and θ_{III}, obtained in the third

approach/approximation,

$$x = x_0 + \int_0^t v_{p\text{III}} \cos\theta_{\text{III}} dt. \qquad (7.84)$$

In certain cases proves to be advisable the detail with linear function (7.77) to use analytical dependence for the determination of distance between the rocket and the target/purpose. Function for r can be undertaken in the form of the sum three first members of Taylor series, comprised according to the degrees of time t [43],

$$r = r_0 + \dot{r}_0 t + 0{,}5\ddot{r}_0 t^2.$$

The derivatives $\dot{r}_0$ and $\ddot{r}_0$ are determined depending on guidance method from initial conditions. For example, with pursuit guidance to the target/purpose, driving/moving towards rocket it is rectilinear at constant height/altitude, for flat/plane trajectory of guidance, let us have on formula (7.81)

$$\dot{r}_0 = -(v_{p0} + v_{\text{ц}} \cos\theta_0).$$

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The second derivative at the constant velocity of target/purpose after differentiation of last/latter equality and substitution (7.78) will take the form

$$\ddot{r}_0 = -\left(\dot{v}_{p0} - \frac{v_{\text{ц}}^2}{r_0} \sin^2\theta_0\right). \qquad (7.85)$$

The use of dependences (7.77) and (7.85) under the conditions, stipulated above, it makes it possible to solve the task of guidance to target/purpose completely in quadratures [43]. A similar method of performance calculation of motion can be used also with other methods of guidance on three-point curve or with parallel approach.

It is necessary to keep in mind that the described principle can be applied with the sufficiently simple cases of moving the target/purpose. It gives satisfactory coincidence with the results of precise methods of calculation (numerical integration) when in advance taken dependences (7.77) and (7.85) prove to be close to the expected real.

3.4. Approximate analytical methods of determining the motion characteristics of the rockets with guidance.

The approximate analytical calculation methods of trajectory of guidance are based on the assumptions, which considerably simplify the real/actually occurring process. It is suggested, that the rocket and target/purpose move in one plane, that the target/purpose moves rectilinearly, target speeds and rocket are known and constant in the process of guidance. Rocket is replaced by point, and it is suggested

that the kinematic constraint between the rocket and the target/purpose, superimposed by guidance method, is realized ideally. Thus, from examination is eliminated the action of all forces and torque/moments, but investigation itself in this case bears purely kinematic character.

Subsequently let us consider that the rocket and target/purpose move in the first fourth of the right starting system of coordinates Cxy under initial conditions $x_{p0}<x_{u0};\ y_{p0}<y_{u0}$.

Let us examine the case of guidance with fixed-lead angle $\alpha_p=\alpha_{p0}=\text{const}$. For rectilinear motion of target/purpose $a_{n\,u}=0$ and $v_u\neq 0$. Then from (4.10)

$$\frac{d\alpha_u}{dt}+\frac{d\gamma}{dt}=0.$$

Utilizing (4.9) and replacing α_p on α_{p0}, we will obtain

$$\frac{d\alpha_u}{dt}=\frac{v_p\sin\alpha_{p0}-v_u\sin\alpha_u}{r}. \qquad (7.86)$$

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After factoring out v_u in right side and after designating $p=\frac{v_p}{v_u}$, we will obtain

$$\frac{d\alpha_u}{dt}=\frac{v_u(p\sin\alpha_{p0}-\sin\alpha_u)}{r}. \qquad (7.87)$$

From the condition of ideal advance/prevention (4.6) let us find angle α_u, which corresponds to angle α_{p0}. let us designate it through $\alpha_{u\,B}$ and let us call/name the angle of necessary rendezvous. Then

$$\sin \alpha_{u\,B} = p \sin \alpha_{p0}. \qquad (7.88)$$

After conducting replacement in (7.87), we will obtain

$$\frac{d\alpha_u}{dt} = v_u \frac{(\sin \alpha_{u\,B} - \sin \alpha_u)}{r}. \qquad (7.89)$$

Replacing in (4.8) α_p on α_{p0}, let us have

$$\frac{dr}{dt} = v_u \cos \alpha_u - v_p \cos \alpha_{p0}. \qquad (7.90)$$

After dividing (7.90) to (7.89), we will obtain

$$\frac{dr}{r} = \frac{\cos \alpha_u - p \cos \alpha_{p0}}{\sin \alpha_{u\,B} - \sin \alpha_u} d\alpha_u. \qquad (7.91)$$

From this equation can be obtained the formula for the determination of distance between the rocket and the target/purpose in the process of guidance.

The integration of equation (7.91) has common sense under condition $\alpha_{p0} < \arcsin \frac{1}{p}$. otherwise the rocket target/purpose will not overtake.

Analytical dependence for r can be obtained by integration (7.91)

$$\int_{r_0}^{r} \frac{dr}{r} = -\int_{\alpha_{u0}}^{\alpha_u} \frac{\cos \alpha_u - p \cos \alpha_{p0}}{\sin \alpha_u - \sin \alpha_{u\,B}} d\alpha_u$$

and further

$$\int_{r_0}^{r}\frac{dr}{r}=-\int_{\alpha_{u0}}^{\alpha_u}\frac{\cos\alpha_u\,d\alpha_u}{\sin\alpha_u-\sin\alpha_{u.B}}+p\cos\alpha_{p0}\int_{\alpha_{u0}}^{\alpha_u}\frac{d\alpha_u}{\sin\alpha_u-\sin\alpha_{u.B}}. \tag{7.92}$$

The first integral of right side is equal to

$$\int_{\alpha_{u0}}^{\alpha_u}\frac{\cos\alpha_u\,d\alpha_u}{\sin\alpha_u-\sin\alpha_{u.B}}=|\ln(\sin\alpha_u-\sin\alpha_{u.B})|_{\alpha_{u0}}^{\alpha_u}.$$

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For the solution of the second integral, it is necessary to introduce substitution $\operatorname{tg}\frac{\alpha_u}{2}=z$, from which

$$d\alpha_u=\frac{2dz}{1+z^2} \text{ and } \sin\alpha_u=\frac{2z}{1+z^2}.$$

After conducting replacement in the second integral of right side, we will obtain

$$p\cos\alpha_{p0}\int_{\alpha_{u0}}^{\alpha_u}\frac{d\alpha_u}{\sin\alpha_u-\sin\alpha_{u.B}}=p\cos\alpha_{p0}\int_{z_0}^{z}\frac{2dz}{2z-\sin\alpha_{u.B}-z^2\sin\alpha_{u.B}}.$$

After integration and transformations, introducing the

designation

$$k = p\frac{\cos\alpha_{p0}}{\cos\alpha_{u\,B}},$$

we will obtain

$$p\cos\alpha_{p0}\int\limits_{\alpha_{u0}}^{\alpha_u}\frac{d\alpha_u}{\sin\alpha_u-\sin\alpha_{u\,B}} = -k\ln\left.\frac{\operatorname{tg}\frac{\alpha_u}{2}-\operatorname{ctg}\frac{\alpha_{u\,B}}{2}}{\operatorname{tg}\frac{\alpha_u}{2}-\operatorname{tg}\frac{\alpha_{u\,B}}{2}}\right|_{\alpha_{u0}}^{\alpha_u}.$$

Uniting both the integral of right side (7.92) and substituting the apparitors of integration, we will obtain

$$\frac{r}{r_0}=\left(\frac{\sin\alpha_{u0}-\sin\alpha_{u\,B}}{\sin\alpha_u-\sin\alpha_{u\,B}}\right)\left(\frac{\operatorname{tg}\frac{\alpha_u}{2}-\operatorname{tg}\frac{\alpha_{u\,B}}{2}}{\operatorname{tg}\frac{\alpha_{u0}}{2}-\operatorname{tg}\frac{\alpha_{u.B}}{2}}\right)^k\times$$
$$\times\left(\frac{\operatorname{tg}\frac{\alpha_u}{2}-\operatorname{ctg}\frac{\alpha_{u\,B}}{2}}{\operatorname{tg}\frac{\alpha_{u0}}{2}-\operatorname{ctg}\frac{\alpha_{u.B}}{2}}\right)^{-k}.$$

After multiplying right and left parts on $\frac{\cos^2\frac{\alpha_u}{2}}{\cos^2\frac{\alpha_{u0}}{2}}$, after transformations we will obtain the final formula, which determines the distance between the rocket and the target/purpose according to the line of the sighting

$$r=r_0\frac{\cos^2\frac{\alpha_{u0}}{2}}{\cos^2\frac{\alpha_u}{2}}\left(\frac{\operatorname{tg}\frac{\alpha_u}{2}-\operatorname{tg}\frac{\alpha_{u\,B}}{2}}{\operatorname{tg}\frac{\alpha_{u0}}{2}-\operatorname{tg}\frac{\alpha_{u\,B}}{2}}\right)^{k-1}\times$$
$$\times\left(\frac{\operatorname{tg}\frac{\alpha_u}{2}-\operatorname{ctg}\frac{\alpha_{u.B}}{2}}{\operatorname{tg}\frac{\alpha_{u0}}{2}-\operatorname{ctg}\frac{\alpha_{u\,B}}{2}}\right)^{-k-1}. \qquad (7.93)$$

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The solution of equation (7.92) can be written, also, in other form, after replacing $\sin\alpha_{u\,B}$ by (7.88) [39]:

$$r=r_0\left(\frac{\sin\alpha_u-p\sin\alpha_{p0}}{\sin\alpha_{u0}-p\sin\alpha_{p0}}\right)^{\frac{p\cos\alpha_{p0}}{\sqrt{1-p^2\sin^2\alpha_{p0}}}-1}\times$$

$$\times\left(\frac{1-p\sin\alpha_{p0}\sin\alpha_{u0}+\cos\alpha_{u0}\sqrt{1-p^2\sin^2\alpha_{p0}}}{1-p\sin\alpha_{p0}\sin\alpha_u+\cos\alpha_u\sqrt{1-p^2\sin^2\alpha_{p0}}}\right)^{\frac{p\cos\alpha_{p0}}{\sqrt{1-p^2\sin^2\alpha_{p0}}}}. \quad (7.94)$$

It is possible to derive also formula for determining the approach time of rocket for target/purpose. Utilizing (7.88), (7.89) and (7.90), we will obtain

$$v_u dt=\frac{dr}{\cos\alpha_u-p\cos\alpha_{p0}}=\frac{r\,d\alpha_u}{p\sin\alpha_{p0}-\sin\alpha_u}.$$

Carrying out appropriate substitution [39], it is possible to obtain

$$v_u dt=\frac{1}{(1-p^2)\cos\alpha_{p0}}\left(p\,dr+d\left[r\cos(\alpha_u-\alpha_{p0})\right]\right).$$

Integrating right side from r_0 to r, we will obtain dependence for determination of approach time:

$$t=\frac{r_0\,[p+\cos(\alpha_{u0}+\alpha_{p0})]-r\,[p+\cos(\alpha_u+\alpha_{p0})]}{v_u\,(p^2-1)\cos\alpha_{p0}}. \qquad (7.95)$$

tav the time of interception (i.e. the time of the motion of rocket from initial position to point of impact) will be determined from last/latter formula with r=0

$$t_{в}=\frac{r_0\,[p+\cos(\alpha_{u0}+\alpha_{p0})]}{v_u\,(p^2-1)\cos\alpha_{p0}}. \qquad (7.96)$$

The normal accelerations, experience/tested by rocket in the process of guidance, are determined by formula (4.11). Remembering that for case $\frac{d\alpha_{p}}{dt}=0$, in question and $\frac{d\gamma}{dt}$ is located through formula (4.9), we will obtain

$$a_{n\,p}=-v_p v_u\,\frac{\sin\alpha_u-p\sin\alpha_{p0}}{r}. \qquad (7.97)$$

After substituting into last/latter formula expression for r of (7.93) and after conducting transformations, we will obtain

$$a_{n\,p}=\frac{2v_p^2}{pr_0}\left(\sin\frac{\alpha_u-\alpha_{u.B}}{2}\right)^{2-k}\times\left(\cos\frac{\alpha_u+\alpha_{u.B}}{2}\right)^{2+k}\times$$
$$\times\left(\sin\frac{\alpha_{u0}-\alpha_{u.B}}{2}\right)^{-1+k}\times\left(\cos\frac{\alpha_{u0}+\alpha_{u.B}}{2}\right)^{-1-k}. \qquad (7.98)$$

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Let us examine the case of guidance according to parallel approach method. Integrating (4.8), we can write:

$$\int_{r_0}^{r} dr = \int_{0}^{t} (v_u \cos \alpha_u - v_p \cos \alpha_p)\, dt. \qquad (7.99)$$

With the horizontal motion of target/purpose with the constant velocity and preservation/retention/maintaining in the process of guidance p=const, we will obtain:

$$r = r_0 - (v_u \cos \alpha_u - v_p \cos \alpha_p)\, t. \qquad (7.100)$$

Utilizing (4.7), let us write

$$\cos \alpha_p = \sqrt{1 - \frac{\sin^2 \alpha_u}{p^2}}, \qquad (7.101)$$

and then

$$r = r_0 - \left(v_u \cos \alpha_u - v_p \sqrt{1 - \frac{\sin \alpha_u}{p^2}}\right) t. \qquad (7.102)$$

Flight time let us determine from (7.100) with r=0

$$t = \frac{r_0}{v_u \cos \alpha_u - v_p \cos \alpha_p}. \qquad (7.103)$$

During the rectilinear and uniform motion of target/purpose and

p=const the rocket will move also rectilinearly and is evenly, and consequently, $a_{n\,p}=0$. With the maneuvering of target/purpose from (4.10) and (4.11) let us write

$$a_{n\,u}=v_u\frac{d\alpha_u}{dt} \text{ and } a_{n\,p}=v_p\frac{d\alpha_p}{dt}. \qquad (7.104)$$

Differentiating (4.7) and utilizing (7.104), we will obtain

$$a_{n\,p}=a_{n\,u}\frac{\cos\alpha_u}{\cos\alpha_p}. \qquad (7.105)$$

Replacing $\cos\alpha_p$ and substituting (7.101), we will obtain communication/connection between the normal accelerations of rocket and target/purpose

$$a_{n\,p}=a_{n\,u}\sqrt{\frac{1-\sin^2\alpha_u}{1-\frac{\sin^2\alpha_u}{p^2}}}. \qquad (7.106)$$

During the proportional approach of equation (4.8) and (4.15) they are integrated in final form only in the case of the uniform horizontal motion of target/purpose for the particular value of factor of proportionality a=2 [39]. The general case requires numerical solution.

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With pursuit guidance $\alpha_p=0$, then from (4.6), (4.10) and (4.11) it is possible to write:

$$\frac{dr}{dt}=v_{\text{u}}(\cos\alpha_{\text{u}}-p), \qquad (7.107)$$

$$a_{n\,\text{u}}=v_{\text{u}}\left(\frac{d\alpha_{\text{u}}}{dt}+\frac{v_{\text{u}}\sin\alpha_{\text{u}}}{r}\right), \qquad (7.108)$$

$$a_{n\,\text{p}}=v_{\text{p}}\frac{d\gamma}{dt}. \qquad (7.109)$$

During rectilinear motion of target/purpose $a_{n\,\text{u}}=0$ and from (7.108)

$$\frac{d\gamma}{dt}=\frac{d\alpha_{\text{u}}}{dt}=-\frac{v_{\text{u}}\sin\alpha_{\text{u}}}{r}. \qquad (7.110)$$

After dividing (7.107) to (7.110), we will obtain

$$\frac{dr}{r}=-\frac{(\cos\alpha_{\text{u}}-p)\,d\alpha_{\text{u}}}{\sin\alpha_{\text{u}}}. \qquad (7.111)$$

Last/latter equation corresponds to guidance to the driven out target/purpose. Integrating it, we will obtain:

$$r=k\,\frac{(\sin\alpha_{\text{u}})^{p-1}}{(1+\cos\alpha_{\text{u}})^{p}}. \qquad (7.112)$$

Integration constant k will be determined according to the initial conditions r_0 and $\alpha_{\text{u}0}$

$$k=\frac{r_0\,(1+\cos\alpha_{\text{u}0})^{p}}{(\sin\alpha_{\text{u}0})^{p-1}}. \qquad (7.113)$$

Recall that during the horizontal motion of the target/purpose

$$\alpha_{\text{u}}=\varphi=\gamma.$$

In the case of guidance to the target/purpose, driving/moving

towards,

$$\frac{d\gamma}{dt} = \frac{d\alpha_u}{dt} = \frac{v_u \sin \alpha_u}{r} \qquad (7.114)$$

and

$$\frac{dr}{r} = -\frac{(\cos \gamma + p)\, d\alpha_u}{\sin \alpha_u}. \qquad (7.115)$$

Integrating, we will obtain

$$r = k' \frac{(1+\cos\gamma)^p}{(\sin\gamma)^{p+1}}, \qquad (7.116)$$

where the integration constant will be equal to

$$k' = \frac{r_0 (\sin \alpha_{u0})^{p+1}}{(1+\cos\gamma)^p}. \qquad (7.117)$$

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Let us determine the time of motion. For the case of the driven out target/purpose, let us multiply (7.107) on $\cos\gamma = \cos\alpha_u$, a (7.110) on $\sin\gamma = \sin\alpha_u$. Let us deduct of the second product the first and we will obtain after transformation the following differential equation:

$$(\cos\gamma + p)\, dr - r \sin\gamma\, d\gamma = v_u (1 - p^2)\, dt.$$

After integration we will obtain

$$t = \frac{r_0 (\cos\gamma_0 + p) - r(\cos\gamma + p)}{v_u (p^2 - 1)}. \qquad (7.118)$$

For the target/purpose, which flies towards, it is possible to

obtain

$$t=\frac{r(\cos\gamma-p)-r_0(\cos\gamma_0-p)}{v_u(p^2-1)}. \quad (7.118)$$

Total flying time to encounter with target let us determine with r=0; respectively let us have from (7.118):

$$t_*=\frac{r_0(\cos\gamma_0+p)}{v_u(p^2-1)}, \quad (7.119)$$

a from (7.118)*

$$t_*=\frac{r_0(p-\cos\gamma_0)}{v_u(p^2-1)}. \quad (7.120)$$

the normal acceleration of rocket let us determine, after substituting in (7.109) for the case of the driven out target/purpose of equation (7.110) and (7.112) and for the case of target/purpose, driving/moving towards, (7.114) and (7.116). We will obtain with guidance to the driven out target/purpose

$$a_{n\,p}=-\frac{v_p^2(1+\cos\gamma)^p}{pk(\sin\gamma)^{p-2}}; \quad (7.121)$$

with guidance to the target/purpose, driving/moving towards,

$$a_{n\,p}=\frac{v_p^2}{pk'}\cdot\frac{(\sin\gamma)^{p+2}}{(1+\cos\gamma)^p}. \quad (7.122)$$

We will obtain the motion characteristics of the rocket with matching guidance. After equating r_p+dr_p to r_p (see Fig. 4.5), we can write

$$(r_p d\gamma)^2+(dr_p)^2=(v_p dt)^2.$$

After dividing last/latter relationship/ratio on $(d\gamma)^2$, we will obtain:

$$\left(\frac{dr_p}{d\gamma}\right)^2+r_p^2=v_p^2\left(\frac{dt}{d\gamma}\right)^2. \qquad (7.123)$$

For horizontal rectilinear motion of target/purpose $\alpha_u=\gamma$ and along Fig. 4.5 let us determine

$$\frac{d\gamma}{dt}=\frac{v_u \sin\gamma}{r_u}. \qquad (7.124)$$

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Then from (7.123) it is possible to write

$$\left(\frac{dr_p}{d\gamma}\right)^2+r_p^2=\frac{r_u^2 p^2}{\sin^2\gamma}. \qquad (7.125)$$

Since $y_u=\text{const}$, then it is convenient to introduce replacement $r_u=\frac{y_u}{\sin\gamma}$ and then

$$\left(\frac{dr_p}{d\gamma}\right)^2+r_p^2=\frac{(y_u p)^2}{\sin^4\gamma}. \qquad (7.126)$$

The obtained differential equation is not integrated in quadratures. It can be solved approximately (see, for example [39]). The time of motion let us define from the following equality (see Fig. 4.5):

$$t=\frac{y_u(\operatorname{ctg}\gamma_0-\operatorname{ctg}\gamma)}{v_u}. \qquad (7.127)$$

Normal acceleration of rocket we determine as usual, according to the formula

$$a_{np}=v_p\frac{d\varphi}{dt}. \qquad (7.128)$$

Let us find derivative

$$\frac{d\varphi}{dt}=\frac{d\varphi}{d\gamma}\,\frac{d\gamma}{dt}. \qquad (7.129)$$

Utilizing Fig. 4.5, let us write:

$$\operatorname{tg}\varphi=\frac{dy}{dx}=\frac{d\,(r_p\sin\gamma)}{d\,(r_p\cos\gamma)}.$$

Let us accomplish the process/operation of differentiation of right side with respect to γ let us divide numerator and denominator into $d\gamma$; after this

$$\operatorname{tg}\varphi=\frac{r'_{p\gamma}\sin\gamma+r_p\cos\gamma}{r'_{p\gamma}\cos\gamma-r_p\sin\gamma}. \qquad (7.130)$$

We differentiate last/latter equality on γ

$$\frac{1}{\cos^2\varphi}\,\frac{d\varphi}{d\gamma}=\frac{r_p^2+2\,(r'_{p\gamma})^2-r_{p\gamma}r''_{p\gamma}}{(r'_{p\gamma}\cos\gamma-r_p\sin\gamma)^2}. \qquad (7.131)$$

Let us find value $\frac{1}{\cos^2\varphi}=1+\operatorname{tg}^2\varphi$, using (7.130),

$$\frac{1}{\cos^2\varphi}=\frac{r_p^2+(r'_{p\gamma})^2}{(r'_{p\gamma}\cos\gamma-r_p\sin\gamma)^2}. \qquad (7.132)$$

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After substituting (7.132) in (7.131), we will obtain

$$\frac{d\varphi}{d\gamma}=\frac{r_p^2+2(r'_{p\gamma})^2-r_{p\gamma}r''_{p\gamma}}{r_p^2+(r'_{p\gamma})^2}. \quad (7.133)$$

Value $r''_{p\gamma}$ let us find from equation (7.126), it differentiated it. Preliminarily let us designate the numerator of right side (7.126) through k^2

$$r'_{p\gamma}r''_{p\gamma}+r_pr'_{p\gamma}=-2\frac{k^2\operatorname{ctg}\gamma}{\sin^4\gamma} \quad (7.134)$$

Directly from (7.126) we will obtain

$$r'_{p\gamma}=\sqrt{\frac{k^2}{\sin^4\gamma}-r_p^2}. \quad (7.135)$$

Substituting last/latter equality in preceding/previous, we will obtain

$$r''_{p\gamma}=-\frac{2k^2}{\sin^4\gamma}\frac{\operatorname{ctg}\gamma}{\sqrt{\frac{k^2}{\sin^4\gamma}-r_p^2}}-r_p. \quad (7.136)$$

Substituting (7.135) and (7.136) in (7.133), we will obtain

$$\frac{d\varphi}{d\gamma}=2\left(1+\frac{r_p\operatorname{ctg}\gamma}{\sqrt{\frac{k^2}{\sin^4\gamma}-r_p^2}}\right). \quad (7.137)$$

Utilizing (7.128), (7.129) and (7.137), we will obtain

$$a_{np}=\frac{2pv^2\sin\gamma}{r_u}\left(1+\frac{r_p\operatorname{ctg}\gamma}{\sqrt{\frac{k^2}{\sin^4\gamma}-r_p^2}}\right). \quad (7.138)$$

Introducing the constant height/altitude of the motion of target/purpose $y_{\text{ц}}$, we will obtain

$$a_{np}=\frac{2pv_p^2\sin^2\gamma}{y_{\text{ц}}}\left(1+\frac{r_p\operatorname{ctg}\gamma}{\sqrt{\frac{(y_{\text{ц}}p)^2}{\sin^4\gamma}-r_p^2}}\right). \qquad (7.13)$$

The kinematic analysis of the possible conditions of the encounter of the rocket with target with ignorance by different methods will be examined in chapter IX.

§4. Similarity of trajectories and the tabular methods of solution.

In the practice of ballistic calculations, are applied the so-called tabular methods of the solutions with the aid of which it is possible to find the cell/elements of characteristic points in the trajectory, for example, of apex/vertex or impact point.

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Especially widely are utilized ballistic tables for the calculation of the trajectory elements of the flight of the projectiles of cannon-type artillery. These tables can be used also for calculation

of the inactive legs of rockets. The study of systems of equations (5.8) and (5.10) shows that the trajectory elements of definition by three parameters: initial velocity v_0, by the ballistic coefficient c; by angle of departure θ_0. After calculating a large quantity of trajectories, it is possible to make table of the cell/elements of trajectoriesafter taking as the entries of value v_0, c and θ_0. In tables are usually given the values of complete distance - x_c, trajectory height - y_s, total flying time - t_c, to the velocity in impact point - v_c and the angle of tangent inclination to trajectory in impact point - θ_c. Are well known the ballistic tables of ANII (artillery scientific research institute) and the ballistic collection of artillery academy [59]. Calculations according to tables are reduced to the determination of the unknown values by simple interpolation on v_0, c and θ_0.

For the trajectory calculation of antiaircraft firing to three intake parameters is added the fourth - time of motion.

Using the tables of antiaircraft fire, it is possible for the trajectory, determined by v_0, c and θ_0, to find x_t, y_t and v_t that correspond to different missile flight time t_1, t_2, t_3 and of so forth.

Ballistic tables successfully are utilized for the solution of

the inverse designed problems of external ballistics. For example, in terms of designed range x_0 and values c and θ_0 it is possible to simply find the necessary initial velocity of projectile - v_0.

Ballistic tables are utilized for trajectory calculation, which correspond to normal meteorological conditions. The limits of the applicability of tables can be expanded, if we use theory they can be expanded, if to use the theory of similitude of trajectories. French scholarly P. Langevin established the dependence between the characteristics of two trajectories of the projectiles of the constant mass, for which the values of temperature and pressure surrounding air, corresponding to the beginning of trajectories, were different. After assuming that the temperature of air differs by a constant value from temperature, determined by normal linear dependence, and change of air pressure is subjected to hypothesis about the vertical equilibrium of the atmosphere, it is possible to show that the trajectories will be similar, if of them are identical three determining parameters:

$$c^* = c\,\frac{h_0}{h_{0N}};\quad v_{\tau_0} = v_0\sqrt{\frac{\tau_{0N}}{\tau_0}} \text{ and } \theta_0,$$

where h_0 and τ_0 - pressure and temperature in initial point in the trajectory. Two Similar trajectories of coordinate x and y, will have to be related as temperatures in the initial point of trajectory τ_0;

and τ_{02}; velocities and flight time will be related as square roots of the relation of the temperatures in initial point; the angles of tangent inclination will be equal, i.e.,

$$\frac{x_{t1}}{x_{t2}} = \frac{\tau_{01}}{\tau_{02}};\quad \frac{y_{t1}}{y_{t2}} = \frac{\tau_{01}}{\tau_{02}};$$

$$\frac{t_{t1}}{t_{t2}} = \sqrt{\frac{\tau_{01}}{\tau_{02}}};\quad \frac{v_{t1}}{v_{t2}} = \sqrt{\frac{\tau_{01}}{\tau_{02}}};\quad \frac{\theta_{t1}}{\theta_{t2}} = 1. \qquad (7.14)$$

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Langevin's formulas are applied, for example, in the case of the trajectory calculation of the projectiles or barrel systems for the conditions, which correspond to the conditions of firing in mountains, for the calculation of the inactive legs of rockets, etc. the beginning of the inactive leg of the rocket of class "surface, surface," sometimes is located on considerable height/altitude; therefore the direct use of ballistic tables can give significant errors. Let us designate the horizontal range of the inactive leg, calculated from ballistic tables, through x_2, the cell/elements of the beginning of this section let us supply with index "k", and end/lead - by index "k′" (Fig. 7.9). From similarity condition of trajectories, it is possible to obtain the following formulas

$$\left.\begin{aligned} x_2 &= \frac{\tau_\kappa}{\tau_{0N}}\,\Phi_x(c^*, v_{\tau\kappa}, \theta_\kappa), \\ y_S &= \frac{\tau_\kappa}{\tau_{0N}}\,\Phi_y(c^*, v_{\tau\kappa}, \theta_\kappa), \\ \theta_{\kappa'} &= \Phi_\theta(c^*, v_{\tau\kappa}, \theta_\kappa), \\ v_{\kappa'} &= \sqrt{\frac{\tau_\kappa}{\tau_{0N}}}\,\Phi_v(c^*, v_{\tau\kappa}, \theta_\kappa), \\ t_{\kappa'} &= \sqrt{\frac{\tau_\kappa}{\tau_{0N}}}\,\Phi_t(c^*, v_{\tau\kappa}, \theta_\kappa), \end{aligned}\right| \qquad (7.141)$$

where $c^* = c_{\text{II}}\frac{h_\kappa}{h_{0N}}$ and $v_{\tau\kappa} = v_\kappa\sqrt{\frac{\tau_{0N}}{\tau_\kappa}}$.

In these formulas by symbols $v_\kappa, \theta_\kappa, h_\kappa$ and τ_κ are designated respectively the velocity, angle of departure, barometric pressure and virtual temperature in the point of the beginning of inactive leg, i.e., at the end of the active section.

Ballistic coefficient for a passive section is equal to

$$c_{\text{II}} = \frac{id^2}{Q_0 - Q_\tau}\,10^3.$$

The values of functions $\Phi_x, \Phi_y, \Phi_\theta, \Phi_v$, and Φ_t are taken from usual ballistic tables in input values c^*, $v_{\tau\kappa}$ and θ_κ.

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Thus, for instance, $\Phi_x(c^*, v_{\tau\kappa}, \theta_\kappa)$ is equal to the distance, determined

by ballistic tables for the numerical values of quantities, indicated in brackets. similarity conditions of trajectories are strictly valid within the limits of a linear change in the temperature with height/altitude. At the same time numerical calculations show that the dependences of similarity it is possible to use, also, if peak of the trajectory exceeds the limits of the applicability of linear change of temperature with height/altitude.

§ 5. Formula of K. Ye. Tsiolkovskiy. Calculation of the powered flight trajectory of the unguided rocket.

Simplest analytical solution is in the case of rectilinear movement of rocket, if we do not consider air resistance and gravitational force. In this case the equation of motion will take the form

$$m\dot{v}=P. \qquad (7.142)$$

Substituting in (7.142) values of P and m from formulas (2.126) and (1.24), we will obtain

$$dv=w_e\frac{Q_{сек}\,dt}{Q_0-\int\limits_0^t Q_{сек}\,dt}.$$

Introducing new variable $z=Q_0-\int\limits_0^t Q_{сек}dt$, integrating and carrying out replacement, we will obtain the velocity of rocket at the moment

of time t

$$v = w_e \ln \frac{Q_0}{Q_0 - \int_0^t Q_{\text{сек}}\, dt}, \qquad (7.143)$$

or

$$v = w_e \ln \frac{m_0}{m} = v_u, \qquad (7.144)$$

where m_0 and m - are initial and flowing the masses of rocket.

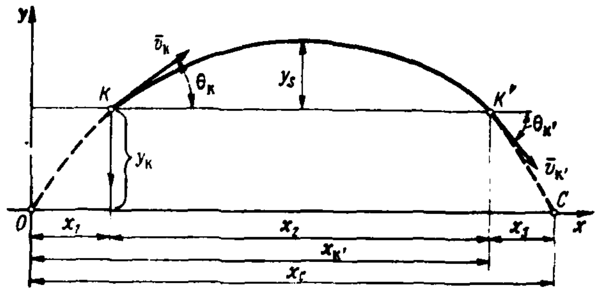

Fig. 7.9. Dividing circuit of trajectory into separate calculated sections.

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Last/latter formula can be written in other form

$$\frac{m}{m_0}=e^{-\frac{v}{w_e}}. \qquad (7.145)$$

Designating full rate of propellant flow toward the end of the operation of engine t_K through $Q_T=\int_0^{t_K} Q_{sec}\,dt$, we will obtain the formula, which determines the maximum velocity which can have the rocket without the account of the action on it of the gravity force and air resistance

$$v_{max}=w_e \ln\frac{Q_0}{Q_0-Q_T}. \qquad (7.146)$$

If we present the initial weight of rocket as

$$Q_0=Q_n+Q_T,$$

where $Q_п$ - passive weight of rocket, then

$$v_{max}=w_e \ln\left(1+\frac{Q_T}{Q_п}\right), \tag{7.147}$$

or

$$v_{max}=-w_e \ln\left(1-\frac{Q_T}{Q_0}\right). \tag{7.148}$$

For practical calculations it is possible to use tabulated function

$$k(\lambda)=-\ln(1-\lambda),$$

where $\lambda=\frac{\int\limits_0^t Q_{сек}dt}{Q_0}$.

Then

$$v=w_e k(\lambda), \tag{7.149}$$

Formula (7.144) was for the first time derived by K. E. Tsiolkovskiy and was named his name. It with the success is used for theoretical studies in the field of rocket engineering.

We will use the first equation of system (3.82). Besides the assumptions, accepted during obtaining of system itself (3.82), they

additionally consider that the gas flow rate per second, which escape through the nozzle, is constant on time and it is equal to

$$Q_{сек}=\frac{Q_т}{t_к}. \quad (7.150)$$

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The function of change of air density with height/altitude they replace by constant value $H(y_{cp})$.

After the appropriate transformations it is possible to obtain from the first equation of system (3.82)

$$\frac{dv}{d\lambda}=\frac{w_e}{1-\lambda}-\frac{aF(v)}{1-\lambda}-b\sin\theta, \quad (7.151)$$

where

$$a=c_0H(y_{cp})\frac{Q_0t_к}{Q_т}; \qquad b=\frac{Q_0gt_к}{Q_т}.$$

Last/latter equation can be integrated. If we accept angle θ for constant value, then

$$v=w_e\ln\frac{1}{1-\lambda}-a\int_0^\lambda\frac{F(v)}{1-\lambda}d\lambda-b\lambda\sin\theta. \quad (7.152)$$

First term of right side, which represents by itself the modified formula of K. E. Tsiolkovskiy, determines the velocity of rocket without the account of the effect of the air resistance and weight. Second term considers the effect of the air resistance, and

the third - a gravity effect.

Second term of equation (7.152) contains under integral the experimental function F(v), given by table; for calculating the third term, it is necessary to select constant value of sin θ. There are difficulties during the solution also of other equations of system (3.82); therefore calculation is conducted with the application/use of special tables on the individual sections of trajectory several approach/approximations. Detailed calculation procedures are presented in works [13] and [60].

In the first approximation, of the characteristic of movement they find without the account of the air resistance and gravitational force. From (7.152)

$$v_u = w_e \ln \frac{1}{1-\lambda}.$$

the displacement of rocket on powered flight trajectory is equal

$$S_1 = \int_0^t v_u\, dt.$$

Substituting in this formula v_u and dt from (3.79), with constant flow rate per second $Q_{\text{сек}}$ let us have

$$S_1 = \frac{w_e Q_0 t_k}{Q_T} k_1(\lambda), \qquad (7.153)$$

where

$$k_1(\lambda) = \int_0^\lambda \ln \frac{1}{1-\lambda}\, d\lambda.$$

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From the second equation of system (3.82) we will obtain

$$\frac{d\theta}{\cos\theta}=-\frac{Q_0 t_{\kappa} g}{Q_{\tau}}\,\frac{d\lambda}{v}.$$

Equalizing v of velocity, determined by formula of Tsiolkovskiy, we will obtain

$$\int\limits_{\theta_0}^{\theta}\frac{d\theta}{\cos\theta}=-\frac{Q_0 g t_{\kappa}}{Q_{\tau} w_e}\int\limits_{\lambda_0}^{\lambda}\frac{d\lambda}{\ln\frac{1}{1-\lambda}},$$

where λ_0 it corresponds to the torque/moment of the descent of rocket from guides. After designating

$$k_2(\lambda)=\int\limits_0^{\lambda}\frac{d\lambda}{\ln\frac{1}{1-\lambda}}$$

and approximately replacing

$$\int\limits_{\theta_0}^{\theta}\frac{d\theta}{\cos\theta}\approx\frac{\theta-\theta_0}{\cos\theta_{cp}},$$

we will obtain dependence for determining the flight path angle at the end of the active section

$$\theta_{\kappa_1}=\theta_0-\frac{Q_0 g t_{\kappa}\cos\theta_{cp}}{Q_{\tau} w_e}\left[k_2(\lambda_{\kappa})-k_2(\lambda_0)\right]. \qquad (7.154)$$

During calculations as the first approximation, it is possible to accept

$$\cos\theta_{cp} \approx \cos\theta_0.$$

Initial value λ_0 is located approximately through the formula

$$\lambda_0 = \frac{Q_т}{Q_0 t_к} t_0,$$

where the time of the action of rocket along guides is equal

$$t_0 = \sqrt{\frac{2L_н}{a_н}}.$$

Average acceleration during the action of rocket along guides and the velocity of descent from guides find under the condition

$$m = m_0 = \text{const}; \quad P = \text{const}.$$

Then $a_н = \dot{v} = \frac{P}{m_0} - g\sin\theta_0$ and $v_0 = \sqrt{2a_н L_н}$, where $L_н$ - path of rocket on guides.

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Let us return to equation (7.152)'. in second term of right side, let us replace

$$d\lambda = \frac{Q_{сек}}{Q_0} dt = \frac{Q_т}{t_к Q_0} dt \text{ and } 1 - \lambda = \frac{m}{m_0}.$$

Of the condition of first approximation

$$\frac{dv_{u}}{dt}=\frac{P}{m}=\frac{Q_{сек}}{g}\frac{w_e}{m}.$$

Then

$$dt=\frac{gm}{Q_{сек}w_e}dv_{u}.$$

After this, keeping in mind (3.79), let us rewrite (7.152) in this form:

$$v_2=v_{u}-\frac{a}{w_e}\int_0^v F(v_{u})dv_{u}-gt\sin\theta_{cp}. \qquad (7.155)$$

Designating

$$L_1(v_{u})=\int_0^v F(v_{u})dv_{u},$$

let us write

$$v_2=v_{u}-\frac{a}{w_e}L_1(v_{u})-gt\sin\theta_{cp}.$$

The path of projectile on the active section of trajectory will be determined according to the formula

$$S_2=S_1-\frac{a}{w_e}\int_0^t L_1(v_{u})dt-\frac{gt^2}{2}\sin\theta_{cp}. \qquad (7.156)$$

Utilizing a formula of Tsiolkovskiy, we convert integrand in

second term of the right side

$$\int_0^t L_1(v_u)\,dt = \frac{Q_0 t_\kappa}{Q_\tau w_e} N(v_u, w_e).$$

Accepting $w_e = 2000$ m/s, let us replace $N(v_\text{п}, w_e) = N(v_\text{п})$ and, opening in (7.156). content of a, let us write

$$S_2 = S_1 - c_0 H(y_{cp}) \frac{Q_0^2 t_\kappa^2}{Q_\tau^2 w_e^2} N(v_u) - \frac{gt^2}{2} \sin \theta_{cp}. \qquad (7.157)$$

Are known further approach/approximations (the third and the fourth) regarding the characteristics of end of powered flight trajectory. The order of iterations is retained previous - the data of previous approach/approximation they are utilized for that follow.

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Functions $k(\lambda)$, $k_1(\lambda)$, $k_2(\lambda)$ and $N(v_\text{п})$ are tabulated and placed in [13]. Tables $L_1(v_\text{п})$ and $N(v_\text{п})$ are comprised in connection with the standard function of "resistance of air 1930".

The correction for a change in angle θ_κ, which depends on the air resistance, they frequently disregard, determining θ_κ according to formula (7.154). During repeated calculation in right side, it is taken $\cos\theta_{cp}$, where

$$\theta_{cp} = \frac{\theta_0 + \theta_{\kappa 1}}{2}.$$

The coordinates of the end/lead of powered flight trajectory approximately are determined on the formulas

$$x_к = S_к \cos\theta_{ср}; \quad y_к = S_к \sin\theta_{ср}. \qquad (7.158)$$

The method presented gives acceptable for approximate computations results with the relatively low maximum speeds of motion and the short operating time of engine.

§ 16. Determination of the velocity at the end of the powered flight trajectory of the guided missile of class "surface -surface."

The range of flight of ballistic missiles can be presented in the form of the functional dependence

$$L = f(v_к, \theta_к, x_к, y_к).$$

Under given conditions of firing, the greatest interest represents the velocity at the end of the active section - $v_к$. The analysis of the trajectories of ballistic missiles makes it possible to develop the procedure of the calculation of corrections into the velocity, obtained on formula K. E. Tsiolkovskiy, determined by air resistance and by gravitational force. The generalization of the results of investigations is facilitated by the fact that the rockets of the class in question, depending on design features and the expected firing distance, have the standard, well studied trajectories with the sufficiently narrow range of the flight path angles at the end of the active section. Let us consider that the program of the flight control of rocket on pitch angle provides the assigned flight path angle at the end of the active section. Setting

aside for the moment a question of the selection of optimum program concerning pitch angle, it is presented here one of the widespread calculation procedures, after using system of equations (3.83) [2]. In first term of the first equation of system (3.83) let us replace values of X and P, using (2.92) and (2.120). Let us designate

$$P_{\pi}=P_{0}+S_{a}p_{0N}. \tag{7.159}$$

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After replacement and transformations, we will obtain

$$dv=-\frac{\tau_{\Phi}}{m_0}P_{n}\frac{d\mu}{\mu}+\tau_{\Phi}g\sin\theta d\mu+$$
$$+\frac{S\varrho v^2}{2m_0}ic_{x\,\text{эт}}\tau_{\Phi}\frac{d\mu}{\mu}+\frac{S_a p_{0N}}{m_0}\tau_{\Phi}\pi(y)\frac{d\mu}{\mu}. \tag{7.160}$$

Integrating within limits from v_0 to v and from μ_0 to μ, let us have

$$v-v_0=\frac{\tau_{\Phi}}{m_0}P_{n}\ln\frac{\mu_0}{\mu}-\tau_{\Phi}\int_{\mu}^{\mu_0}g\sin\theta d\mu-$$
$$-\frac{Si\tau_{\Phi}}{m_0}\int_{\mu}^{\mu_0}\frac{\varrho v^2}{2}c_{x\,\text{эт}}\frac{d\mu}{\mu}-\frac{S_a p_{0N}\tau_{\Phi}}{m_0}\int_{\mu}^{\mu_0}\pi(y)\frac{d\mu}{\mu}. \tag{7.161}$$

in final form can be undertaken, as earlier, the integral only of first term, which corresponds to the velocity, determined on formula K. E. Tsiolkovskiy. Second term determines a decrease in the velocity from the action of gravitational force; the third term determines a decrease in the velocity from the action of the air resistance; the fourth term determines the dependence of velocity v

on change of the engine thrust with height/altitude. The integrals, entering in (7.161), in final form are not taken. Let us introduce for them the designations

$$J_1=\int_{\mu}^{\mu_0} g\sin\theta d\mu;\quad J_2=\int_{\mu}^{\mu_0}\frac{\rho v^2}{2}c_{x\,9T}\frac{d\mu}{\mu};\quad J_3=\int_{\mu}^{\mu_0}\pi(y)\frac{d\mu}{\mu}. \quad (7.162)$$

Additionally let us designate

$$\Delta v_1=\tau_\Phi J_1;\quad \Delta v_2=\tau_\Phi\frac{Si}{m_0}J_2;\quad \Delta v_3=\tau_\Phi\frac{S_a p_{0N}}{m_0}J_3, \quad (7.163)$$

and then

$$v-v_0=\frac{\tau_\Phi}{m_0}P_n\ln\frac{\mu_0}{\mu}-\Delta v_1-\Delta v_2-\Delta v_3. \quad (7.164)$$

When conducting of the approximate computations of the powered flight trajectories of the guided missiles of the class in question usually they take $\mu_0=1$ and $v_0=0$.

Using standard parabolic program for the last/latter equation of system (3.83)

$$\theta=A(\mu-\mu_2)^2+B(\mu-\mu_2)+C, \quad (7.165)$$

it is possible to comprise table of integral J_1. Such tables are comprised with $\mu_0=1$ for different values θ_K [2]. In (7.165) μ_2 it corresponds to the finite value of angle θ on active section; A, B, C - constants of concrete/specific/actual program.

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The velocity in the first approximation, is determined as follows:

$$v_1=\tau_\Phi\left(\frac{P_\text{п}}{m_0}\ln\frac{1}{\mu}-J_1\right). \qquad (7.166)$$

It is possible to show that

$$J_2=f(v_1, \sigma),$$

where

$$\sigma=\tau_\Phi\sqrt{\tilde{w}_e}\sin\theta_\kappa\ 10^{-3} \text{ and } \tilde{w}_e=\frac{1}{2}(w_{e\,\text{п}}+w_{e0}). \qquad (7.167)$$

Here $w_{e\text{п}}$ and w_{e0} - with respect effective exhaust gas velocities for a void and for ground-based standard conditions

$$w_{e\,\text{п}}=\frac{P_\text{п}}{|\dot{m}|};\ w_{e0}=\frac{P_0}{|\dot{m}|}.$$

Value P_0 is calculated from (2.118), a $P_\text{п}$ from (7.159). In both cases the thrust must be decreased by magnitude of losses on controls. Integral J_2 is determined from curve/graph, comprised on the basis of the large number of calculations. Input values into curve/graph are v_1 and σ, calculated for specific conditions for (7.166) and (7.167). Integral J_3 is determined with the aid of curve/graph, comprised also on the basis of the generalization of large calculated material. The entry into curve/graph are the real

operating time of engine t and the parameter

$$\nu_0 = \frac{g\tau_\Phi}{w_{e0}} = \frac{\tau_\Phi}{P_{\text{уд}0}} = \frac{Q_0}{P_0}, \qquad (7.168)$$

where $P_{\text{уд}0}$ will be determined in accordance with (2.123). Passage from parameter η, determined by curve/graph, to integral J_3 is realized on the empirical formula

$$J_3 = \frac{\eta}{0{,}001\,\tilde{w}_e \sqrt[3]{\tilde{w}_e \sin\theta_K}}. \qquad (7.169)$$

The described method of calculation v proves to be highly useful when conducting of ballistic design, but it requires the subsequent refinement of results during the solution of direct problem of external ballistics.

§7. The approximation methods of determining the complete flying range of rockets. Method of equivalent projectile.

Complete flying range is equal (see Fig. 7.9)

$$x_C = x_1 + x_2 + x_3.$$

the first (active) trajectory phase the second (passive) are calculated by one of the methods, presented above.

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The trajectory phase whose beginning corresponds to point K', and end/lead - to a point of intersection with the Earth (to point C) is approximately calculated by the method, known from external ballistics of barrel systems. Calculation is based on power-series expansion of function y=f(x) in the vicinity of point K'

$$y=y_{\kappa'}+(x-x_{\kappa'})\frac{dy_{\kappa'}}{dx}+\frac{(x-x_{\kappa'})^2}{2!}\frac{d^2y_{\kappa'}}{dx^2}+\frac{(x-x_{\kappa'})^3}{3!}\frac{d^3y_{\kappa'}}{dx^3}+\ldots$$

Utilizing the obtained above relationship/ratios, are not difficult to obtain those comprise of the terms of the expansion of ordinate y according to the degrees of the horizontal range of the phase of flight in question

$$\frac{dy_{\kappa'}}{dx}=\operatorname{tg}\theta_{\kappa'};\ \frac{d^2y_{\kappa'}}{dx^2}=-\frac{g}{v_{\kappa'}^2\cos^2\theta_{\kappa'}};$$

$$\frac{d^3y_{\kappa'}}{dx^3}=-\frac{2c_{11}gH(y_{\kappa'})G(v_{\kappa'})}{v_{\kappa'}^3\cos^3\theta_{\kappa'}}.$$

Then initial equation for determining the parameter x with section x_3 is written as follows:

$$y=y_{\kappa'}+\operatorname{tg}\theta_{\kappa'}(x-x_{\kappa'})-\frac{g}{2v_{\kappa'}^2\cos^2\theta_{\kappa'}}(x-x_{\kappa'})^2-$$
$$-\frac{c_{11}gH(y_{\kappa'})G(v_{\kappa'})}{3v_{\kappa'}^3\cos^3\theta_{\kappa'}}(x-x_{\kappa'})^3. \qquad (7.170)$$

Equation is solved with substitution $y=y_C=0$ by the method of iterations.

$$y_{\kappa'}=\mathrm{tg}\,|\theta_{\kappa'}|\,(x_C-x_{\kappa'})+\frac{g}{2v_{\kappa'}^2\cos^2\theta_{\kappa'}}(x_C-x_{\kappa'})^2+$$
$$+\frac{c_{11}gH(y_{\kappa'})G(v_{\kappa'})}{3v_{\kappa'}^3\cos^3\theta_{\kappa'}}(x_C-x_{\kappa'})^3. \quad (7.171)$$

In the first approximation, is considered only first member of right side (7.171)

$$x_{31}=(x_C-x_{\kappa'})_1=\frac{y_{\kappa'}}{\mathrm{tg}\,|\theta_{\kappa'}|}. \quad (7.172)$$

In the second approach/approximation we substitute $x_{31}=(x_C-x_{\kappa'})_1$ in the second and third members of the right side of equation (7.171) and find

$$x_{311}=\frac{y_{\kappa'}}{\mathrm{tg}\,|\theta_{\kappa'}|}-\frac{gx_{31}^2}{2v_{\kappa'}^2\cos^2\theta_{\kappa'}\,\mathrm{tg}\,|\theta_{\kappa'}|}-$$
$$-\frac{c_{11}gH(y_{\kappa'})G(v_{\kappa'})\,x_{31}^3}{3v_{\kappa'}^3\cos^3\theta_{\kappa'}\,\mathrm{tg}\,|\theta_{\kappa'}|}. \quad (7.173)$$

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For the third approach/approximation we substitute the value x_{311} in right side (7.173). Usually calculation in the second and third approach/approximations gives close results and the subsequent approach/approximations they prove to be excessive.

To the approximation methods of the trajectory calculation of the rockets, which have relatively low firing distance, can be attributed the so-called method of equivalent artillery shell. This method assumes finding such initial conditions of the casting of artillery shell (θ_0 and v_0), with which the trajectory of artillery shell would coincide with missile trajectory in value and sense of the vector of the speed in the point of the end/lead of the engine operation, i.e., so that at the point with coordinates x_K and y_K would occur equality $v_{K.э}=v_K$ and $\theta_{K.э}=\theta_K$. It is logical that the equivalent artillery shell must have the same value of ballistic coefficient, as rocket on inactive leg, i.e., $c_{П}$. The calculated trajectory of equivalent projectile in this case must coincide with missile trajectory on inactive leg. To point with coordinates x_K and y_K the missile trajectories and projectile do not coincide. To determine the initial conditions of the trajectory of equivalent projectile is possible by numerical integration or one of the analytical methods of solving the mission objectives of the artillery shells, for example, by the method of pseudovelocity. In the case of small firing distances, the application/use of a method of equivalent projectile is expedient to combine with the use of tables of external ballistics.

Let be known the trajectory elements in the beginning of passive section v_K, θ_K and y_K (Fig. 7.10). For determining the initial

conditions of the trajectory of the equivalent projectile of constant mass $v_{0\infty}$ and $\theta_{0\infty}$ and of horizontal range from the conditional, it began trajectories θ_{0} to the projection of point K on axis 0x (value $x_{к0}$ in Fig. 7.10) it is possible system of equations (5.7) to integrate numerically from point K to point 0_{0}. In this case, the space of integration it is necessary to take negative, i.e., $h_{t}<0$.

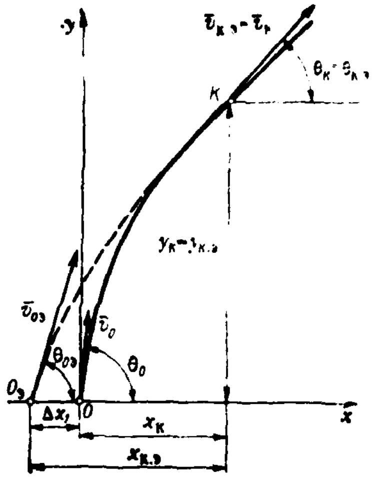

Fig. 7.10. Construction of the initial phase of trajectory of equivalent projectile.

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During integration the speed of the equivalent projectile

$$v_э=\sqrt{u_э^2+w_э^2}$$

will grow/rise from $v_к$ to $v_{0э}$, will grow/rise also the flight path angle $\theta_э$. The ordinates of calculation points in the trajectory will decrease from value $y_к=y_{к.э}$, since in the case in question

$$y_{(i+1)э}=y_{iэ}+(-\Delta y_{iэ}).$$

The termination of integration, and respectively, and values $v_{0э}$, $\theta_{0э}$ and $x_{к.э}$, are determined by condition $y_э=0$.

The initial conditions, which correspond to equivalent projectile, can be used for the approximate computations of trajectories according to the ballistic collection, comprised for the projectiles of constant mass. It is possible, for example, to construct the family of trajectories with different angles of departure. During calculations they must remain constant/invariable $v_{0э}$ and $c_{П}$. One and the same unguided rocket depending on initial angle of departure has different trajectories. To the passive phases of each trajectory, obviously, will correspond their values $v_{0э}$, $\theta_{0э}$ and Δx_1, but their determination in the complete range of a change in the basic parameter is very laborious and the application/use of a method of equivalent projectile will lose sense. Therefore the trajectory, from passive section of which is determined $v_{0э}$ and $\theta_{0э}$, should take the appropriate approximately half of the range of the change in the basic parameter, for example, to the half of distance from $x_{C\,min}$ to $x_{C\,max}$.

For the approximate investigations the trajectory of the center of mass of long range ballistic missile can be also replaced by the trajectory of the equivalent projectile of constant mass. The passive (elliptical) trajectory phase is determined by parameters p, ℓ and $\varphi_н$.

calculated according to the trajectory elements, known for the beginning of passive section (point "N" in Fig. 7.5).

Designating, as earlier, initial conditions at the release point of the equivalent projectile through v_{0e} and θ_{0e} and replacing r on R_3, from (7.11) we will obtain

$$v_{0e}\cos\theta_{0e}=\frac{r_N v_N \cos\theta_N}{R_3}. \qquad (7.174)$$

From (7.15) it is possible to obtain

$$\frac{p}{1-e^2}=\frac{r}{2-\varkappa}=\frac{r_N}{2-\varkappa_N}. \qquad (7.175)$$

In last/latter equality $\varkappa=\frac{rv^2}{K}$ - for current point in the trajectory, $\varkappa_N=\frac{r_N v_N^2}{K}$ - for the beginning of passive section.

The speed in any point in the trajectory is equal to

$$v=\sqrt{\frac{K\varkappa}{r}}. \qquad (7.176)$$

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Utilizing (7.175) and (7.176), it is possible to obtain

$$v=\sqrt{v_N^2+\frac{2K}{r}\left(1-\frac{r}{r_N}\right)}. \qquad (7.177)$$

The initial velocity of equivalent projectile on the surface

Earth.

$$v_{0_{э}}=\sqrt{v_{н}^{2}+\frac{2K}{R_{3}}\left(1-\frac{R_{3}}{r_{н}}\right)}. \qquad (7.178)$$

From (7.174) the angle of departure of equivalent projectile is equal to

$$\theta_{0_{э}}=\arccos\frac{r_{н}v_{н}\cos\theta_{н}}{R_{3}v_{0_{э}}}. \qquad (7.179)$$

After accepting in (7.14) on Fig. 7.5 $\varphi=0$, we will obtain

$$r_{н}=\frac{p}{1-e\cos\varphi_{S}}. \qquad (7.180)$$

From (7.180) it is possible to find angle φ_S and range angle from circle with radius of $r_{н}=r_{н'}$, of passing through the point "N":

$$2\varphi_{S}=2\arccos\frac{1-\frac{p}{r_{н}}}{e}. \qquad (7.181)$$

From the equations, which describe Keplerian motion of rockets, it is possible to obtain one additional formula for determining angle φ_S [2]

$$\varphi_{S}=\operatorname{arctg}\frac{1}{2}\frac{\sin 2\theta_{н}}{\nu_{н}-\cos^{2}\theta_{н}}, \qquad (7.182)$$

where

$$\nu_{н}=\frac{g_{т.н}r_{н}}{v_{н}^{2}},$$

and $g_{т.н}$ - acceleration from attracting force (gravity) at the point "N". By analogy with (7.181) and (7.182) for an equivalent projectile range angle can be found from the formulas

$$2\varphi_{э,S}=2\arccos\frac{1-\frac{p}{R_{3}}}{e}, \qquad (7.183)$$

or

$$2\varphi_{э,s} = 2\operatorname{arc\,tg} \frac{1}{2} \frac{\sin 2\theta_{0э}}{\nu_{0э} - \cos^2 \theta_{0э}}, \qquad (7.184)$$

where $\nu_{0э} = \frac{g_{т0} R_3}{v_{0э}^2}$ and $g_{т0}$ - acceleration from attracting force on the surface of the Earth.

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Linear distance over the surface of the spherical model of the Earth is equal to

$$L_{R_3} = 2R_3\varphi_{э,s}. \qquad (7.185)$$

Initial velocity $v_{0э}$ can be used for the approximate construction of the family of elliptical trajectories with different angles of departure θ_0. In this case, one should remember that the distances, determined on formulas (7.184) and (7.185), will differ somewhat from the real. The value of error depends on the coordinates of the beginning of the inactive leg and motion characteristics - $v_к$ and $\theta_к$ at this point.